고차항 양자중력과 복잡계 등에 응용

남순건 (경희대학교)

KISTI, 2011년 3월 30일

Physics of New Massive Gravity and New Type Black Holes

Soonkeon Nam[1]

with Jong-Dae Park[1], Sang-Heon Yi[2] and Yongjoon Kwon[1]

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March. 30 2011 / KISTI

Outline

1. Introduction
2. New Massive Gravity
   - Gravity in 3D
   - New Massive Gravity
   - Black Holes in NMG
   - Extended New Massive Gravity
3. New Type Black Holes
   - Black Hole Solutions in (E)NMG
   - Warped AdS BH
   - New Type Black Hole
4. QNM of New type black holes
5. Conclusion

Gravity in Three Dimensions

- The Problem of Quantum Gravity
- Gravity in Three Dimensions
- Renormalizablity and Unitarity

Quantum Gravity

Einstein gravity as a theory of interacting massless spin 2 fields around a flat *Minkowski* background

- The theory is non-renormalizable
- We can add higher derivative terms (Stelle, 1977)

$$\mathcal{L} \sim R + a(R_{\mu\nu\lambda\rho})^2 + b(R_{\mu\nu})^2 + cR^2$$

- The theory is Renormalizable but not Unitary

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Why Three Dimensional Gravity

- Quantum Gravity can be studied
- Higher derivative terms can be added
- Dimensional reduction of Higher Dimensional Gravity Theories
- There are black hole solutions : BTZ BH
- Much easier to study AdS_3/CFT_2

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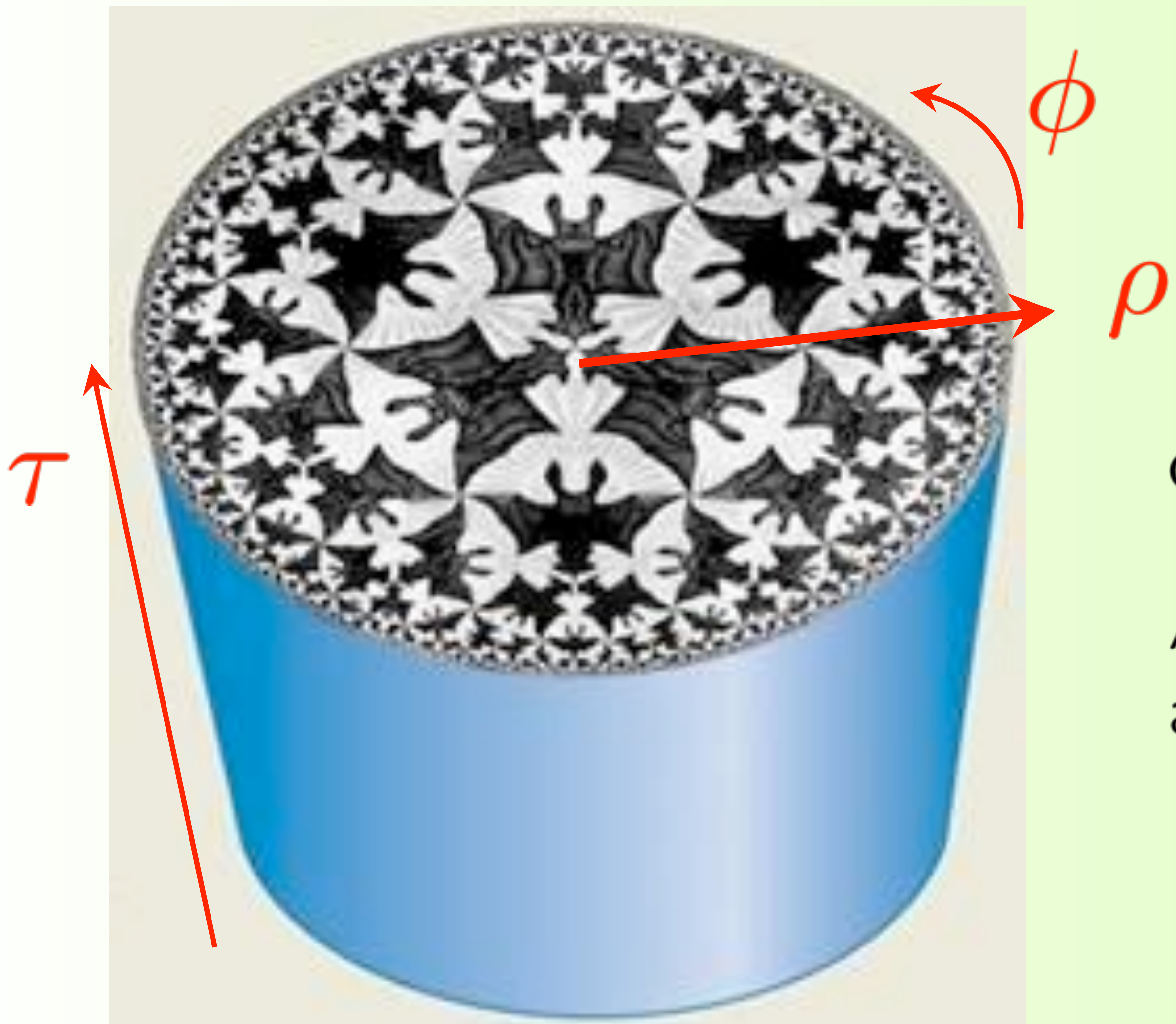
AdS

$$ds^2_{AdS} = \ell^2(-\cosh^2\rho\, d\tau^2 + \sinh^2\rho\, d\phi^2 + d\rho^2)$$



