QCD
(Quantum Chromodynamics)

Syllabus

- Introduction (Chap. 1)
- Special Relativity (Chap. 2)
- Quantum Mechanics (Chap. 3)
- Detector
- Data Processing
- Feynman diagram (Chap. 4)
- QED (Chap. 5) & Weak interaction (Chap. 7)
- QCD (Chap. 6)

QCD

QUANTUM ELECTRODYNAMICS: is the quantum theory of the electromagnetic interaction.

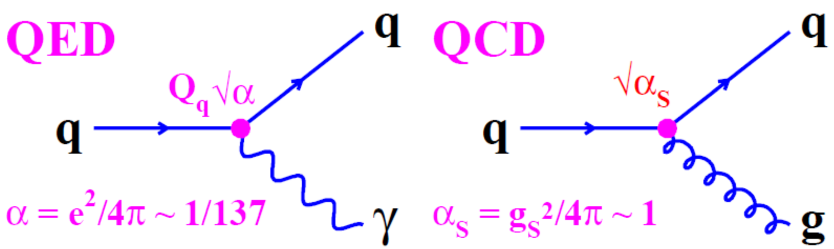
- ★ mediated by massless photons
- ★ photon couples to electric charge, e
- ★ Strength of interaction : $\langle \psi_f | \hat{\mathrm{H}} | \psi_i \rangle \propto \sqrt{\alpha}$.

$$\alpha = \frac{e^2}{4\pi}$$

QUANTUM CHROMO-DYNAMICS: is the quantum theory of the strong interaction.

- ★ mediated by massless gluons, *i.e.* $1/q^2$ propagator
- ★ gluon couples to "strong" charge
- ★ Only quarks have non-zero "strong" charge, therefore only quarks feel strong interaction

Basic QCD interaction looks like a stronger version of QED, $\alpha_S > \alpha_{EM}$



(subscript em is sometimes used to distinguish the α_{em} of electromagnetism from α_S).

COLOUR

In QED:

- ★ Charge of QED is electric charge.
- ★ Electric charge - conserved quantum number.

In QCD:

- ★ Charge of QCD is called "COLOUR"
- ★ COLOUR is a conserved quantum number with 3 VALUES labelled "red", "green" and "blue"

Quarks carry "COLOUR" $r \; g \; b$

Anti-quarks carry "ANTI-COLOUR" $\overline{r} \; \overline{g} \; \overline{b}$

Leptons, γ, $\mathrm{W}^{\pm}$, Z^0 DO NOT carry colour, i.e. "have colour charge zero" $\longrightarrow$ DO NOT participate in STRONG interaction.

Note: Colour is just a label for states in a non-examinable SU(3) representation

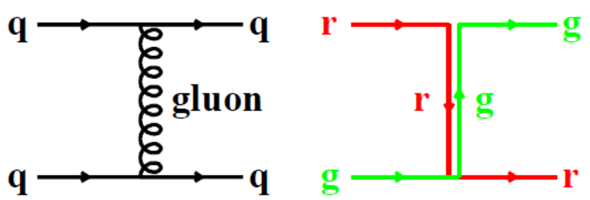
$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

GLUONS

In QCD:

★ Gluons are MASSLESS spin-1 bosons

Consider a red quark scattering off a green quark.
Colour is exchanged but always conserved.



UNLIKE QED:

★ Gluons carry the charge of the interaction.

★ Gluons come in different colours.

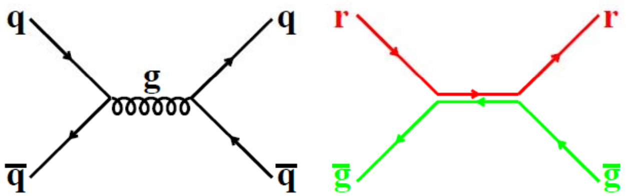
Expect 9 gluons (3 colours $\times$ 3 anti-colours)

$$r\overline{b},\ r\overline{g},\ g\overline{r},\ g\overline{b},\ b\overline{g},\ b\overline{r}$$
$$r\overline{r},\ g\overline{g},\ b\overline{b}$$

However: Real gluons are orthogonal linear combinations of the above states. The combination $\frac{1}{\sqrt{3}}(r\overline{r}+g\overline{g}+b\overline{b})$ is colourless and does not take part in the strong interaction.

Colour at Work

EXAMPLE: $q\overline{q}$ Annihilation



Normally do not show colour on Feynman diagrams - colour is conserved.

QED POTENTIAL:

$$V_{\rm QED} = -\frac{\alpha}{r}$$

QCD POTENTIAL:

At short distances QCD potential looks similar

$$V_{\rm QCD} = -\frac{4}{3}\frac{\alpha_S}{r}$$

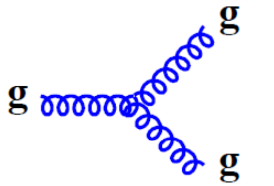
apart from $\frac{4}{3}$ colour factor.

Note: the colour factor (4/3) arises because more than one gluon can participate in the process $q \to qg$. Obtain colour factor from averaging over initial colour states and summing over final/intermediate colour states

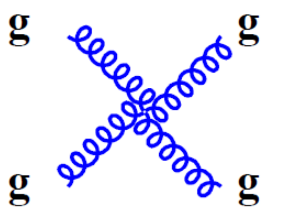
SELF-INTERACTIONS

At this point, QCD looks like a stronger version of QED. This is true up to a point. However, in practice QCD behaves very differently to QED. The similarities arise from the fact that both involve the exchange of MASSLESS spin-1 bosons. The big difference is that GLUONS carry colour "charge".

GLUONS CAN INTERACT WITH OTHER GLUONS:

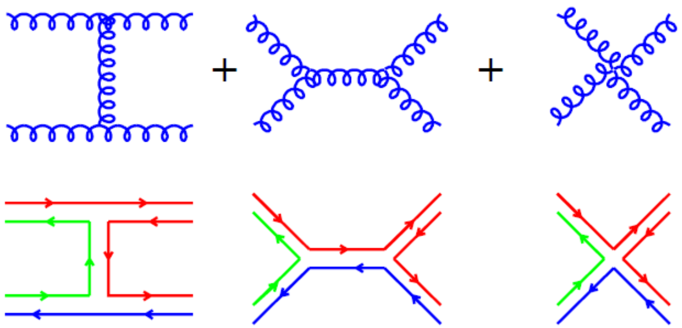


3 GLUON VERTEX



4 GLUON VERTEX

EXAMPLE: Gluon-Gluon Scattering $gg \to gg$



e.g. $r\bar{g} + g\bar{b} \to r\bar{r} + r\bar{b}$

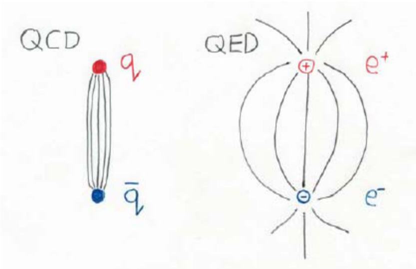
CONFINEMENT

<u>NEVER OBSERVE:</u> single FREE quarks/gluons

★ quarks are always confined within hadrons

★ This is a consequence of the strong self-interactions of gluons.

Qualitatively, picture the colour field between two quarks. The gluons mediating the force act as additional sources of the colour field - they attract each other. The gluon-gluon interaction pulls the lines of colour force into a narrow tube or STRING. In this model the string has a 'tension' and as the quarks separate the string stores potential energy.



Energy stored per unit length $\sim$ constant.

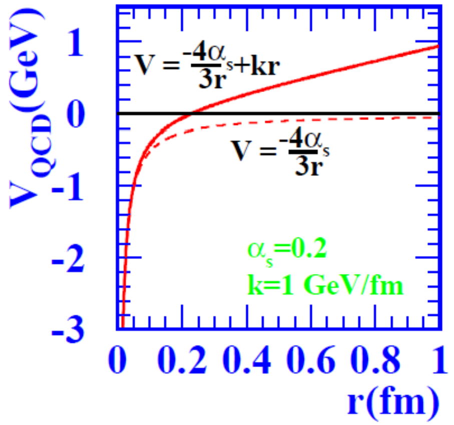
$$V(r) \propto r$$

★ Requires infinite energy to separate two quarks. Quarks always come in combinations with zero net colour charge: CONFINEMENT.

How Strong is Strong ?

QCD Potential between quarks has two components:

- ★ "COULOMB"-LIKE TERM : $-\frac{4}{3}\frac{\alpha_S}{r}$
- ★ LINEAR TERM : $+kr$



Force between two quarks at separated by 10 m:

$$V_{QCD} = -\frac{\alpha_S}{r} + kr$$

with $k \approx 1\,\mathrm{GeV}/\text{fm}$

$$F = -\frac{dV}{dr} = \frac{\alpha_S}{r^2} + k$$

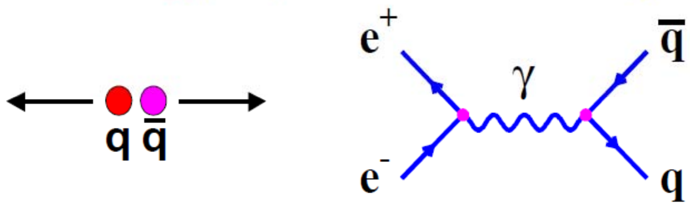
at large r $F = k = \frac{1.6\times 10^{-10}}{10^{-15}} N$

$$= 160000\,N$$

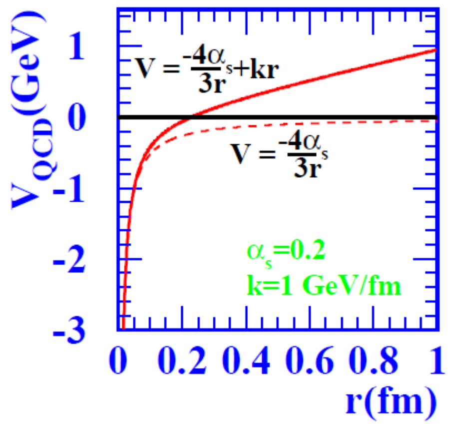
Equivalent to the weight of approximately 65 Widdecombes.

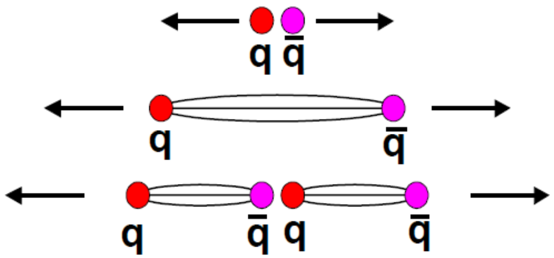
JETS

Consider the $q\overline{q}$ pair produced in $e^+e^- \to q\overline{q}$:



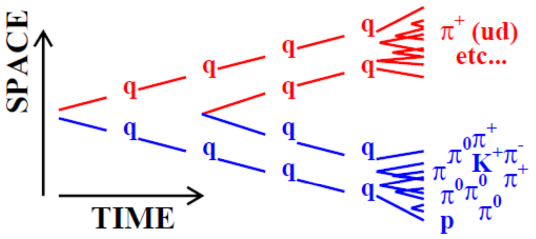
As the quarks separate, the energy stored in the colour field ('string') starts to increase linearly with separation. When $E_{stored} > 2m_q$ new $q\overline{q}$ pairs can be created.

