Quark Model

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Every hadron will be characterized by a set of quantum numbers its mass, electric charge, baryon number, spin, and it may be an eigenstate of parity, charge-conjugation, etc.
The strange quark 's' is a component of the so-called strange particles discovered in cosmic rays in the 1950s. The discovery of 'c' quark resulted from the observation of massive meson states of the type $\Psi = c\bar{c}$ in 1974, and that of the 'b' quark followed from the detection of even heavier mesons $\Upsilon = b\bar{b}$ in 1977, top quark in 1995.

Quantum Numbers of Mesons and Baryons

#Mass

-The **mass** of the hadron will be determined by several contributions.

-The problem of calculating the mass of a hadron, and the related problem of mass of a quark or a gluon in a hadron, are complicated non-perturbative questions

Property \ Quark	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	−1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	−1	0
T – topness	0	0	0	0	0	+1

#Orbital angular momentum

- $\vec{L}$ The **orbital angular momentum** will be 0,1,2,....

#Spin

- $\vec{S}$ will add to give the **total spin** of mesons.

#Isospin

#Parity

-The fermion and antifermion have opposite intrinsic **parity**.

-The meson is an eigenstate of **parity** with eigenvalue.

$$P = -(-1)^L$$

(-1^L) come from th effect of the rotation on the angular wavefunction.
-The system can be returned to its orginal state by rotating and interchanging spins, which

give $(-1)^{S+1}$ since spin-zero is antisymmetric and wpin-one is symmetric.

Charge conjugation

$$C=(-1)(-1)^{L}(-1)^{S+1}=(-1)^{L+S}$$: **charge conjugation**

-The meson states will be labeled by their total angular moment J, by P and C, and by the flavor structure of their quarks($u\bar{u}$ mesons, $s\bar{s}$ mesons, etc)

#Baryon number

u,d,s quark has same baryon numbers, $\frac{1}{3}$

#Charge

$$Q=\frac{1}{2}(B+S)+I_3$$ where I_3 is isospin.

#Hypercharge

$Y\equiv B+S$ is called the hypercharge, it folloes that quarks must also carry fractional charges of $\frac{2}{3}$ and $-\frac{1}{3}$.

Color

The value $J=\frac{3}{2}$ is then obtained by having the quarks in a symmetric spin state, with spins "paralled", as in $\Delta^{++}=u\uparrow u\uparrow u\uparrow$, for example. This clearly violates the Pauli principle, that two or more fermions may not exist in the same quantum state. New quantum number was appeared that another degree of freedom, called **color,** was necessary for other reasons. It is postulated that quarks exist in three colors - say red, green, blue – and that baryons and mesons built from quarks have zero net color, that is, they are color singlets. It is simple a notation for a new property of quarks, quite seperate from the flavor quantum number. The three color specify the same way that the signs + and – specify their electric charges.

Light Mesons, Baryons

The pattern of lighter mesons and baryon was very important in deading to some of the ideas that are part of the Standard Model today.

The u and d quarks are very light, with free masses on the order of 10 MeV.

The constituent mass is mass which results from the binding of massless quarks into a color singlet state.

Quarks are not observed as free particles and hence must be confined in hardrons by the interquark potential.

	I	I_3	S	Meson	Quark combination	Decay	Mass, MeV
octet	1	1	0	π^+	$u\bar{d}$	$\pi^\pm \to \mu\nu$	140
	1	-1	0	π^-	$d\bar{u}$		
	1	0	0	π^0	$(d\bar{d} - u\bar{u})/\sqrt{2}$	$\pi^0 \to 2\gamma$	135
	$\frac{1}{2}$	$\frac{1}{2}$	$+1$	K^+	$u\bar{s}$	$K^+ \to \mu\nu$	494
	$\frac{1}{2}$	$-\frac{1}{2}$	$+1$	K^0	$d\bar{s}$	$K^0 \to \pi^+\pi^-$	498
	$\frac{1}{2}$	$-\frac{1}{2}$	-1	K^-	$\bar{u}s$	$K^- \to \mu\nu$	494
	$\frac{1}{2}$	$\frac{1}{2}$	-1	$\bar{K}^0$	$\bar{d}s$	$\bar{K}^0 \to \pi^+\pi^-$	498
	0	0	0	η_8	$(d\bar{d} + u\bar{u} - 2s\bar{s})/\sqrt{6}$	$\eta \to 2\gamma$	549
singlet	0	0	0	η_0	$(d\bar{d} + u\bar{u} + s\bar{s})/\sqrt{3}$	$\eta' \to \eta\pi\pi$ $\to 2\gamma$	958

