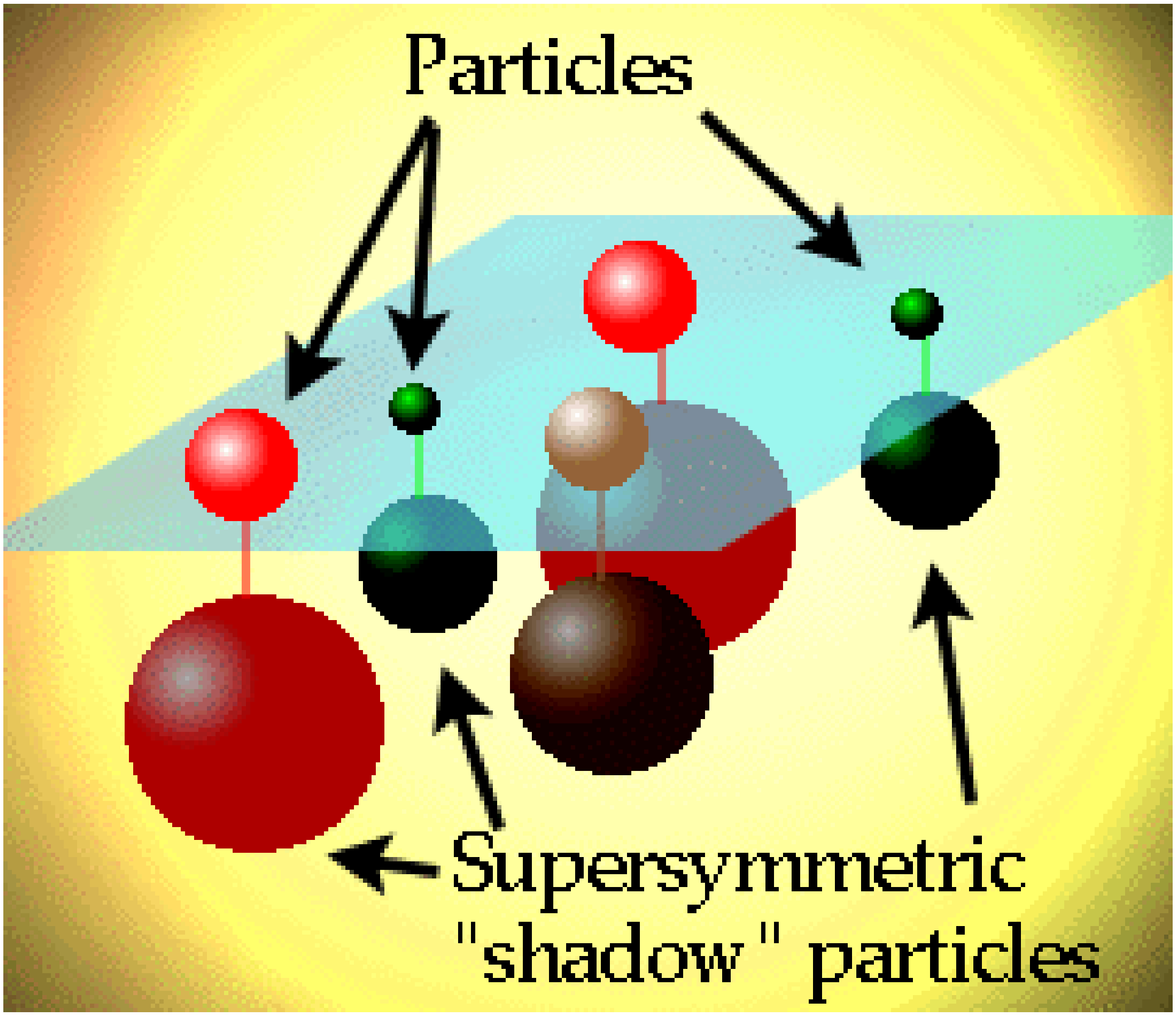

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Supersymmetry

Introduction:

The aim of theoretical physics is to describe as many phenomena as possible by a simple and natural theory. In elementary particle physics, the hope is that we will eventually achieve a unified scheme which combines all particles and all their interactions into one consistent theory. We wish to make further progress on the path which started with Maxwell's unification of magnetism and electrostatics, and which has more recently led to unified gauge theories of the weak and of the electromagnetic, and perhaps also of the strong interaction.

