Resummation of Relativistic Corrections to $J/\psi \rightarrow e^+e^-$

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Outline

• Quarkonium electromagnetic current
• A new method to compute 1-loop NRQCD corrections to all orders in $\nu$
• Resummation of relativistic corrections
• Summary
Quarkonium Electromagnetic Current

- Full QCD definition:

\[ (-ie e_Q) i A_H^\mu = \langle 0 | J_{EM}^\mu | H \rangle, \quad J_{EM}^\mu = (-ie e_Q) \bar{\psi} \gamma^\mu \psi \]

- NRQCD factorization formula

\[ i A_H^i = \sqrt{2m_H} \sum_n c_n \langle 0 | O_n^i | H \rangle \]

\( O_n^i \): NRQCD operators involving \( Q \bar{Q} \) decay in terms of two-component Pauli spinor fields

\( c_n \): Short-distance coefficients that are free of IR divergences and calculable perturbatively

quarkonium EM current
• Short-distance coefficients can be computed by perturbative matching:

\[ iA_{Q\bar{Q}_1}^i = \sum_n c_n \langle 0 | O_n^i | Q\bar{Q}_1 \rangle \]

\[ \langle 0 | O_n^i | Q\bar{Q}_1 \rangle : \text{perturbative NRQCD matrix element} \]

• We only consider $Q\bar{Q}$ operators.

• Matching coefficients at order $\alpha_s^0$ are determined as

order $\alpha_s^0$ : 

\[ iA_{Q\bar{Q}_1}^{i(0)} = \sum_n c_n^{(0)} \langle 0 | O_n^i | Q\bar{Q}_1 \rangle^{(0)} \]

finite

order $\alpha_s^1$ : 

\[ iA_{Q\bar{Q}_1}^{i(1)} = \sum_n c_n^{(1)} \langle 0 | O_n^i | Q\bar{Q}_1 \rangle^{(0)} + \sum_n c_n^{(0)} \langle 0 | O_n^i | Q\bar{Q}_1 \rangle^{(1)} \]

IR sensitive

\[ \left[ iA_{Q\bar{Q}_1}^{i(1)} \right]_{\text{NRQCD}} \]
• General structure of full-QCD amplitude for a color-singlet $Q\bar{Q}$ pair:

$$i A^i_{Q\bar{Q}_1} = \bar{v}(p_2)(G\gamma^i + Hq^i)u(p_1)$$

$$= G\eta^\dagger \sigma^i \xi - \left[ \frac{G}{E(E + m)} + \frac{H}{E} \right] q^i \eta^\dagger q \cdot \sigma \xi.$$ 

$$\mathcal{O}^i_A = \chi^\dagger \sigma^i \psi \quad (S \text{ wave})$$

$$\mathcal{O}^i_B = \chi^\dagger \left( -\frac{i}{2} \nabla^i \right) \left( -\frac{i}{2} \nabla \cdot \sigma \right) \psi \quad (S \& D \text{ wave})$$

• IR divergence emerges from order $\alpha_s$

$$G = 1 + \frac{\alpha_s C_F}{4\pi} \left\{ 2 \left[ (1 + \delta^2)L(\delta) - 1 \right] \frac{1}{\epsilon_{\text{IR}}} + (1 + \delta^2) \left[ \frac{\pi^2}{\delta} + \frac{\pi}{\delta} \frac{1}{\epsilon_{\text{IR}}} \right] + \cdots \right\},$$

$$H = \frac{\alpha_s C_F}{4\pi} \frac{1 - \delta^2}{m} \left[ -\frac{i\pi}{\delta} + \cdots \right].$$

IR divergence

Coulomb divergence

Long-distance contribution must be subtracted.
• \( c_n^{(0)} \) is known to all orders in \( \nu^{2n} \)
  Bodwin, Chung, Kang, Lee, and Yu (PRD77, 094017)