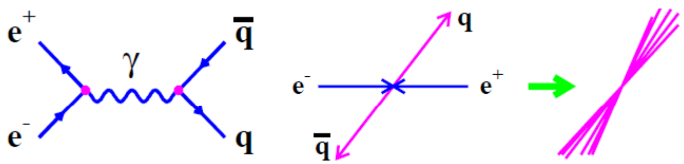




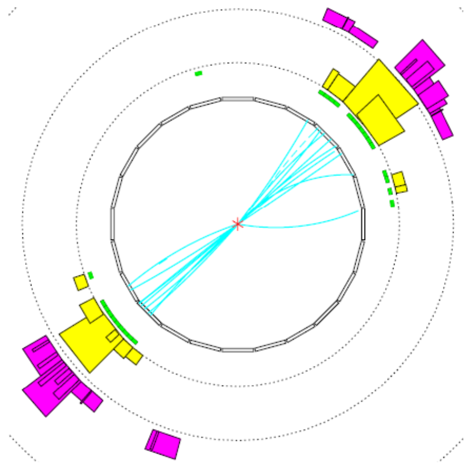
as energy decreases... hadrons freeze out

As quarks separate, more $q\overline{q}$ pairs are produced from the potential energy of the colour field. This process is called HADRONIZATION. Start out with quarks and end up with narrowly collimated JETS of HADRONS



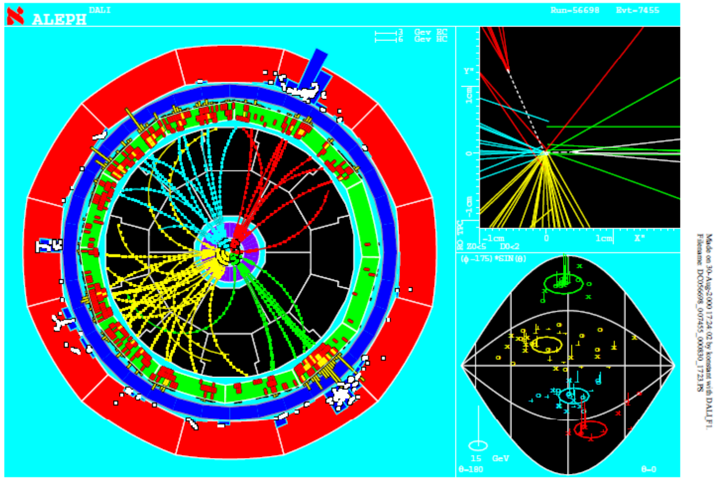


Typical $e^+e^- \rightarrow q\overline{q}$ Event



The hadrons in a quark(anti-quark) jet follow the direction of the original quark(anti-quark). Consequently $e^+e^- \rightarrow q\overline{q}$ is observed as a pair of back-to-back jets of hadrons

aside.....



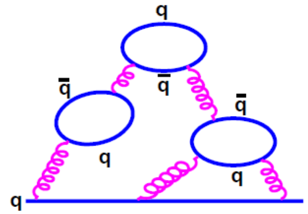
★ You will now recognize the "Higgs" event from the cover of Handout I as

$$e^+e^- \longrightarrow \text{something} \longrightarrow q\bar{q}q\bar{q}$$

Running of α_S

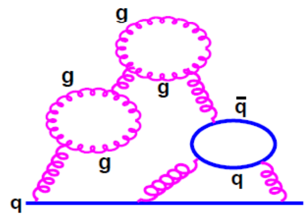
- ★ α_S specifies the strength of the strong interaction
- ★ BUT just as in QED, α_S isn't a constant, it "runs"
- ★ In QED the bare electron charge is screened by a cloud of virtual electron-positron pairs.
- ★ In QCD a similar effect occurs.

In QCD quantum fluctuations lead to a 'cloud' of virtual $q\overline{q}$ pairs



one of many (an infinite set) such diagrams analogous to those for QED.

In QCD the gluon self-interactions ALSO lead to a 'cloud' of virtual gluons

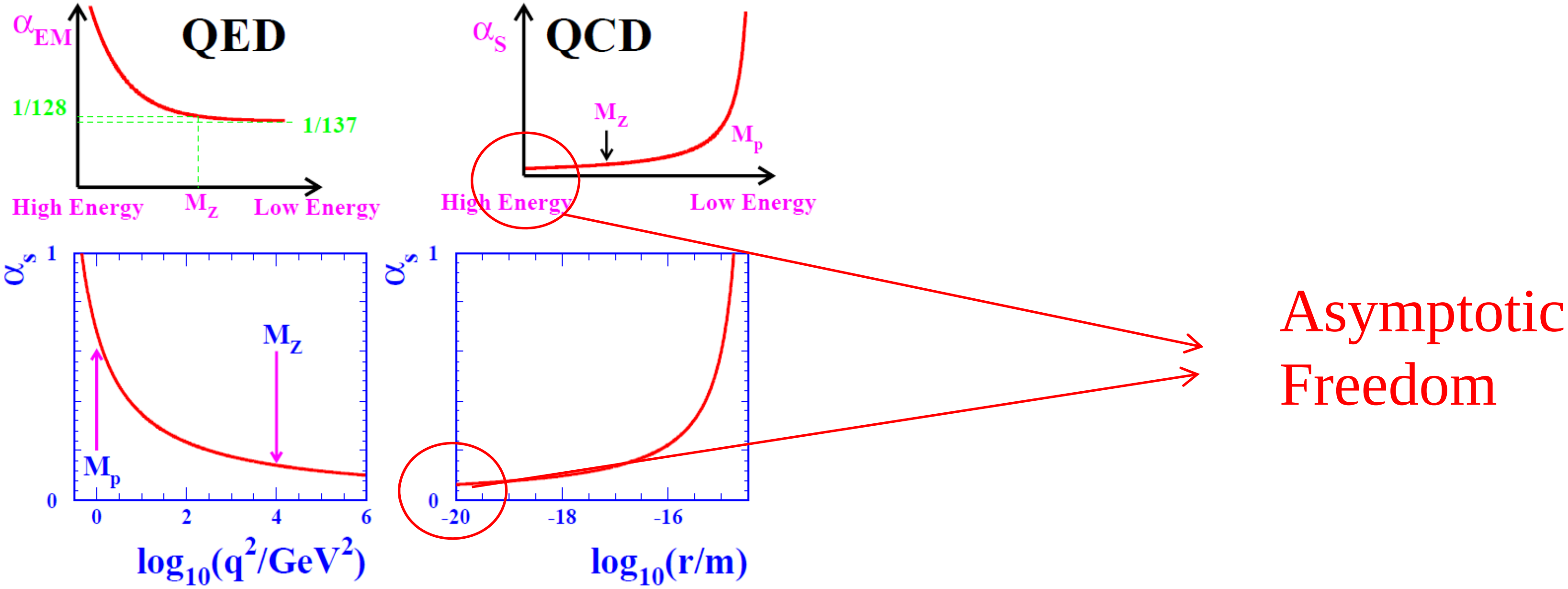


one of many (an infinite set) such diagrams. Here there is no analogy in QED, photons don't have self-interactions since they don't carry the charge of the interaction.

=> 6.3 Nonabelian Nature of Quantum Chronodynamics

Colour Anti-Screening

- ★ Due to the gluon self-interactions bare colour charge is screened by both virtual quarks and virtual gluons
- ★ The cloud of virtual gluons carries colour charge and the effective colour charge INCREASES with distance !
- ★ At low energies (large distances) α_S becomes large $\rightarrow$ can't use perturbation theory (not a weak perturbation)

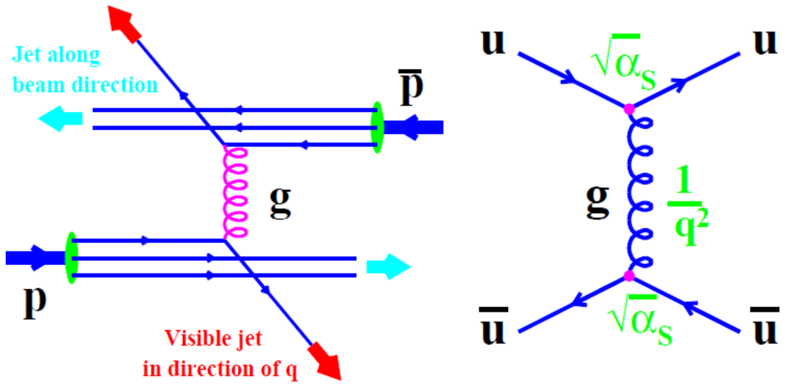


- ★ At High energies (short distances) α_S is small. In this regime treat quarks as free particles and can use perturbation theory
  ASYMPTOTIC FREEDOM
- ★ At $\sqrt{s} = 100$ GeV, $\alpha_S = 0.12$.

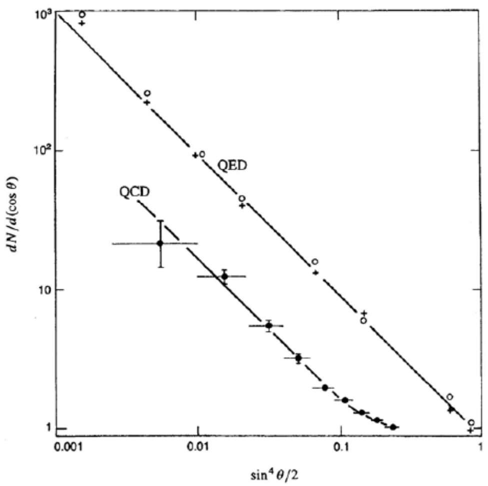
Scattering in QCD

=> 6.2 Form factor and structure function

EXAMPLE: High energy proton-antiproton scattering

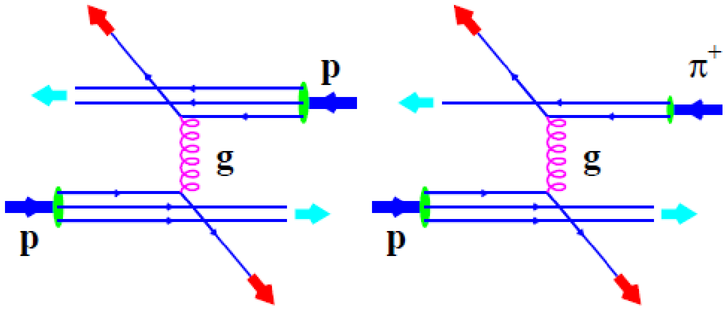


$$M \sim \frac{1}{q^2}\sqrt{\alpha_S}\sqrt{\alpha_S} \;\Rightarrow\; \frac{d\sigma}{d\Omega} \sim \frac{(\alpha_S)^2}{\sin^4\theta/2}$$



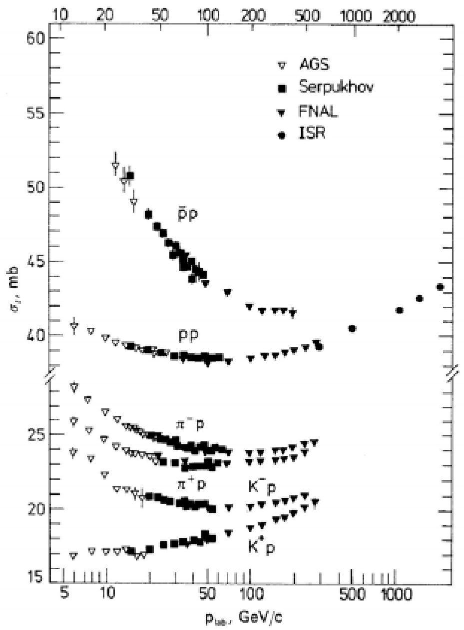
The upper points are the Geiger and Marsden data (1911) for the elastic scattering of α particles as they traverse thin gold and silver foils. The lower points show the angular distribution of the quark jets observed in proton-antiproton scattering at $q^2 = 2000\mathrm{GeV}^2$ Both follow the Rutherford formula for elastic scattering: $\sin^4 \frac{\theta}{2}$.

EXAMPLE: pp vs π^+p scattering



Calculate ratio of $\sigma(pp)_{\text{total}}$ to $\sigma(\pi^+p)_{\text{total}}$

★ QCD does not distinguish between quark flavours, only COLOUR charge of quarks matters.