The purpose of this report is to introduce the reader to a development in theoretical particle physics which carries our hopes of being led further along that path: supersymmetry.

Definition & Explanation :

Supersymmetry is, by definition, a symmetry between fermions and bosons. A supersymmetric field theoretical model consists of a set of quantum fields and of Lagrangian for them which exhibits such a symmetry. The Lagrangian determines, through the Action Principle, the equations of motion and hence the dynamical behaviour of the particles. A supersymmetric model which is covariant under general coordinate transformations or equivalently, a model which possess local (“gauged”) supersymmetry is called a supergravity model. Supersymmetric theories describe model worlds of particles, created from the vacuum by the fields, and the interactions between these particles. The supersymmetry manifests itself in the particle and in stringent relationship between different interaction processes even if these involve particles of different spin and of different statistics.

Both supersymmetry and supergravity aim at a unified description of fermions and bosons, and hence of matter and interaction. Supergravity is particularly ambitious in its attempt at unification of the gravitational with other interactions. All supersymmetric models succeed to some degree in these aims, but they fail in actually describing the world as we experience it and thus are models, not theories. We are still striving to find some contact between one of the models and physical reality so that the model could become an underlying theory for nature at its most fundamental level. By “ most fundamental level” particle physicist mean at present the decomposition of matter into quarks and leptons(**fermions**) and that understanding of all forces between them as arising out of four types of basic interactions, gravitational, weak , electromagnetic and strong. These are described in terms of exchange particles (**bosons**).the framework within which these building blocks make up a physical theory is relativistic quantum field theory. Seen at this level, “unification” ought to include all four interactions. There is however, a quantitative and qualitative difference between the gravitational interaction and thee others which has had profound consequences both for the structure of the universe and for our understanding of it.

The concept of gauge invariance grew out of the observation that if the “charge” (e.g. Electric charge, total energy, isospin, etc.) is conserved in a dynamical system, then the Lagrangian for the system is invariant under “global gauge transformations” of the fields. For example, the electric charge is related to invariance under phase transformation $\psi \to \mathbf{e}^{iq\theta}\psi$ for all fields ψ which describe particles of charge q. similarly, the energy is related to time translations $\psi(\mathbf{t,x}) \to \psi(\mathbf{t+\Delta t,x})$. The converse is also true (Noether's theorem): if the Lagrangian is invariant under some infinitesimal transformation $\psi \to \psi + \delta\psi$, then there is conserved current and a conserved charge associated with this gauge invariance. (“Gauge” is an unfortunate misnomer, originating in an attempt by H. Weyl in 1918 to relate the electric charge to a rescaling transformation $\psi \to \mathbf{e}^{\lambda}\psi$). We call the transformations “global” if their parameters do not depend on the space-time coordinates, if $\theta = $ constant. This relationship between conserved quantum numbers and global symmetries of the Lagrangian led, in the 1960's to search for globally gauge-invariant field theories capable of describing and classifying all elementry particles. The “8-fold way” was a symmetry very much in this vein and it was in this context that quarks were first postulated as building blocks of strongly interacting matter.

The introduction of supersymmetry is not a revolution in the way one view physics. It is an additional symmetry that an otherwise “normal” field theoretical model can have. As we shall see, all that is required for a field theory to be supersymmetric is that it contains specified types and numbers of fields in interaction with each other and that the carious interaction strengths and particles masses have properly related values. As an example, consider the SU(3) gauge theory of gluons, which can be made supersymmetric by including a massless neutral colour octet of spin ½ particles which are their own antiparticles. Jargon has it that such spin ½ partners of the gluons are called “gluinos”. If our model contains not only gluons but also quarks, we must also add corresponding partners for them. These have spin 0 and are commonly called “squarks”.