• **Determination of** \( c_n^{(1)} \)

\[
iA_{QQ_1}^{i(1)} - \left[ iA_{QQ_1}^{i(1)} \right]_{\text{NRQCD}} = \sum_n c_n^{(1)} \langle 0 | \mathcal{O}_n^i | Q\bar{Q}_1 \rangle^{(0)}
\]

\[
iA_{QQ_1}^{i(1)} - \left[ iA_{QQ_1}^{i(1)} \right]_{\text{NRQCD}} = \Delta G^{(1)} \eta^\dagger \sigma^i \xi - \frac{\Delta G^{(1)}}{E(E + m)} + \frac{\Delta H^{(1)}}{E} q^i \eta^\dagger q \cdot \sigma \xi
\]

• **In this work, we compute** \( c_n^{(1)} \) **to all orders in** \( \nu \)

quarkonium EM current
### References for \( c_n^{(i)} \)

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<th>( \alpha_s^{0} )</th>
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[1] Bodwin, Braaten, Lepage, PRD 51, 1125
[3] Barbieri et al., PL 57B, 455
[6] Czarnecki, Melnikov, PRL 80, 2531

[X] Bodwin, Chung, Kang, Lee, Yu, PRD 77, 094017
[Y] This work
Computation of \[
\begin{bmatrix}
iA^{i(1)}_{Q\bar{Q}_1}
\end{bmatrix}_{\text{NRQCD}}
\]

- Calculation of \[
\begin{bmatrix}
iA^{i(1)}_{Q\bar{Q}_1}
\end{bmatrix}_{\text{NRQCD}}
\]
to higher orders in \( \nu \) requires knowledge of a huge number of
  - \( \text{NRQCD operators,} \)
  - the interactions,
  - and the Feynman rules to compute loop corrections.

- This becomes a daunting task at all orders in \( \nu \).
BBL Method to compute \[ i \mathcal{A}_{Q\bar{Q}1}^{i(1)} \]_{NRQCD}

i) Use NRQCD perturbation theory to construct amplitude.

ii) Integrate out \( k_0 \).

iii) Expand the integrand in powers of 3-momenta divided by \( m \).

iv) Discard power-divergent scaleless integrals.

We propose a new method that replaces step i).
A new method of calculation

- Interactions in NRQCD through infinite order in $\nu$ is equivalent to QCD, but with the interactions re-arranged in an expansion in powers of $\nu$.

- Instead of using NRQCD perturbation theory, we expand the full-QCD integrand in powers of 3-momenta divided by $m$ after evaluating $k_0$ integral.
Comparison with Method of Regions

- At lowest order in $\mathcal{V}$, the method of regions [Beneke and Smirnov (NPB 1998)] has been used to compute order-$\alpha_s^2$ correction to quarkonium EM current
  

- In MoR, contribution from each region can be computed separately by expanding small parameters

  • $\sim m v^2$
  • $\sim m v$
  • $\sim m$
Comparison with Method of Regions

• With MoR, it is difficult to obtain a closed-form formula for the hard part valid to all orders in $\mathcal{V}$.

• In our method, the NRQCD contribution becomes a simple series that can be resummed easily, yielding very compact expressions for the hard part.

• In dimensional regularization, our method is equivalent to calculating “soft + ultrasoft + potential” part by MoR.

• Our method also works for hard cut-off like lattice, but MoR does not.
Our final results

- IR pole cancels in \( i\mathcal{A}_{QQ_1}^{i(1)} - [i\mathcal{A}_{QQ_1}^{i(1)}]_{\text{NRQCD}} \)
- UV pole is absorbed by redefining NRQCD operators in \( \overline{\text{MS}} \) scheme:

\[
\Delta G_{\text{MS}}^{(1)} = \frac{\alpha_s C_F}{4\pi} \left\{ 2 \left[ (1 + \delta^2) L(\delta) - 1 \right] \log \frac{\mu^2}{m^2} + 6\delta^2 L(\delta) - 4(1 + \delta^2) K(\delta) - 4 \right\}
\]

\[
\Delta H^{(1)} = \frac{\alpha_s C_F}{4\pi} \frac{2(1 - \delta^2)}{m} L(\delta)
\]

\[
L(\delta) = \frac{1}{2\delta} \log \left( \frac{1 + \delta}{1 - \delta} \right) = 1 + \frac{\delta^2}{3} + O(\delta^4)
\]

\[
K(\delta) = \frac{1}{4\delta} \left[ \text{Sp} \left( \frac{2\delta}{1 + \delta} \right) - \text{Sp} \left( -\frac{2\delta}{1 - \delta} \right) \right] = 1 + \frac{4\delta^2}{9} + O(\delta^4)
\]
Exact cancellation of IR and non-analytic contributions to all orders in $\mathcal{V}$