At high energy (E $\gg$ binding energy of quarks within hadrons) ratio of pp and $p\pi$ total cross sections depends on number of possible quark-quark combinations.

Predict

$$\frac{\sigma(\pi p)}{\sigma(pp)} = \frac{2\times 3}{3\times 3} = \frac{2}{3}$$

Experiment

$$\frac{\sigma(\pi p)}{\sigma(pp)} \approx \frac{24\ mb}{38\ mb}$$

$$\approx 0.63$$

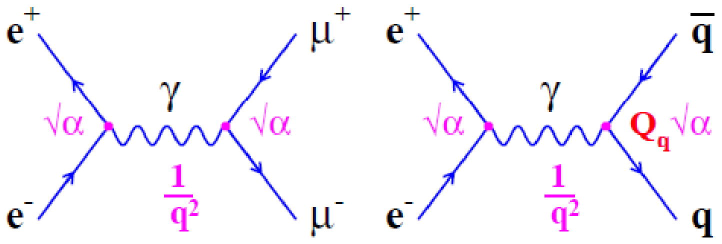
QCD in e^+e^- Annihilation

⇒Section 6.1 Quark and antiquark pair production in e+e- annihilation

Direct evidence for the existence of colour comes from e^+e^- Annihilation.

★ Compare $e^+e^- \to \mu^+\mu^-, e^+e^- \to q\overline{q}$:

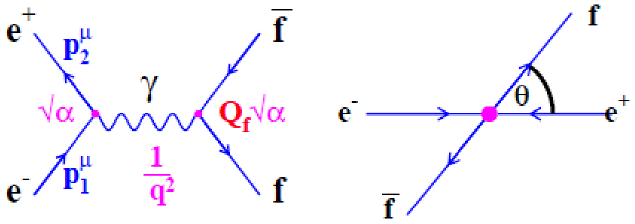
$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



If we neglect the masses of the final state quarks/muons then the **ONLY** difference is the charge of the final state particles ($Q_\mu = -1, Q_q = +\frac{2}{3}$ or $-\frac{1}{3}$)

Start by calculating the cross section for the process $\sigma(e^+e^- \to f\overline{f})$. ($f\overline{f}$ represent a fermion-antifermion pair *e.g.* $\mu^+\mu^-$ or $q\overline{q}$).

see **Handout II** for the case where $f\overline{f} = \mu^+\mu^-$



Electron/Positron beams along z-axis

$$\begin{aligned} p_1^\mu &= (E, p_x, p_y, p_z) \\ p_1^\mu &= (E, 0, 0, E) \quad \text{neglecting } m_e \\ p_2^\mu &= (E, 0, 0, -E) \\ q^\mu &= p_1^\mu + p_2^\mu \\ &= (2E, 0, 0, 0) \\ q^2 &= 4E^2 = s \end{aligned}$$

where s is (centre-of-mass energy)^2.

Fermi's Golden rule and Born Approximation

$$\frac{d\sigma}{d\Omega} = 2\pi |M|^2 \frac{d\rho(E_f)}{d\Omega}$$

Matrix element M:

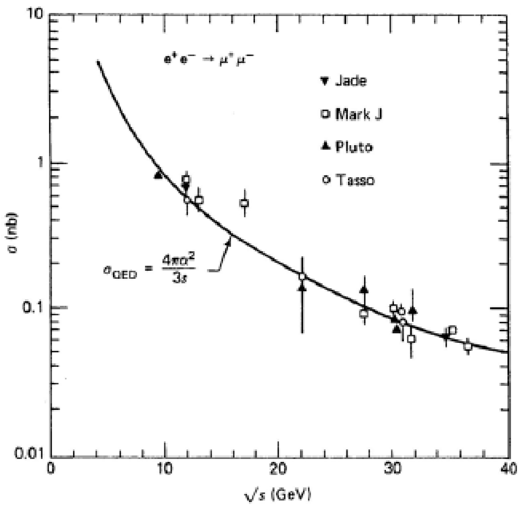
$$\begin{aligned} M &= \langle v_{e+} | Q_e e | u_{e-} \rangle \frac{1}{q^2} \langle v_{\overline{f}} | Q_f e | u_f \rangle \\ &= \frac{-4\pi\alpha Q_e Q_f}{q^2} \quad \text{with} \quad \alpha = \frac{e^2}{4\pi} \\ \frac{d\sigma}{d\Omega} &= 2\pi \frac{(-4\pi\alpha Q_e Q_f)^2}{q^4} \frac{E^2}{(2\pi)^2} \frac{1}{4} (1 + \cos^2\theta) \\ &= \frac{\alpha^2 Q_f^2}{4s} (1 + \cos^2\theta) \end{aligned}$$

★ $(1+\cos^2\theta)$ comes from spin-1 photon "decaying" to two spin-half fermions. see lecture on Dirac equation

Total cross section for $e^+e^- \to f\bar{f}$

$$\begin{aligned}\sigma &= \int \frac{d\sigma}{d\Omega} d\Omega \\ &= \int_0^{2\pi}\int_0^{\pi} \frac{\alpha^2 Q_f^2}{4s}(1+\cos^2\theta)\sin\theta d\theta d\phi \\ &= \frac{\pi\alpha^2 Q_f^2}{2s}\int_{-1}^{+1}(1+y^2)dy \qquad (y=\cos\theta) \\ &= \frac{4\pi\alpha^2 Q_f^2}{3s}\end{aligned}$$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$



$\sigma(e^+e^- \to \mu^+\mu^-)$ for e^+e^- collider data at centre-of-mass energies 8-36 GeV

Back to

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

For a single quark flavour of a given colour

$$R = Q_q^2$$

However, we measure $e^+e^- \to \text{jets}$ not $e^+e^- \to u\bar{u}$. A jet from a u-quark looks just like a jet from a d-quark... Need to sum over flavours (u,d,c,s,t,b) and colours (r, g, b).

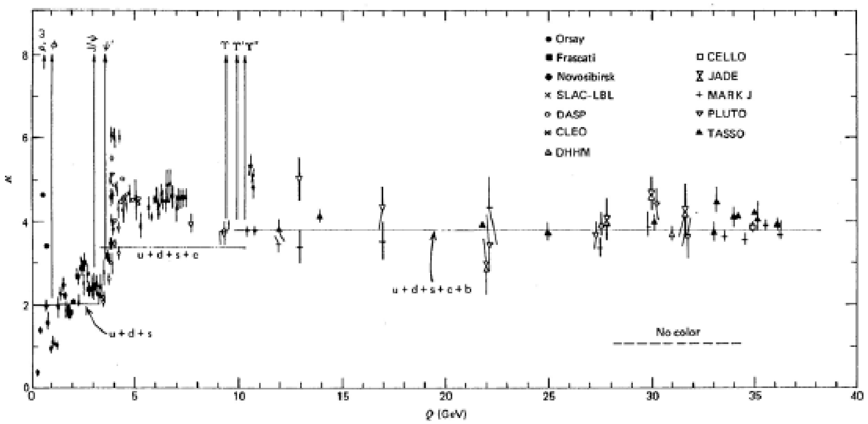
$$R = 3\sum_i Q_i^2 \qquad (3\text{colours})$$

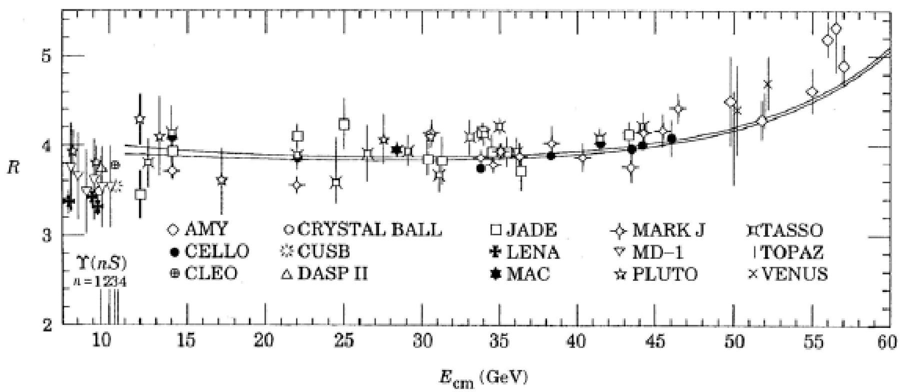
where the sum is over all quark flavours kinematically accessible at centre-of-mass energy, $\sqrt{s}$, of collider.

Energy	Ratio R	
$\sqrt{s} > 2m_s$ ~1 GeV	$3(\frac{4}{9}+\frac{1}{9}+\frac{1}{9})$ u,d,s	= 2
$\sqrt{s} > 2m_c$ ~4 GeV	$3(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9})$ u,d,s,c	= $3\frac{1}{3}$
$\sqrt{s} > 2m_b$ ~10 GeV	$3(..+\frac{1}{9})$ u,d,s,c,b	= $3\frac{2}{3}$
$\sqrt{s} > 2m_t$ ~350 GeV	$3(..+\frac{4}{9})$ u,d,s,c,b,t	= 5

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Data: $\sqrt{s}$ from $0 - 40$ GeV





★ R_μ increases in steps with $\sqrt{s}$

★ $\sqrt{s} < 11\ GeV$ region complicated by resonances: charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$).

★ R_μ Data exclude 'no colour' hypothesis.

STRONG EVIDENCE for COLOUR

Experimental Evidence for Colour

★ R_μ

★ The existence of the $\Omega^-(sss)$

The $\Omega^-(sss)$ is a (L=0) spin-$\frac{3}{2}$ baryon consisting of 3 strange-quarks. The wave-function

$$\psi = s\uparrow s\uparrow s\uparrow$$

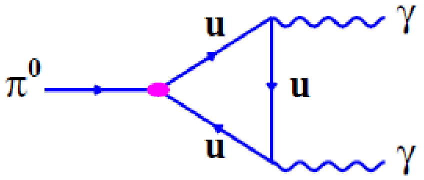
is SYMMETRIC under particle interchange. However quarks are FERMIONS, therefore require an ANTI-SYMMETRIC wave-function, *i.e.* need another degree of freedom, namely COLOUR.

$$\psi = (s\uparrow s\uparrow s\uparrow)\psi_{colour}$$

$$\psi_{colour} = \frac{1}{\sqrt{6}}(rgb+gbr+brg-grb-rbg-bgr)$$

★ $\pi^0 \to \gamma\gamma$ decay rate

Need colour to explain $\pi^0 \to \gamma\gamma$ decay rate.



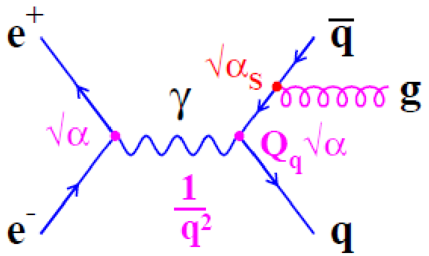
$$\Gamma(\pi^0 \to \gamma\gamma) \propto N^2_{colour}$$

EXPT : $N_{colour} = 2.99 \pm 0.12$

Evidence for Gluons

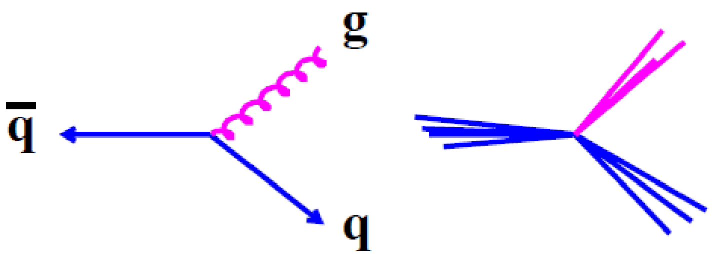
In QED, electrons can radiate photons. In QCD quarks can radiate gluons.