Supersymmetric theories, and particularly supergravity theories, “unite” fermions and bosons into multiplets and lift the basic distinction between matter and interaction. The gluinos, for example, are thought of as carriers of the strong force as much as the gluons, except that as fermions they obey ab exclusion principle and thus will never conspire to form a coherent, measurable potential. The distinction between forces and matter becomes phenomenological: bosons – and particularly massless ones-manifest themselves as forces because they can build up coherent classical fields; fermions are seen as matter because no two identical ones can occupy the same point in space- an intuitive definition of material existence.

For some time it was thought that symmetries which would naturally relate forces and fermionic matter would be in conflict with field theory. The progress is understanding elementary particles through the **SU(3)** classification of the “eight fold way” (a global symmetry) had led to attempts to find a unifying symmetry which would directly relate to each other several of the **SU(3)** multiplets (baryon octet, decuplet, etc.), even if these had different spins. The failure of attempts to make those “spin symmetries” relativistically covariant led to the formulation of a series of no-go theorems, culminating impossible, within the theoretical framework of relativistic field theory, to unify space-time symmetry with internal symmetries. More precisely, the theorem says that the charge operators whose eigenvalues represent “internal” quantum numbers such as electric charge, isospin, hypercharge, etc. must be translationally and rotationally invariant. This means that these operators commute with the energy, the momentum and the angular momentum operators. Indeed, the only symmetry generators which transform at all under both translations and rotations are those of the Lorentz transformations themselves (rotations and transformations to coordinate systems which move with constant velocity).

Supersymmetry transformations are generated by quantum operators **Q** which change fermionic states into bosonic ones and vice versa. **Q |fermion> = |boson>**; **Q |boson> = |fermion>** which particular bosons and fermions are related to each other by the operation of some such **Q**, how many **Q's** there are, and which properties other than the statistics of the states are changed by that operation depends on the supersymmetric model under study. There are, however, a number of properties which are common to **Q's** in all supersymmetric models.

By definition, the **Q's** change the statistics and hence the spin of the states. Spin is related to behaviour under spatial rotations, and thus supersymmetry is -in some sense -a space-time symmetry. Normally, an d particularly so in models of “extended supersymmetry” (**N=8** supergravity being one example), the **Q's** also affect some of the internal quantum numbers of the states. It is this property of combining internal with space-time behaviour that makes supersymmetric field theories interesting in the attempt to unify all fundamental interactions.

Non-trivial space-time properties of the **Q's** consider the following.

Because fermions and bosons behave differently under rotations, the **Q** cannot be invariant under such rotations. We can, for example, apply the unitary operator **U** which, in Hilbert space, represents a rotation of configuration space by **360°** around some axis. Then

$$UQ\,|\text{boson}> = UQU^{-1}U\,|\text{boson}> = U\,|\text{fermion}>$$

$$UQ\,|\text{fermion}> = UQU^{-1}U\,|\text{fermion}> = U\,|\text{boson}>\ .$$

Since fermionic states pick up a minus sign when rotated through 360° and bosonic stats do not,

$U\,|\text{fermion}> = -|\text{fermion}>;\qquad U\,|\text{boson}> = |\text{boson}>$, and since all fermionic and bosonic states, taken together, form a basis in the Hilbert space, we easily see that we must have

$$UQU^{-1} = -Q$$

the rotated supersymmetry generator picks up a minus sign , just as a fermionic state does.