- QCD 1-loop corrections

\[
G^{(1)} = \frac{\alpha_s C_F}{4\pi} \left\{ 2 \left[ (1 + \delta^2) L(\delta) - 1 \right] \frac{1}{\epsilon_{\text{IR}}} + (1 + \delta^2) \left[ \frac{\pi^2}{\delta} - \frac{i\pi}{\delta} \frac{1}{\epsilon_{\text{IR}}} \right] + \cdots \right\},
\]

\[
H^{(1)} = \frac{\alpha_s C_F}{4\pi} \frac{1 - \delta^2}{m} \left[ - \frac{i\pi}{\delta} + \cdots \right].
\]

- NRQCD 1-loop corrections

\[
G_{\text{NRQCD}}^{(1)} = \frac{\alpha_s C_F}{4\pi} \left\{ 2 \left[ (1 + \delta^2) L(\delta) - 1 \right] \left( \frac{1}{\epsilon_{\text{IR}}} - \frac{1}{\epsilon_{\text{UV}}} \right) 
+ (1 + \delta^2) \left[ \frac{\pi^2}{\delta} - \frac{i\pi}{\delta} \frac{1}{\epsilon_{\text{IR}}} \right] + \cdots \right\},
\]

\[
H_{\text{NRQCD}}^{(1)} = \frac{\alpha_s C_F}{4\pi} \frac{1 - \delta^2}{m} \left( - \frac{i\pi}{\delta} \right).
\]
Resumation to all orders in $\mathcal{V}$

- Generalized Gremm-Kapustin relation for spin-independent potential models

$$\left[ \langle q^{2n} \rangle_{H(3S_1)} \right]_{\text{MS}} = \left[ \langle q^2 \rangle_{H(3S_1)} \right]_{\text{MS}}^n.$$  

Bodwin, Kang, and Lee (PRD 2006)

$$\langle q^{2n} \rangle_{H(3S_1)} = \frac{\langle 0 \left| O_{An}^i \right| H(3S_1) \rangle}{\langle 0 \left| O_{A0}^i \right| H(3S_1) \rangle}$$

- This allows us to resum the relativistic corrections

$$iA_{H(3S_1)}^i = \sqrt{2m_H} \ C(\delta) \ \langle 0 \left| \chi^\dagger \sigma^i \psi \right| H \rangle$$

$$C(\delta) = \left[ 1 - \frac{\langle q^2 \rangle}{3E(E + m)} \right] \left( 1 + \Delta G_{\text{MS}}^{(1)} \right) - \frac{\langle q^2 \rangle}{3E} \Delta H^{(1)}$$

$$\delta^2 \quad \Longrightarrow \quad \frac{\langle q^2 \rangle}{m^2 + \langle q^2 \rangle}$$
Order-$\alpha_s$ coefficient of EM current, resummed to $\nu^{2n}$

MEs are from Bodwin, Chung, Kang, Lee, and Yu (PRD77, 094017)

- From the closed-form results, we can show that there is a finite radius of convergence $\nu < 1$. 
Summary

- Order-$\alpha_s$ corrections to the quarkonium EM current at all orders in $v$ have been computed.
- Only $Q\bar{Q}$ contribution is considered. Neglect of gauge-field contribution results in errors of order $v^4$.
- A new method is introduced to compute NRQCD 1-loop corrections directly from QCD amplitude.
- This allows us to obtain very compact expressions for the $Q\bar{Q}$ contributions valid to all orders in $v$.
- We can use this method to resum a class of relativistic corrections.
- The method may be applied to compute short-distance coefficients for other effective theories.
Supplementary
A glance at a traditional fixed order calculation...