$e^+e^- \rightarrow q\bar{q}g$



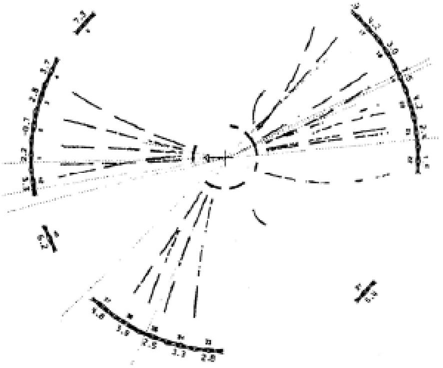
giving an extra factor of $\sqrt{\alpha_S}$ in the matrix element, i.e. an extra factor of α_S in cross section.

In QED we can detect the photons. In QCD we never see free gluons due to confinement. Experimentally detect gluons as an additional jet: 3-Jet Events.

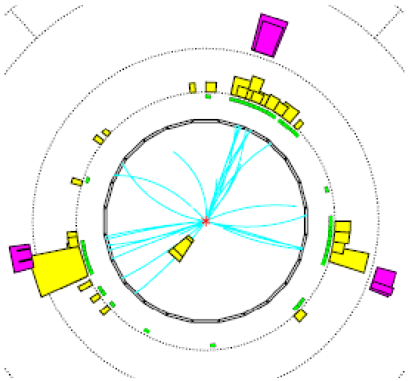


★ Angular distribution of gluon jet depends on gluon spin

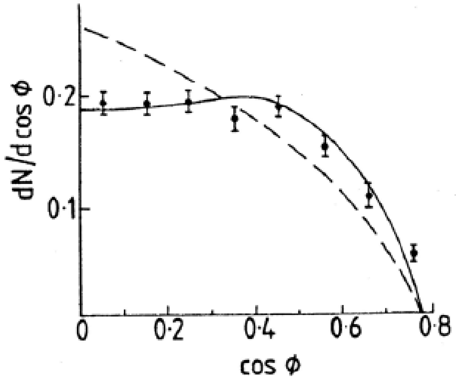
3-Jet Events and Gluon Spin



JADE $\sqrt{s} = 31$ GeV
Direct Evidence for Gluons (1978)



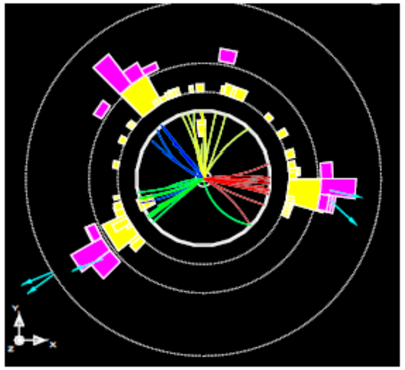
OPAL $\sqrt{s} = 91$ GeV
(1990)



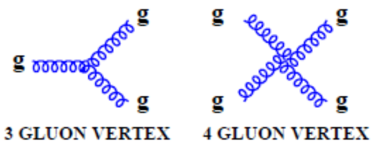
Distribution of the angle, ϕ between the highest energy jet (assumed to be one of the quarks) relative to the flight direction of the other two (in their cms frame). ϕ depends on the spin of the gluon.

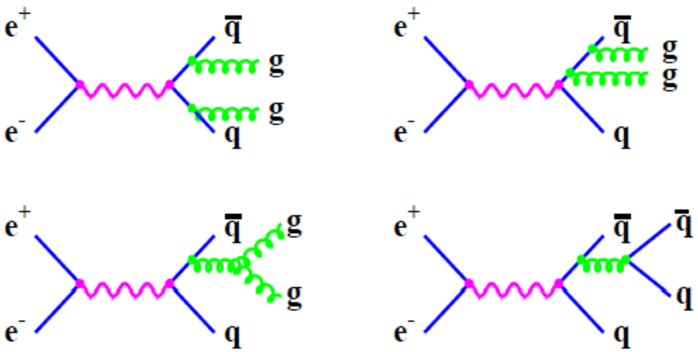
$\Rightarrow$ GLUON is SPIN-1

Gluon Self-Interactions



Direct Evidence for the existence of the the gluon self-interactions from 4-JET events.



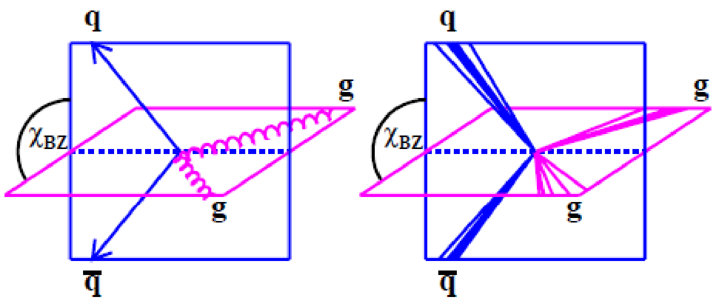


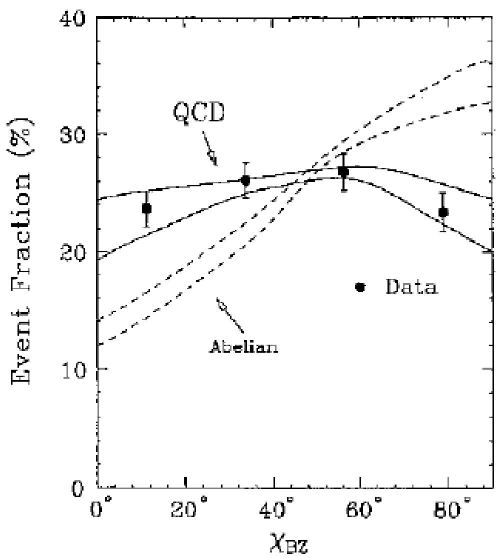
★ Angular distribution of jets is sensitive to existence triple gluon vertex:

- ★ $q\bar{q}g$ vertex consists of 2 spin-$\frac{1}{2}$ quarks and a spin-1 gluon.
- ★ ggg vertex consists of 3 spin-1 gluons, $\therefore$ different angular distribution.

Experimentally:

- ★ Define the two lowest energy jets as the gluons. (gluon jets are more likely to be low energy than quark jets)
- ★ Measure angle between the plane containing the 'quark' jets and the plane containing the 'gluon' jets, χ_{BZ}

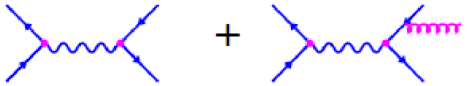




Gluon self-interactions are required to describe the experimental data. Theory without self-interactions (ABELIAN) is inconsistent with observations

Measuring α_S

α_S can be measured in many ways. The cleanest is from R_μ: In practice measure



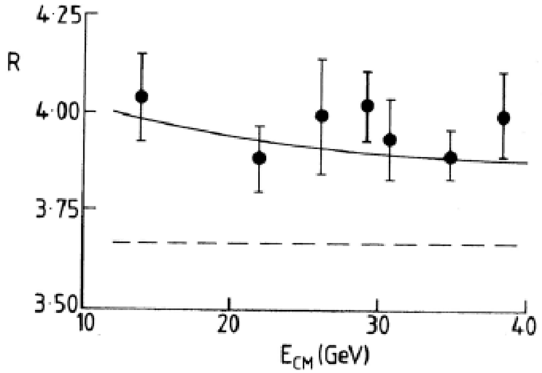
i.e. don't distinguish 2/3 jets. So measure

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\text{not} \quad R_\mu = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

When gluon radiation is included :

$$R_\mu = 3\sum Q_q^2\left(1 + \frac{\alpha_S}{\pi}\right)$$



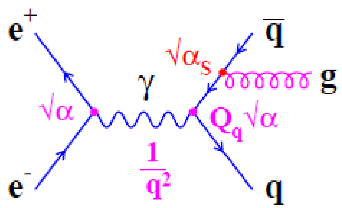
$$\sum_q Q_q^2 = 3\tfrac{2}{3}$$

Therefore $(1 + \frac{\alpha_S}{\pi}) \approx \frac{3.9}{3.66}$

giving $\alpha_S(q^2 = 25^2) \approx 0.20$

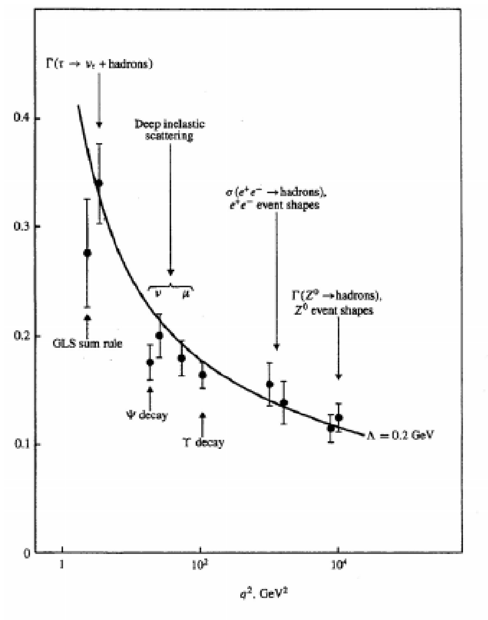
Many other ways to measure α_S ....

_e.g._ 3 jet rate : $e^+e^- \rightarrow q\overline{q}g$



$$\frac{\sigma(3\text{ jet})}{\sigma(2\text{ jet})} = \frac{\sigma(q\overline{q}g)}{\sigma(q\overline{q})} \propto \alpha_S$$

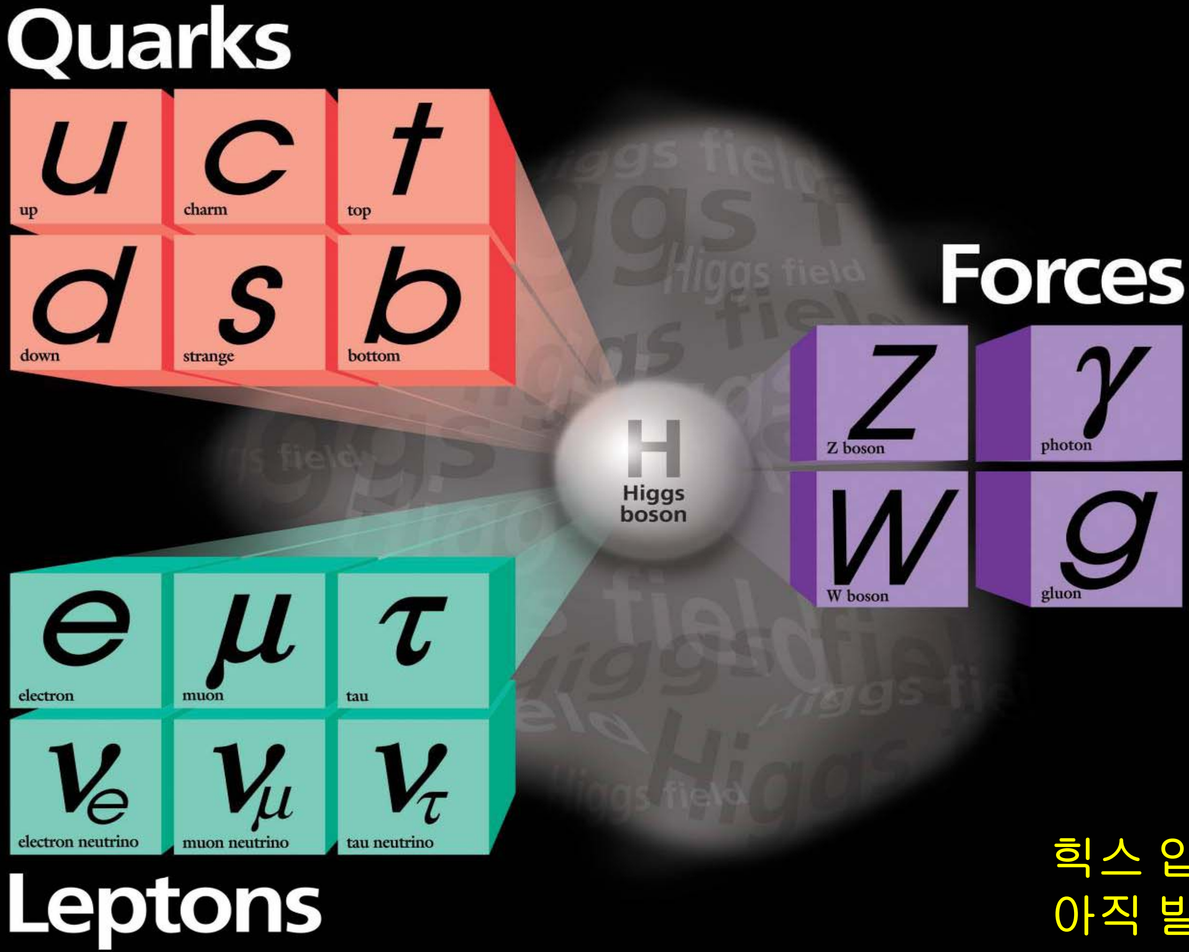
α_S Summary



Summary of α_S measurements

α_S RUNS !