For the generation of space-time translation with vanishing commutator of **Q** with energy and momentum operators E and P,

$$[Q,E] = [Q,P] = 0$$

let us consider the anticommutator of some of **Q** with its Hermitian adjoint $Q^\dagger$. As spinor components the **Q's** are in general not Hermitian, but $\{Q,Q^\dagger\} \equiv QQ^\dagger + Q^\dagger Q$ is a Hermitian operator with positive definite eigenvalues.

$$<...|QQ^\dagger|...> + <...|Q^\dagger Q|...> = |Q^\dagger\,|...>|^2 + |\,Q|\,...>|^2 \geqslant 0$$

this can only be zero for all states $|...>$ if $Q = 0$.

A more detailed investigation will show that $\{Q,Q^\dagger\}$ must be a linear combination of the energy and momentum operators:

$$\{Q,Q^\dagger\} = \alpha\ \ + \beta P$$

when summing this equation over all supersymmetry generators, we find that the βP terms cancel while the α terms add up, so that

$$\Sigma_{\text{all } Q} \{Q,Q^\dagger\} \propto$$

depending on the sign of the proportionality factor,the spectrum for the energy would have to be either ⩾0 or ≤0

some important points to summarize supersymmetry

1. the spectrum of the energy operator **E** (the Hamiltonian) in a theory with supersymmetry contains no negative eigenvalues.

We denote the state with the lowest energy by **|0>** and call it the vacuum. The vacuum will have zero energy

E|0> = 0 if and only if Q|0> = 0 and $Q^\dagger$|0> = 0 for all Q.

any state whose energy is not zero, e.g. Any one-particle state, cannot be invariant under supersymmetry transformations. This means that there must be one (or more) superpartner state **Q|1> or $Q^\dagger$|1>** for every one-particle state **|1>.** the spin of these partner states will be different by ½ from that of **|1>.** thus

2. Each supermultiplet must contain at least one boson and one fermion whose spin differ by ½.

3. All states in a multiplet of unbroken supersymmetry have the same mass.

4. supersymmetry is spontaneously broken if and only if the energy of the lowest lying state (the vacuum) is not exactly zero).

$$\{Q_i,(Q_j)^\dagger\} = \delta_{ij}(\alpha + \beta P)$$

extended supersymmetry → N=1 supersymmetry → no supersymmetry
(at large **E)** **(**at medium **E)** **(**at low **E)**

the fundamental relationship between the generators of supersymmetry is now replaced by

$$\Sigma_{\alpha=1}{}^{4}\{Q_{\alpha i},(Q_{\alpha i})^\dagger\}\propto \text{ for each } i.$$

It is the name given to a hypothetical symmetry of nature.

Basically it is a symmetry which relates fermions and bosons. Just as there are operators that change **neutron** → **proton**, **or** $\mathbf{e}^- \to \upsilon_{\mathbf{e}}$, we can postulate the existence of operators that change bosons into fermions,

$$\mathbf{Q}\ |\mathbf{b}> = |\mathbf{f}> ,$$

with a conjugate operator going the opposite way. **Q** leaves all quantum numbers unchanged except spin. It has been shown that mathematically consistent, supersymmetric, quantum field theories can be constructed. The motivations for studying supersymmetric theories, and for hoping that nature utilizes them, are quite strong. However, at the present time there is no experimental evidence that nature is supersymmetric. Partly it is a typical example of how a Standard Model gives us the tools to quantitatively test whether additional physics is present.

If the Standard Model were part of a supersymmetric theory, with the symmetry not broken at all, it would be very obvious. Every one of the quarks, leptons, and gauge bosons would have a partner, generated by using the above equation or its equivalent for fermions, that differed in spin but was otherwise identical. Some of the states are listed in the Table 1.