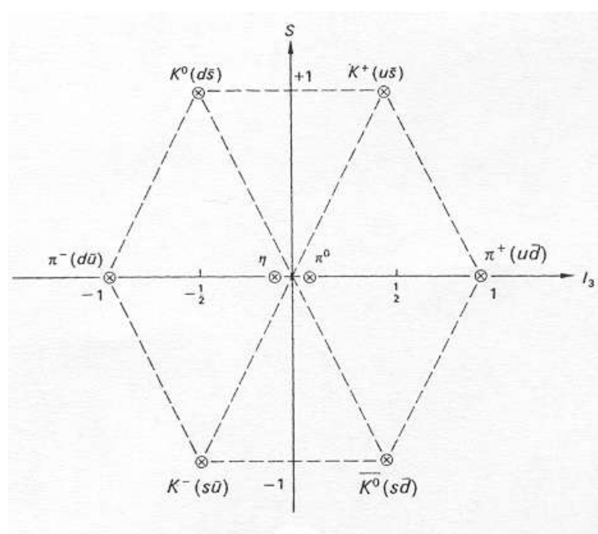
Given spin - $\frac{1}{2}$ for quarks and antiquarks, we might expect both spin triplet ($\uparrow\uparrow$)states of $J=1$ (the vector mesons) and spin singlet ($\downarrow\uparrow$)states of $J=0$ (the pseudoscalar mesons).

#pseudoscalar mesons

Now we are dealing with quarks and antiquarks- thus the interchange $u-\bar{u}$,for example. It is necessary therefore to consider the effect of change conjugation applied to quark wavefunctions. If the baryon number B is conserved, there is no actual physical process $Q \to \bar{Q}'$, as a result of the operation of charge conjugation, or particle-antiparticle conjugation.

These correspond th pseudocsalar mesons, so called because the wavefunctions have $J=0$, have odd parity, and change sign under spatial inversion.

The $L=S=0$ states are pseudocsalar mesons, i,e they have odd parity and spin-zero.

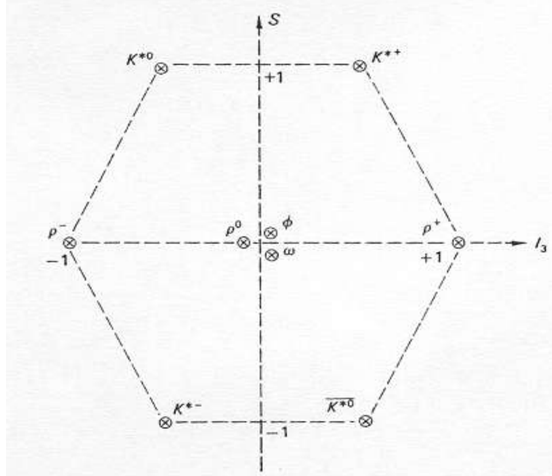


#Vector mesons

The $L=0$ and $S=1$ mesons have $J=1$ and still odd parity; they transform under rotations lide a vecter, and are called vector mesons.

$L=0$ and $S=1$ states

Particle	Quark Content	Mass (MeV)
ρ^+	$u\overline{d}$	770
ρ^-	$\overline{u}d$	”
ρ^0	$(u\overline{u}+d\overline{d})/\sqrt{2}$	”
ω^0	$(u\overline{u}-d\overline{d})/\sqrt{2}$	780
K^{*+}	*etc.*, as above	890
K^{*-}		”
K^{*0}		”
$\overline{K}^{*0}$		”
ϕ		1020



(b)

#Baryon : ex) proton(uud), neutron(udd)