세상은 무엇으로 만들어져 있는가? 표준모형

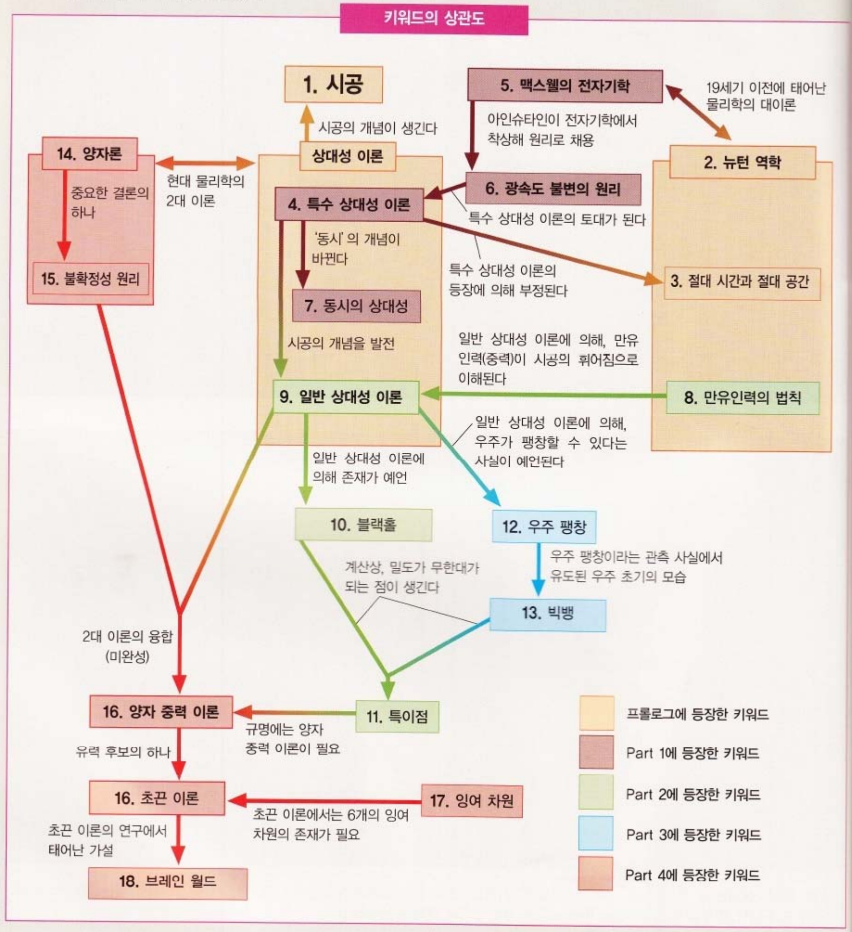


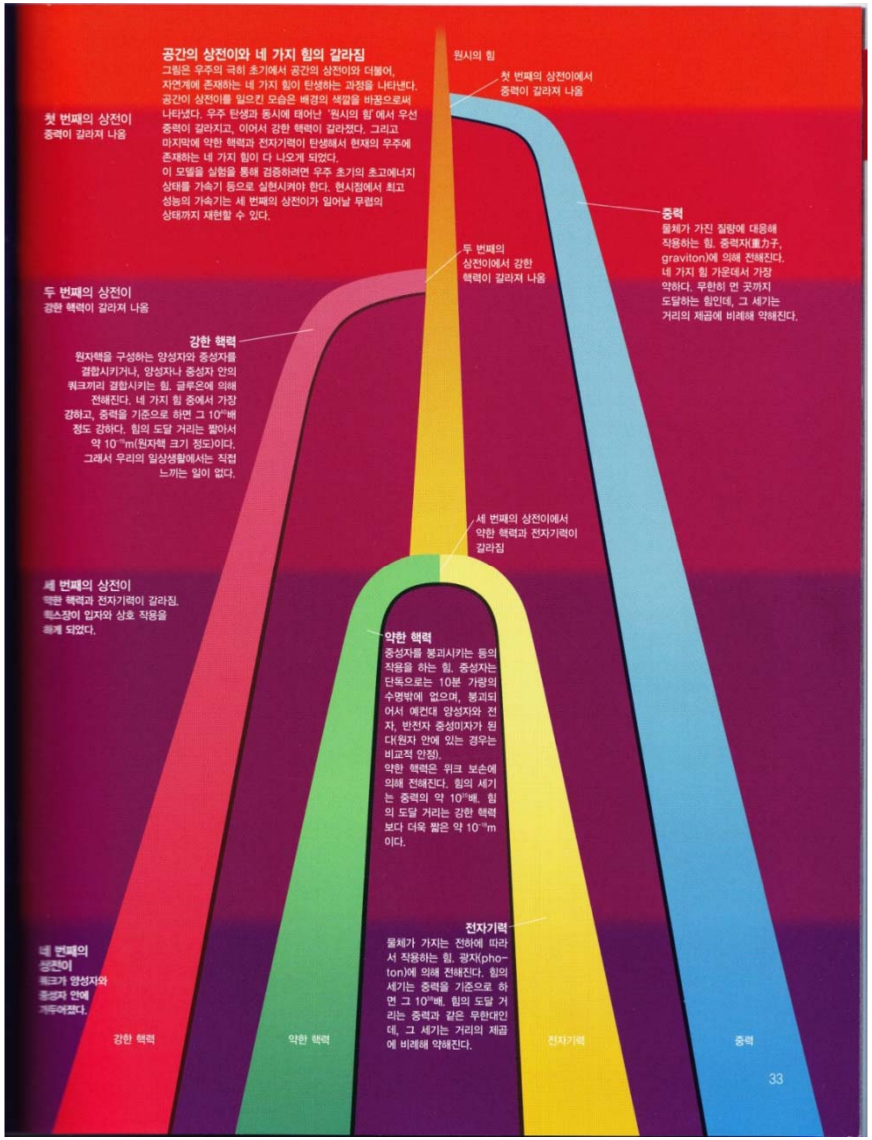
힉스 입자는
아직 발견 안됐음

주요 사항의 관계를 한눈에 알 수 있다

'시공'을 이해하기 위한 키워드

마지막으로 특집 기사에 등장하는 중요한 키워드를 정리한다. 왼쪽 페이지에서는 키워드끼리의 관계성을 도해하고, 오른쪽 페이지에서 키워드를 해설했다.





Thank you.

Quantum Chromodynamics

Quantum Chromodynamics (QCD) is the theory of the strong interaction.

QCD is a non-abelian gauge theory invariant under SU(3) and as a result:

a) The interaction is governed by massless spin 1 objects called "gluons".

b) Gluons couple only to objects that have "color": quarks and gluons

c) There are three different charges ("colors"): red, green, blue.

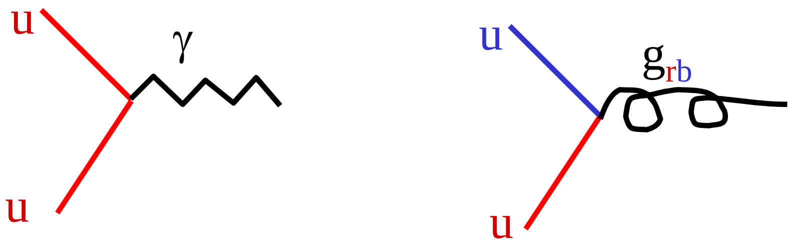
Note: in QED there is only one charge (electric).

M&S 6.3, 7.1

d) There are eight different gluons.

gluon exchange can change the color of a quark but not its flavor.

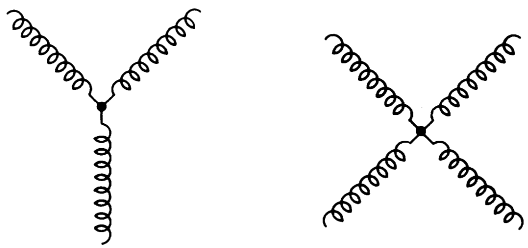
e.g. a red u-quark can become a blue u-quark via gluon exchange.



e) Since gluons have color there are couplings involving 3 and 4 gluons.

Note: In QED the 3 and 4 photon couplings are absent since the photon

does not have an electric charge.



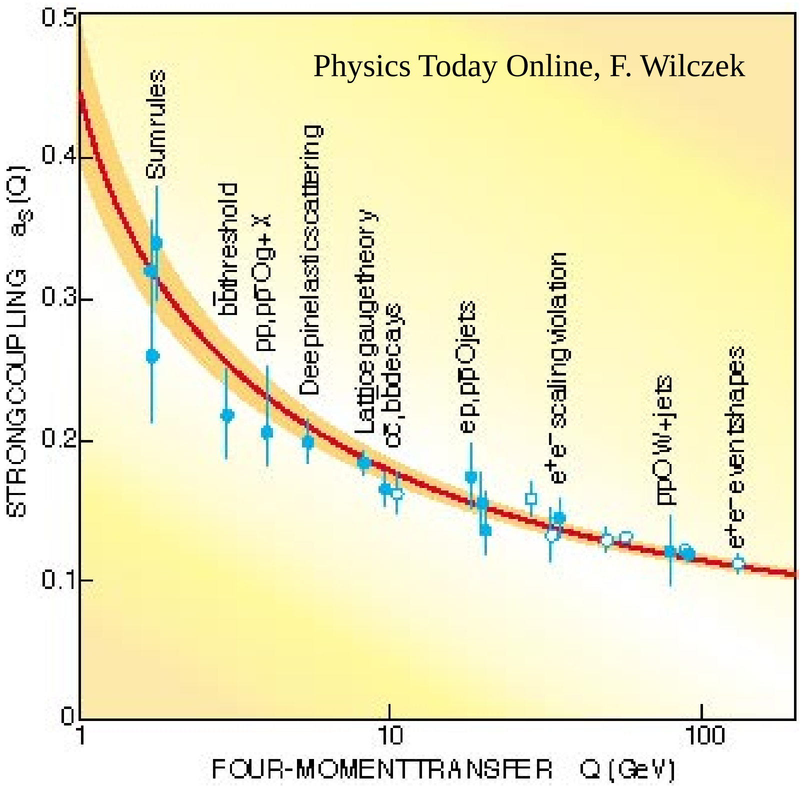
Quantum Chromodynamics

There are several interesting consequences of the SU(3), non-abelian nature of QCD:

a) Quarks are confined in space.
   We can never "see" a quark the way we can an electron or proton.
   Explains why there is no experimental evidence for "free" quarks.

b) All particles (mesons and baryons) are color singlets.
   This "saves" the Pauli Principle.
   In the quark model the Δ^{++} consists of 3 up quarks in a totally symmetric state.
   Need something else to make the total wavefunction anti-symmetric $\Rightarrow$ color!

c) Asymptotic freedom.
   The QCD coupling constant changes its value ("runs") dramatically as function of energy.
   As a result quarks can appear to be "free" when probed by high energy (virtual) γ's and yet be tightly bound into mesons and baryons (low energy).



d) In principle, the masses of mesons and baryons can be calculated using QCD.
   But in reality, very difficult to calculate (almost) anything with QCD.