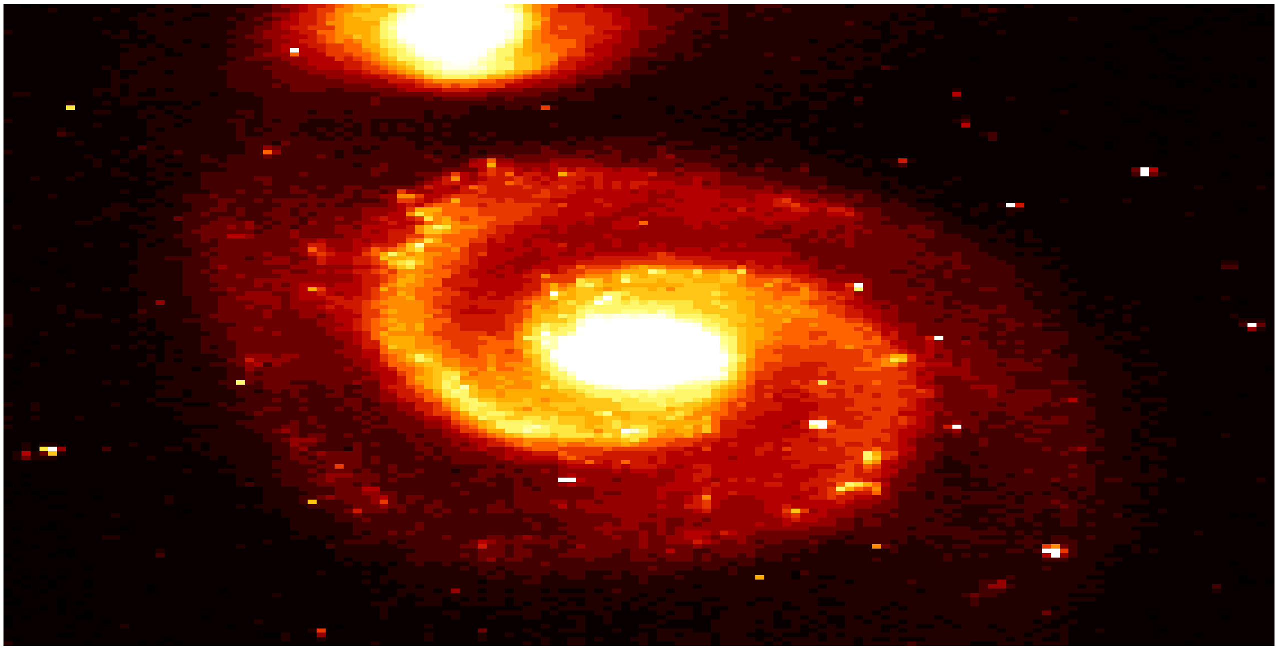
Supersymmetric partners are denoted by **a** ˜. They are usually named by attaching -ino for a gauge boson, or **s-** for a fermion.

If there were an unbroken supersymmetry, then many phenomena would occur. There would be a super-hydrogen atom with $e^{\sim}$ bound to a proton. The chemistry of multiselectron atoms,with bosons rather than fermions bound to the nucleus, would be very different. There would be additional weak interactions, with $W^{\sim}$ and $\mathbf{Z}^{\sim}$ exchanged, and so on. Clearly none of these things happen, and nature does not have an unbroken supersymmetry.

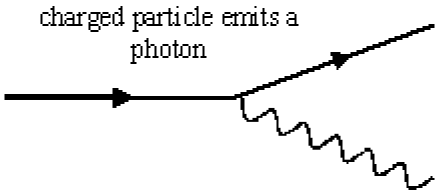
Supersymmetric states

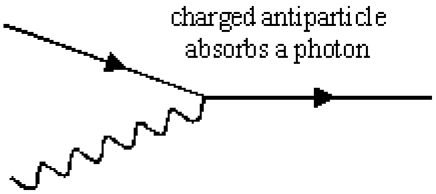
Particle	***Supersymmetric partner***	***Spin of partner***	***Name***
γ	$\tilde{\gamma}$	½	photino
e_L	$\tilde{e}_L$	0	Selectron
u_R	$\tilde{u}_R$	0	up squark
g	$\tilde{g}$	½	gluino
ν_μ	$\tilde{\nu}_\mu$	0	Muon sneutrino
:	:	:	:

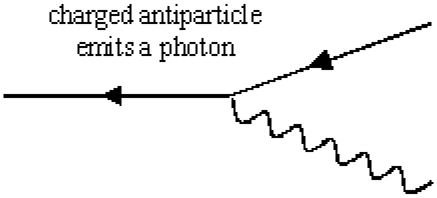
Since we know of the broken symmetry of the electroweak theory, perhaps it is reasonable to also assume that the supersymmetry is broken. Just as with the fermion masses in the Standard Model,a supersymmetric theory can be written that allows the superpartners to have arbitrary masses, but no one has found a way to calculate the masses. At present one can only search for the superpartners in whatever mass range is accessible to experiment. Just as in the Standard Model, once one assumes mass values for the superpartners, the theory is fully predictive; all rates can be calculated.

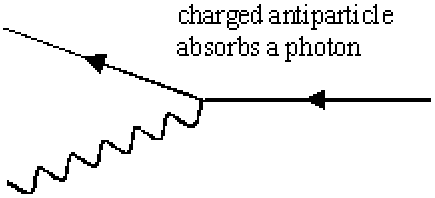


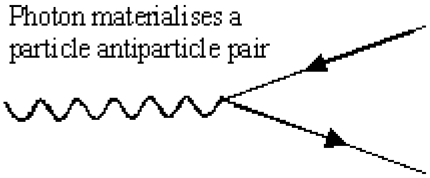
To calculate in the supersymmetric Standard Model, we need the Feynman rules. It is clear what they are. We just take the rules for the Standard Model and replace the particles by their partners in pairs, keeping the coupling strengths the same. The replacements has to be in pairs since otherwise the number of half-integral spin particles would be odd, and it would be impossible to conserve angular momentum in a transition. Then we see, for example, that the full theory

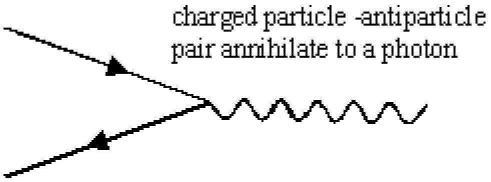












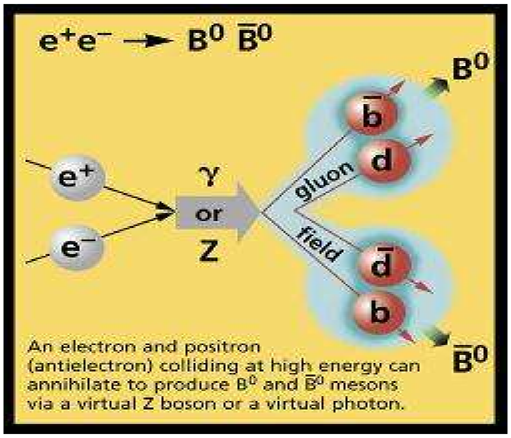
In addition to the interaction of a photon with quarks, there is a quark-squark-photino interaction and a photon-squark-squark interaction. The strengths of all the gauge couplings are just the measured ones we already know, because the measured couplings would know about the existence of the supersymmetric theory even if we don't. Because the couplings change with momentum transfer, if the superpartners were very much heavier than M_w there would be be differences in the couplings.

Since all vertices involve superpartners in pairs, we can draw three important conclusions for a normal supersymmetric theory,

1. supersymmetric partners will be produced in pairs starting from normal particles,
2. the decay of supersymmetric partners will contain a supersymmetric partner,
3. the lightest supersymmetric partner will be stable.