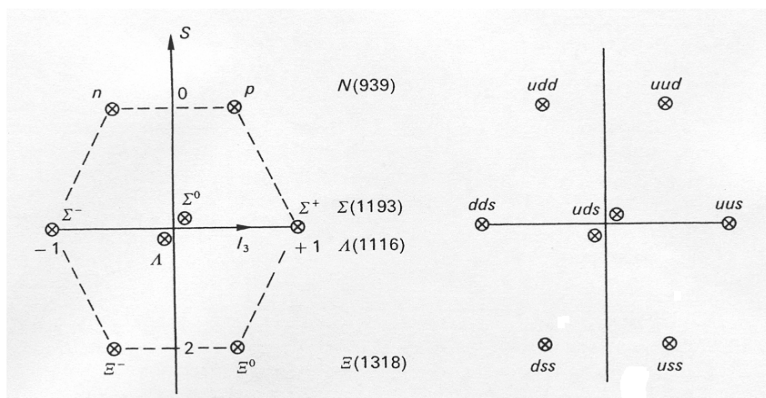
Given the baryon is to consist of 3 quarks chosen from any 3 flavors, 27 combinations are possible.

#The Baryon Octet

The members of the baryon octet of $J^P = \frac{1}{2}^+$ can be worked out in similar fashion. This octet is followed, where the wavefuctions have been indicated as uud,ssu, etc., The eight members consist of the n and p(939) nucleon isospin doublet ($I=\frac{1}{2}$, $S=0$), the $\Sigma(1193)$ isotriplet ($I=1$, $S=-1$), the $\Xi(1318)$ isodoublet ($I=\frac{1}{2}$, $S=-2$), and the $\Delta(1116)$ isosinglet ($I=0$, $S=-1$).

$L_1 = L_2 = 0$ and $S = 1/2$ states

Particle	Quark Content	Mass (MeV)
p	uud	939
n	udd	940
Λ	uds	1115
Σ^+	uus	1193
Σ^-	dds	1197
Σ^0	uds	1189
Ξ^0	uss	1315
Ξ^-	dss	1321

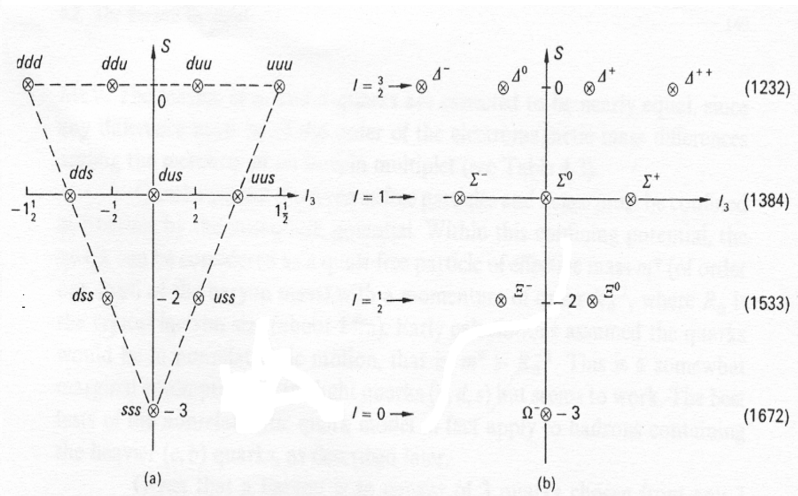


#The Baryon Decuplet

Follewing figure indecates the 10 baryon states of lowst mass and of spin-parity $J^P = \frac{3}{2}^+$, where we plot the strangness S against the third component of isospin, I_3 , for each of the 10 members. Working downward, these consist of as $S=0$ $I=\frac{3}{2}$ isospin quadruplet, the $\Delta(1232)$,existing in the charge substates Δ^{++} , Δ^{+} , Δ^{0} , Δ^{-} . The number 1232 in parentheses indicates the central resonance mass in MeV. Next com an $I=1$ isospin triplet of $S=-1$, the $\Sigma(1384)$;an $S=-2$, $\frac{1}{2}$ isospin doublet.

$L_1 = L_2 = 0$ and $S = 3/2$ states

Particle	Quark Content	Mass (MeV)
Δ^{++}	uuu	1232
Δ^{+}	uud	”
Δ^{0}	udd	”
Δ^{-}	ddd	”
Σ^{*+}	suu	1382
Σ^{*0}	sud	”
Σ^{*-}	sdd	1387
Ξ^{*0}	ssu	1315
Ξ^{*-}	ssd	1321
Ω^{-}	sss	1672



#Reference

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