QED Vs QCD

QED is an abelian gauge theory with U(1) symmetry: $\psi'(\bar{x},t) = e^{-ief(\bar{x},t)}\psi(\bar{x},t)$

QCD is a non-abelian gauge theory with SU(3) symmetry: $\psi'(\bar{x},t) = e^{-ig\sum_{i=1}^{8}\frac{\lambda_i\omega_i(\bar{x},t)}{2}}\psi(\bar{x},t)$

Both are relativistic quantum field theories that can be described by Lagrangians:

QED: $L = \bar{\psi}(i\gamma^u\partial_u - m)\psi + e\bar{\psi}\gamma^u A_u\psi - \frac{1}{4}F^{uv}F_{uv}$

m=electron mass
ψ=electron spinor

electron-γ interaction

A_u=photon field (1)
$F_{uv}=\partial_u A_v - \partial_v A_u$

QCD: $L = \bar{q}_{jk}(i\gamma^u\partial_u - m)q_{jk} + g(\bar{q}_{jk}\gamma^u\lambda_a q_{jk})G^a_u - \frac{1}{4}G^a_{uv}G^{uv}_a$

m=quark mass
j=color (1,2,3)
k=quark type (1-6)
q=quark spinor

quark-gluon interaction

gluon-gluon interaction (3g and 4g)

G^a_u=gluon field (a=1-8)
$G^a_{uv}=\partial_u G^a_v - \partial_v G^a_u - g f_{abc} G^b_u G^c_v$

$[\lambda_a, \lambda_b] = i f_{abc}\lambda_c$

λ_a's (a=1-8) are the generators of SU(3).
λ_a's are 3x3 traceless hermitian matrices.
See M&S Eq. 6.36b for a representation.

f_{abc} are real constants (256)
$f_{abc} \equiv$ structure constants of the group
(M&S Table C.1)

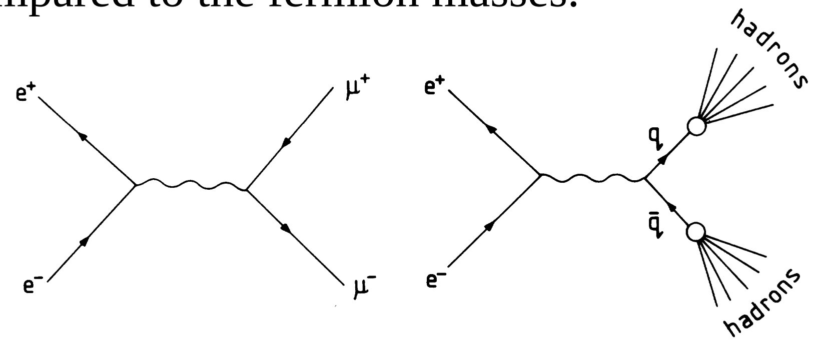
6.1 Quark and Antiquark pair production in e+e-

What is the Evidence for Color?

One of the most convincing arguments for color comes from a comparison of the cross sections for the two processes:

$$e^+e^- \to \mu^+\mu^- \quad \text{and} \quad e^+e^- \to q\bar{q}$$

If we forget about how the quarks turn into hadrons then the graphs (amplitudes) for the two reactions only differ by the charge of the final state fermions (muons or quarks). We are assuming that the CM energy of the reaction is large compared to the fermion masses.



If color plays no role in quark production then the ratio of cross sections should only depend on the charge (Q) of the quarks that are produced.

$$R \equiv \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_{i=1}^{n} Q_i^2$$

However, if color is important for quark production then the above ratio should be multiplied by the number of colors (3).

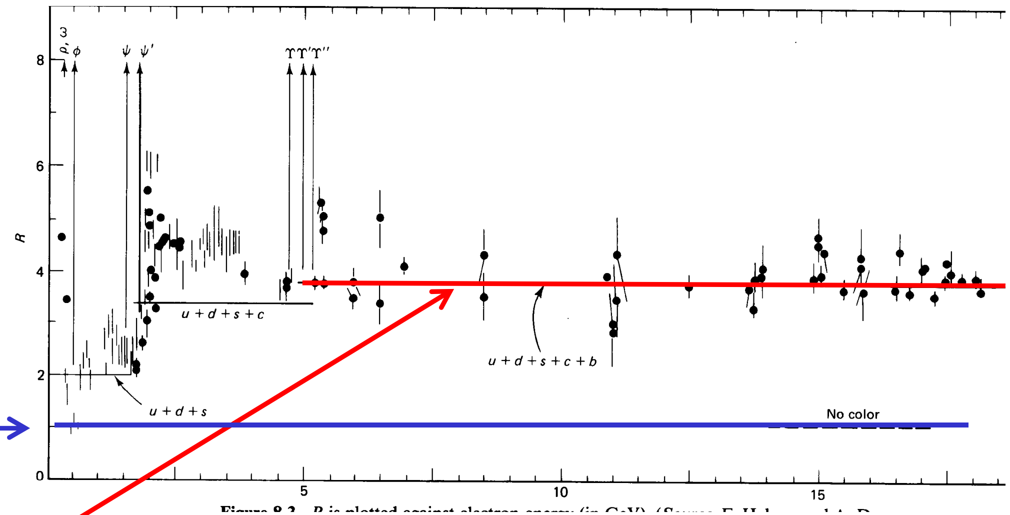
$$R \equiv \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_{i=1}^{n} Q_i^2$$

For example, above b-quark threshold but below top-quark threshold we would expect:

$$R = [(-\frac{1}{3})^2 + (\frac{2}{3})^2 + (-\frac{1}{3})^2 + (\frac{2}{3})^2 + (-\frac{1}{3})^2] = \frac{11}{9}$$

or, if color exists:

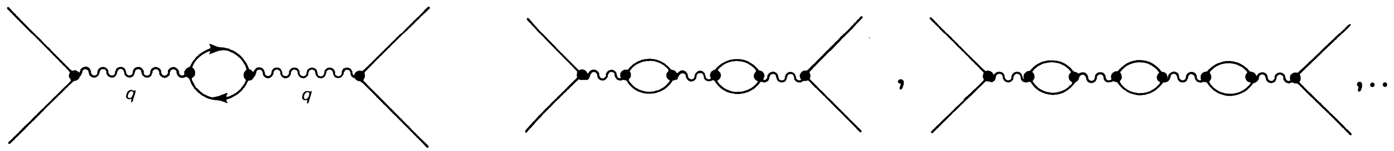
$$R = 3[(-\frac{1}{3})^2 + (\frac{2}{3})^2 + (-\frac{1}{3})^2 + (\frac{2}{3})^2 + (-\frac{1}{3})^2] = \frac{33}{9} = 3.67$$



6.3 Nonabelian Nature of QCD

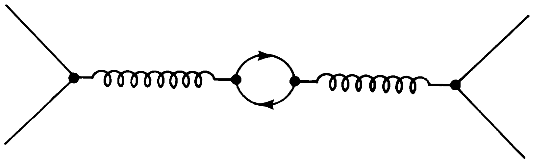
QED Vs QCD

In QED higher order graphs with virtual fermion loops play an important role in determining the strength of the interaction (coupling constant) as a function of distance scale (or momentum scale).



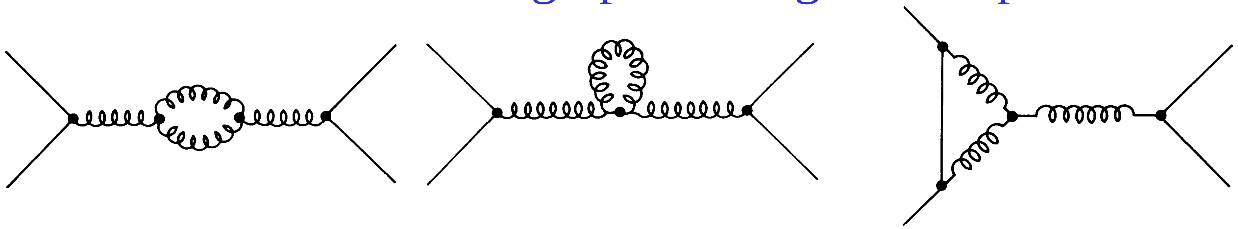
Examples of graphs with virtual fermion loops coupling to virtual photons.

In QCD similar higher order graphs with virtual loops also play an important role in determining the strength of the interaction as a function of distance scale.



A graph with a virtual fermion loop coupling to virtual gluons.

In QCD we can also have a class of graphs with gluon loops since the gluons carry color.



Because of the graphs with gluon loops the QCD coupling constant behaves differently than the QED coupling constant at short distances.

QED Vs QCD

For both QED and QCD the effective coupling constant α depends on the momentum (or distance) scale that it is evaluated at:

$$\alpha(p^2) = \frac{\alpha(0)}{1 - X(p^2)}$$

$\alpha(0)$=fine structure constant $\approx$1/137

For QED it can be shown that:

$$X(p^2) = \left(\sum_{i=1}^{N_f} \left(\frac{q_i}{e}\right)^2\right) \frac{\alpha(\mu^2)}{3\pi} \ln\left(\frac{p^2}{\mu^2}\right)$$

QED: $X(p^2)>0$

Here N_f is the number of fundamental fermions with masses below ½|p| and μ is the mass of the heaviest fermion in the energy region being considered.

The situation for QCD is very different than QED. Due to the non-abelian nature of QCD we find:

$$X(p^2) = \frac{\alpha_s(\mu^2)}{12\pi} \ln\left(\frac{p^2}{\mu^2}\right)[2N_f - 11N_c]$$

Here N_f is the number of quark flavors with masses below ½|p|, μ is the mass of the heaviest quark in the energy region being considered, and N_c is the number of colors (3). For 6 flavors and 3 colors $2N_f$-$11N_c$ < 0 and therefore $\alpha(p^2)$ decreases with increasing momentum (or shorter distances).

The force between quarks decreases at short distances and increases as the quarks move apart!!

asymptotic freedom

QCD and Confinement

The fact that gluons carry color and couple to themselves leads to asymptotic freedom:

strong force decreases at very small quark-quark separation.

strong force increases as quarks are pulled apart.

But does asymptotic freedom lead to quark confinement?

YES

Quark confinement in an SU(3) gauge theory can only be proved analytically for the 2D case of 1 space+1 time.

However, detailed numerical calculations show that even in 4D (3 space+time) quarks configured as mesons and baryons in color singlet states are confined.

In SU(3) there are two conserved quantities (two diagonal generators).