Luke and Savage (PRD 1998), order $\alpha_s v^2$

$\alpha_s v^0$

$\alpha_s v^2$

Corrections to $\langle 0 | \chi^\dagger \sigma^i \psi | Q \bar{Q} \rangle$:

(a) $A_0$ temporal
(b) $\nabla \cdot A$ spatial
(c) $\sigma \cdot B$ spatial
(d) $D \cdot E$ temporal
(e) $D^4$ quark propagator correction

Corrections to $\langle 0 | \chi^\dagger \left( \hat{\nabla} \cdot \sigma \hat{\nabla}^i + \hat{\nabla} \cdot \sigma \hat{\nabla}^i \right) \psi | Q \bar{Q} \rangle$:

(f) $A_0$ temporal

• It is non-trivial to extend to all orders in $v$
A glance at a traditional fixed order calculation...

Luke and Savage (PRD 1998), order $\alpha_s v^2$

\begin{equation}
(a) = c_1 \frac{g^2}{12\pi} u_h^\dagger \alpha_s u_h \left[ \frac{m}{|p|} \pi^2 + i\pi \left( \gamma_E + \frac{2}{d-4} + \ln \frac{p^2}{\pi^2} \right) - \frac{i\pi}{4} \right].
\end{equation}

(4.22)

This reproduces the $O(1/v)$ term in the full amplitude.

As discussed above, there are no graphs at $O(\alpha_s v^2)$ in NRQCD from radiation gluon loops. At $O(\alpha_s v)$ there are contributions from the leading relativistic corrections to Coulomb scattering. In addition, since Coulomb exchange scales as $v^{-1}$, the dressing of $O_2$ with a single $A^0_\mu$ exchange also contributes at $O(\alpha_s v)$. $A^0_\mu$ exchange contributes both via the $p \cdot A$ coupling [Fig. 5(b)]

\begin{equation}
(b) = c_1 \frac{g^2}{12\pi} u_h^\dagger \alpha_s u_h \left[ \frac{|p|}{m} \pi^2 + i\pi \left( \gamma_E - 1 + \frac{2}{d-4} + \ln \frac{p^2}{\pi^2} \right) \right].
\end{equation}

(4.23)

and the Fermi coupling

\begin{equation}
(c) = c_1 \frac{g^2}{12\pi} \left( u_h^\dagger \alpha_s u_h \left[ \frac{|p|}{m} + \frac{m}{|p|} \frac{p \cdot \alpha p^\dagger u_h}{m^2} \right] - \frac{i\pi}{2} \right).
\end{equation}

(4.24)

Coulomb exchange is corrected by the Darwin vertex,

\begin{equation}
(d) = c_1 \frac{g^2}{12\pi} u_h^\dagger \alpha_s u_h \left[ \frac{|p|}{m} - i\pi \right],
\end{equation}

(4.25)

while the spin-orbit coupling does not contribute. The relativistic corrections to the quark and antiquark propagators give

\begin{equation}
(e) = c_1 \frac{g^2}{12\pi} u_h^\dagger \alpha_s u_h \left[ \frac{|p|}{2m} \pi^2 + i\pi \left( \gamma_E + \frac{1}{2} + \frac{2}{d-4} + \ln \frac{p^2}{\pi^2} \right) \right].
\end{equation}

(4.26)

and finally, the one-loop correction to $O_2$ in Fig. 5(f) gives

\begin{equation}
(f) = -c_2 \frac{g^2}{12\pi} \frac{u_h^\dagger \alpha_s u_h}{m^2} \left[ \frac{|p|}{2|p|} \pi^2 + i\pi \left( \gamma_E + \frac{3}{2} + \frac{2}{d-4} + \ln \frac{p^2}{\pi^2} \right) \right] - c_2 \frac{g^2}{12\pi} \frac{u_h^\dagger \alpha_s u_h}{m} \left[ \frac{|p|}{2} \right].
\end{equation}

(4.27)

Our results agree with Luke & Savage at order $\alpha_s v^2$. 