Production and detection of supersymmetric partners

Starting from beams of quarks and leptons, we can draw a variety of diagrams to produce superpartners. One is shown here.
The production cross sections involve the same couplings we are used to, so the cross sections are typical of production rates for W's quarks, etc., except that there is phase space suppression if the superpartners are heavy. Next we have to ask how the partners would act once they are produced. For simplicity let us assume that gluinos are heavier than squarks and than zinos and winos. Then the dominant decays for any sfermion with electric charge will be



$f^{\sim} \rightarrow f + \gamma^{\sim}$; **e.g** $\mu^{\sim} \rightarrow \mu + \gamma^{\sim}$ **or d** $\rightarrow$ **d** $+\gamma^{\sim}$ **.**

As we have learned, typical decay widths for a superpartner of mass $M^{\sim}$ will be $\Gamma^{\sim}$ _**~**$\alpha M^{\sim}$. With $M^{\sim}$ **~** tens of **GeV**, $\Gamma^{\sim}$ is of order **0.1-1 GeV**, so the associated lifetimes are short compared to **10**$^{-20}$ sec,and only the decay products emerge into detector.

To complete the analysis, it is necessary to decide which will be the lightest supersymmetric particle since all the others will decay into it. There are several possibilities; we will assume it is photino for simplicity. If some other superpartner were lighter than the photino we could go through a similar analysis; details change, but qualitative conclusions do not.

Since all the superpartners that are produced will decay in a very short time, only normal particles plus the photino will enter the detector. To detect the presence of supersymmetry we must be able to detect the $\gamma^{\sim}$.

The $\gamma^{\sim}$ will interact by hitting a quark in the detector. The $q^{\sim}$ could be real or virtual depending on the available energy. For simplicity we assume the $\tilde{q}$ is real. The cross section for this is

$$\sigma = \sum_q \int dx\, q(x)\, \hat{\sigma}(\hat{s})$$

where x is the fraction of the proton's momentum carried by the quark,q(x) is the quark structure function and $\hat{\sigma}$ the constituent cross section for

$\tilde{\gamma} + q \to \tilde{q}$. there is a sum over all the quarks in the proton. The square of the center of mass energy of the $\tilde{\gamma}$ and the q is $\hat{s}$, so $\hat{s} = \tilde{M}^2$ where $\tilde{M}$ is the squark mass. Also, $\hat{s} = xs$, where s is the square of the center of mass energy of the photino and proton. The matrix element is approximately $M \simeq e_q\, e\, \bar{u}u$ where e_q is the quark charge (**2/3 or -1/3**). as usual we can replace the spinors by the appropriate mass, $\bar{u}u \simeq \tilde{M}$.

Writing $\hat{s} = xs$, this is $\hat{\sigma} = \pi e_q^2 e^2 \delta(x - \tilde{M}^2/s)/s$.

$$\sigma \approx 4\pi^2\alpha/\tilde{M}^2 \sum_q e_q^2 x\, q(x)$$

where we replaced s by $\tilde{M}^2/x$. The factor $\sum_q e_q^2 x\, q(x)$ is just the structure function $F_2(x)$.

$$\sigma(\tilde{\gamma}p) \approx 4\pi^2\alpha/\tilde{M}^2 F_2(\tilde{M}^2/s).$$

Although we are working in a hypothetical theory, we have calculated the photino interaction cross section in terms of familiar quantities, plus an assumed squark mass. By calculating the value of $\sigma(\tilde{\gamma}p)$, we find then $\sigma(\tilde{\gamma}p) \simeq 2.5\text{x}10^{-33}\ \text{cm}^2$. This is typical of neutrino cross section, about 10^{-7} of a pion cross section. A typical $\tilde{\gamma}$ will not interact in a detector it will escape, carrying away momentum. Thus the experimental signature of supersymmetry is an event where apparently momentum is not conserved. Such events can also occur if neutrinos are produced, for example in decays of W's or of heavy quarks, but then charged lepton is also produced. If events are ever discovered with apparent failure of conservation of momentum, and no charged leptons, they could be the signal of supersymmetry. Then detailed analysis can establish whether they could in fact come from production of super partners.

The relative rates for various processes, the distribution of missing momentum from large to small, and a number of other quantitative predictions can all test whether a supersymmetric interpretation is possible.

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