For $SU(3)_{color}$ these quantities are:

Y^c = "color hypercharge"

I_3^c = the 3rd component of "color isospin"

The quark color hypercharge and isospin assignments are (M&S P143)

	I_3^c	Y^c
r	1/2	1/3
g	-1/2	1/3
b	0	-2/3

Y^c and I_3^c change sign for anti-quarks

By assuming that hadrons are color singlets we are requiring $Y^c = I_3^c = 0$

$$\text{mesons}: \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

$$\text{baryons}: \frac{1}{\sqrt{6}}(r_1 g_2 b_3 - g_1 r_2 b_3 + b_1 r_2 g_3 - b_1 g_2 r_3 + g_1 b_2 r_3 - r_1 b_2 g_3)$$

anti-symmetric under color exchange

QCD and Confinement

The color part of the quark wavefunction can be represented by:

$$r=\begin{pmatrix}1\\0\\0\end{pmatrix}\quad g=\begin{pmatrix}0\\1\\0\end{pmatrix}\quad b=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

Using the M&S representation for the SU(3) generators (eq 6.36b) we have:

$$I_3^c=\frac{1}{2}\lambda_3=\frac{1}{2}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\qquad Y^c=\frac{1}{\sqrt{3}}\lambda_8=\frac{1}{3}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

As shown in M&S eq. 6.33 the only quark and anti-quark combinations with $Y^c = I_3^c = 0$ are of the form:

$$(3q)^p(q\bar{q})^n\ \ (p,n\geq 0)$$

Our baryons are the states with p=1, n=0 and our mesons are states with p=0, n=1.
States such as qqqq and qq are forbidden by the singlet (confinement) requirement.
However, the following states are allowed:

$$qqqqqq \quad p=2, n=0$$
$$q\bar{q}q\bar{q} \quad p=0, n=2$$
$$qqqq\bar{q} \quad p=1, n=1$$

What about bound states of gluons? "glueballs"

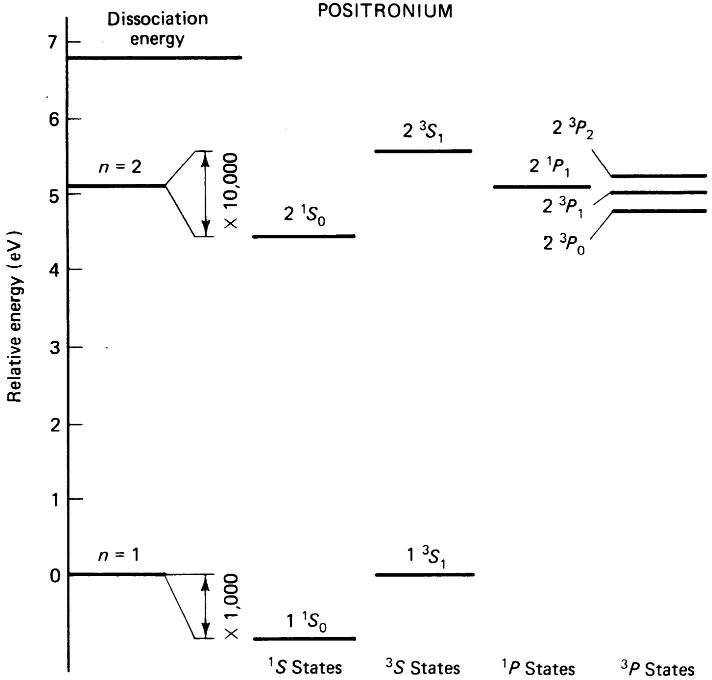
There have been many experimental searches for quark bound states other than the conventional mesons and baryons. To date there is only evidence for the existence of the conventional mesons and baryons.

Quark Anti-Quark Bound States and QCD

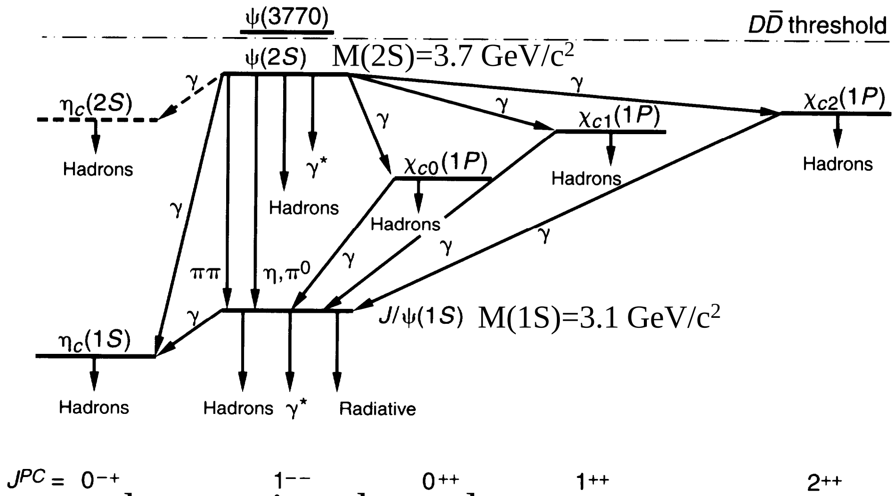
The spectrum of bounds states of heavy $q\bar{q}$ (=$c\bar{c}$ or $b\bar{b}$) states can be calculated in analogy with positronium (e^+e^-). The "QCD" calculations work best for non-relativistic (i.e. heavy states) where the potential can be approximated by:

$$V(r) = -\frac{a}{r} + br$$

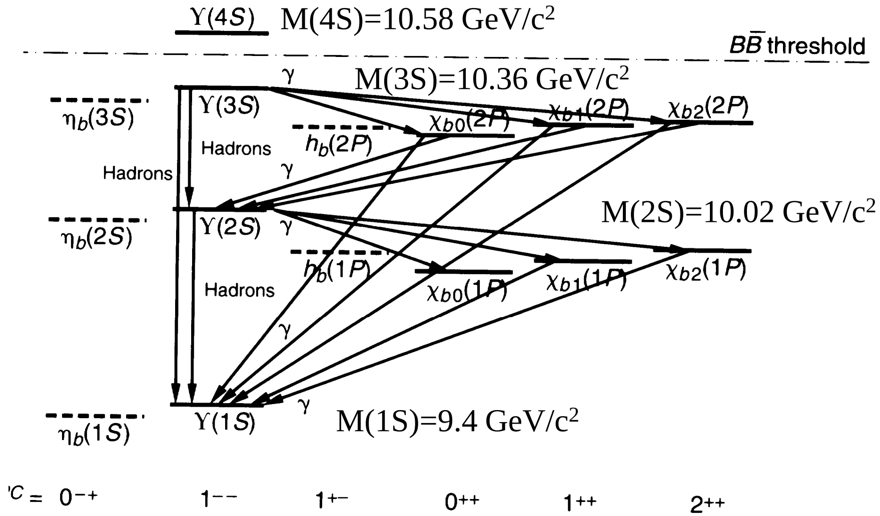
Both *a* and *b* are constants calculated from fitting data or a model.



positronium bound state spectrum



charmonium bound state spectrum



bottomonium bound state spectrum

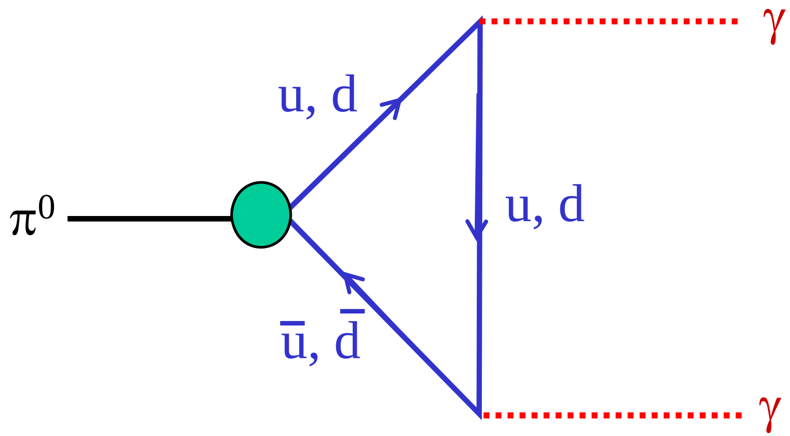
QCD, Color, and the decay of the π^0

1949-50 The decay $\pi^0 \rightarrow \gamma\gamma$ calculated and measured by Steinberger.

1967 Veltman calculates the π^0 decay rate using modern field theory and finds that the π^0 does not decay!

1968-70 Adler, Bell and Jackiw "fix" field theory and now π^0 decays but decay rate is off by factor of 9.

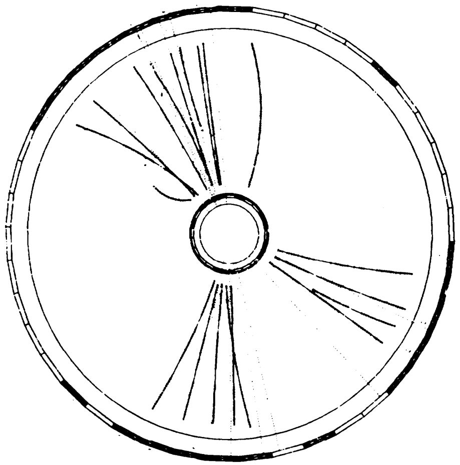
1973-4 Gell-Mann and Fritzsch (+others) use QCD with 3 colors and calculate the correct π^0 decay rate.



Triangle Diagram
Each color contributes one amplitude. Three colors changes the decay rate by 9.

QCD and Jets

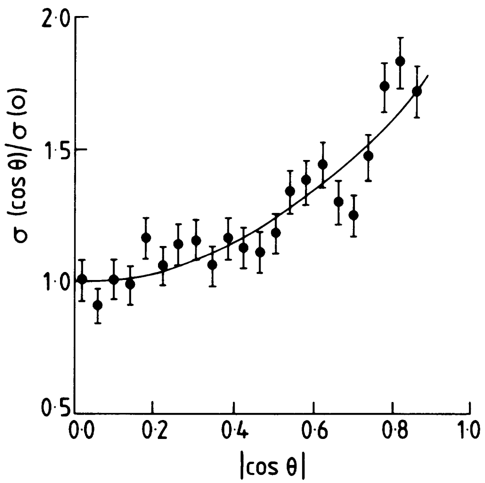
QCD predicts that we will not see isolated quarks. However, the hadrons that we do see in a high energy collision should have some "memory" of its parent quark (or gluon). About 25 years ago someone used the term "jet" to describe the collimation of a group of hadrons as they are Lorentz boosted along the direction of the parent quark.



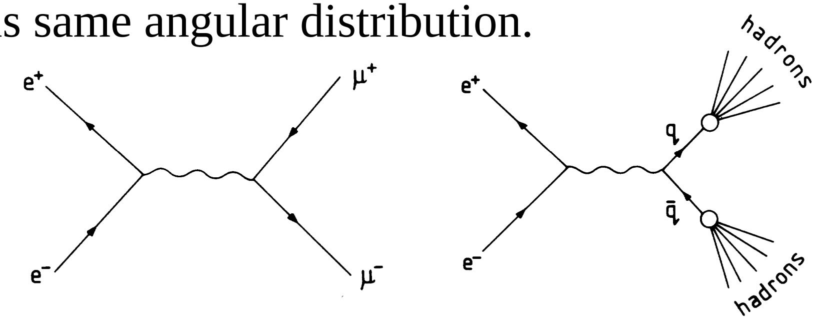
An example of $e^+e^- \to$hadrons with three jets.The lines represent the trajectories of charged tracks in the magnetic field of the central detector of the JADE experiment. The beams are ⊥ to the page.

Again we can compare muon pair production to quark production. The angular distribution (cosθ) with respect to the beam line ("z-axis") for the μ^- is:

$$\frac{d\sigma}{d\cos\theta} = A(1+\cos^2\theta) \text{ for } e^+e^- \to \mu^+\mu^-$$



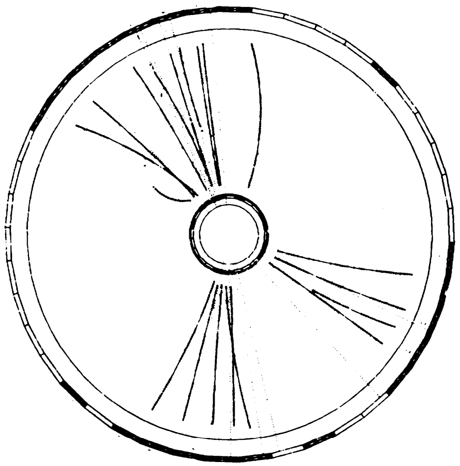
Since the parent quarks are also spin 1/2 fermions we would expect the quarks to have the same angular distribution as muons. We would also expect the momentum vector of the quark jets to have this same angular distribution.



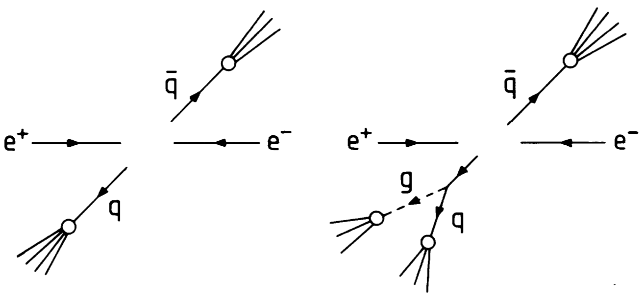
Data from "2 jet" events. Solid line is $1+\cos^2\theta$. Good agreement!!

QCD and Jets

Perhaps even more interesting than the “two jet” events are “three jet” events.
One of the jets is due to a gluon forming hadrons, the other two jets are from the parent quarks.

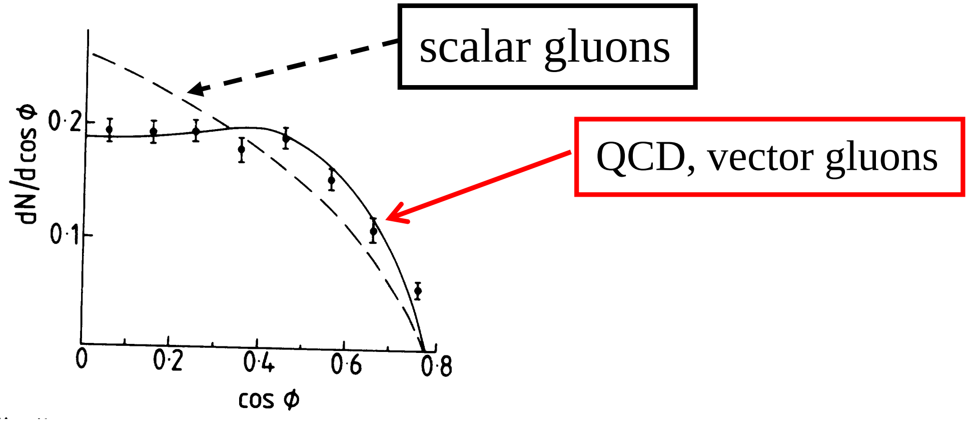


A “three jet” event



Cartoon comparison of “two jet” and “three jet” events.

Using QCD it is possible to calculate the angular distribution (cosθ) of the three jets with respect to the “z-axis”. Data (from the TASSO experiment) is in very good agreement with the QCD.



Applications of QCD

The "OZI" rule: in the 1960's it was noted that the ϕ meson decayed (strongly) into kaons more often than expected:

$BR(\phi \to K^+K^-) = 49.1 \pm 0.6\%$
$BR(\phi \to K_L K_S) = 34.1 \pm 0.5\%$
$BR(\phi \to \pi^+\pi^-\pi^0) = 15.6 \pm 1.2\%$

$\phi = s\bar{s}$
mass=1020 MeV/c^2
$I^G(J^{PC}) = 0^-(1^{--})$

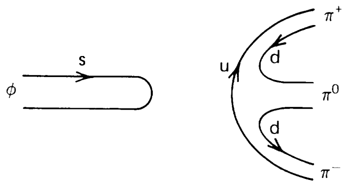
M&S 6.1

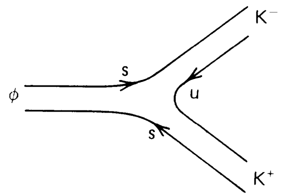
Naively, one would expect the ϕ to preferentially decay into π's over K's since there is much more phase space available for this decay.
Phase space is related to the mass difference between parent and daughters:

$\Delta m(\phi \to \pi^+\pi^-\pi^0) = (1020\text{-}415) = 605 \text{ MeV}/c^2$
$\Delta m(\phi \to K^+K^-) = (1020\text{-}990) = 30 \text{ MeV}/c^2$

$\phi \to \pi^+\pi^-$ is forbidden by "G-Parity" conservation

Okubo, Zweig, and Iizuka (OZI) independently suggested a rule: strong interaction processes where the final states can only be reached through quark anti-quark annihilation are suppressed.



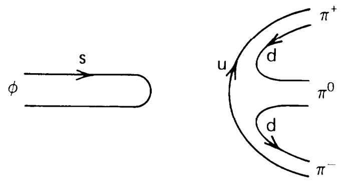


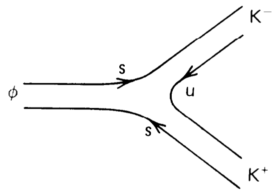
The gluons are not drawn in these diagrams.

Therefore according to their rule $\phi \to \pi^+\pi^-\pi^0$ should be suppressed relative to $\phi \to KK$.
QCD provides an explanation of this behavior.

OZI Rule

QCD explains the OZI rule as follows:
For decays involving quark anti-quark annihilation the initial and final states are connected by gluons. Since gluons carry color and mesons are colorless there must be more than one gluon involved in the decay.





The gluons involved in the decay must combine in a way to conserve all strong interaction quantum numbers. For example, in terms of charge conjugation (C):

two gluon state: C = +1

three gluon state: C = -1

Vector mesons such as the ϕ, ψ, and Y have C=-1 and thus their decays involving quark anti-quark annihilation must proceed through three gluon exchange.

Since these mesons are fairly massive (>1 GeV) the gluons must be energetic (“hard”) and therefore due to asymptotic freedom, the coupling constant for each gluon will be small. Thus the amplitude for $\phi \to \pi^+ \pi^- \pi^0$ will be small since it depends on α_s^3.
Although the amplitude for $\phi \to KK$ also involves gluon exchange it will not be suppressed as these gluons are low energy (“soft”) and therefore α_s is large here.

Although QCD explains the OZI rule it is still very difficult (impossible?) to perform precise rate calculations since the processes are in the regime where α_s large.

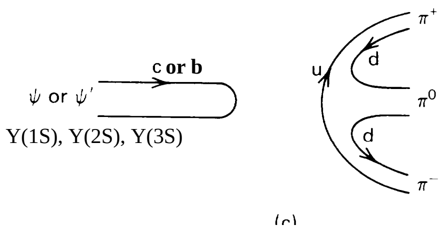
OZI Rule and “Onium”

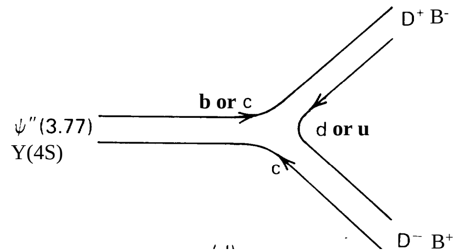
Both charmonium (ψ, ψ′, ψ″) and bottomonium (Υ(1S), Υ(2S), Υ(3S), Υ(4S)) provide examples of the OZI rule in action.

The lower mass charmonium (ψ, ψ′) and bottomonium states (Υ(1S), Υ(2S), Υ(3S)) differ in one important way from the ϕ. While the ϕ is massive enough to decay into strange mesons (ϕ→KK), the ψ and ψ′ are below threshold to decay into charmed mesons while the Υ(1S), Υ(2S), Υ(3S) are below threshold to decay into B-mesons.

$$\psi = c\bar{c}$$

$$\Upsilon = b\bar{b}$$





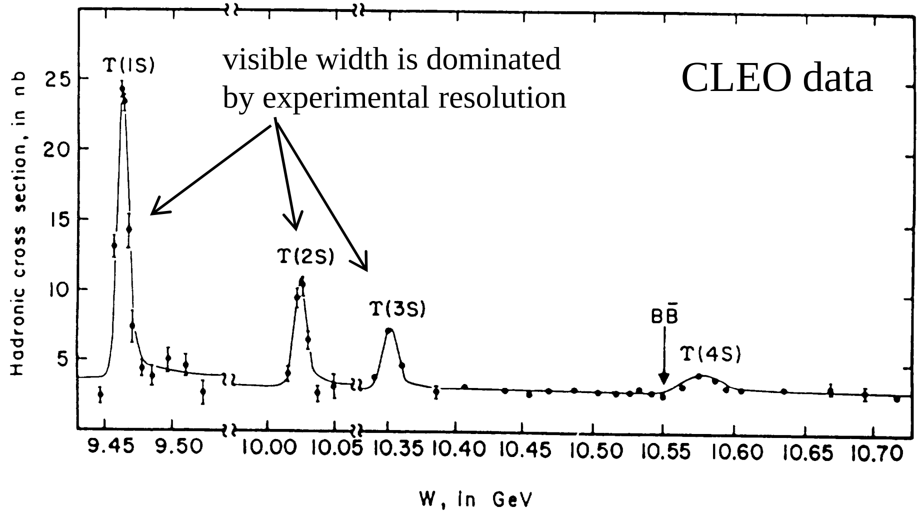
Therefore the decays of the (ψ, ψ′) and (Υ(1S), Υ(2S), Υ(3S)) have to proceed through the annihilation diagram and as a result these states live ≈250-1000X longer than expected without the OZI suppression.

Lifetime of state = $(\text{width})^{-1}$

$\tau = \Gamma^{-1}$

Γ(ψ)=87 keV Γ(ψ″)=24 MeV

Γ(Υ(3S))=26 keV Γ(Υ(4S))=21 MeV



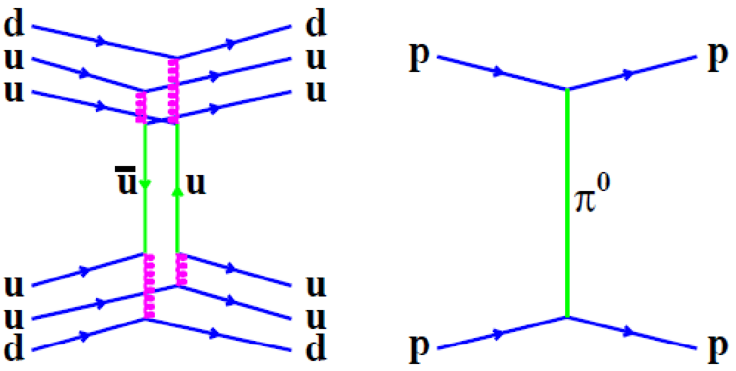
Nucleon-Nucleon Interactions

★ Bound qqq states (e.g. Protons and Neutrons) are COLOURLESS (COLOUR SINGLETS).

★ They can only interact via COLOURLESS intermediate states - i.e. not by single gluons. (conservation of colour charge)

★ Interact by exchange of PIONS

★ One possible diagram shown below :



★ Nuclear potential is YUKAWA potential with

$$V(\tilde{r}) = -\frac{g^2}{4\pi r}e^{-m_\pi r}$$

★ Short range force : range $\sim (m_\pi)^{-1}$

$$\begin{aligned} \text{Range} \quad R &= (0.140\ \mathrm{GeV})^{-1} \\ &= 7\ \mathrm{GeV}^{-1} \\ &= 7\hbar c/(1.6 \times 10^{-10})\ m \\ &= 1.4\ fm \end{aligned}$$

References

- Richard Kass
- Thompson