Constant curvature $\mathcal{R} < 0$.

Asymptotic boundary at $\rho = \infty$.

3D gravity

$$S = I_{EH}$$

3D gravity

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Fairly boring... No black holes.

3D gravity

$$S = I_{EH} - \frac{1}{\ell^2} I_{CC}$$

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Achucarra and Townsend

Witten

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Achucarra and
Townsend

Witten

It is Chern–Simons theory!

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It is gauge theory with no local dofs!

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HOT!

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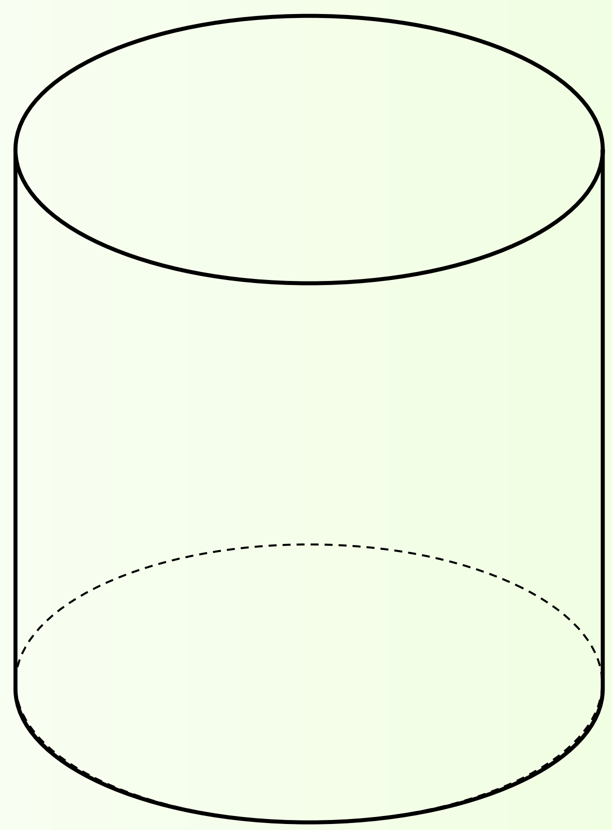
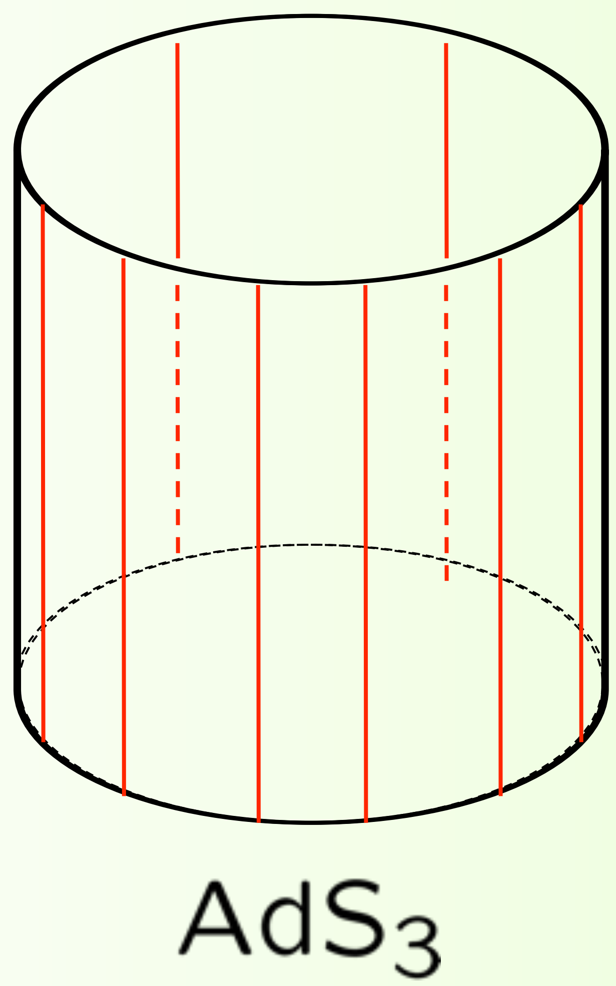
AdS-CFT



AdS_3

AdS-CFT



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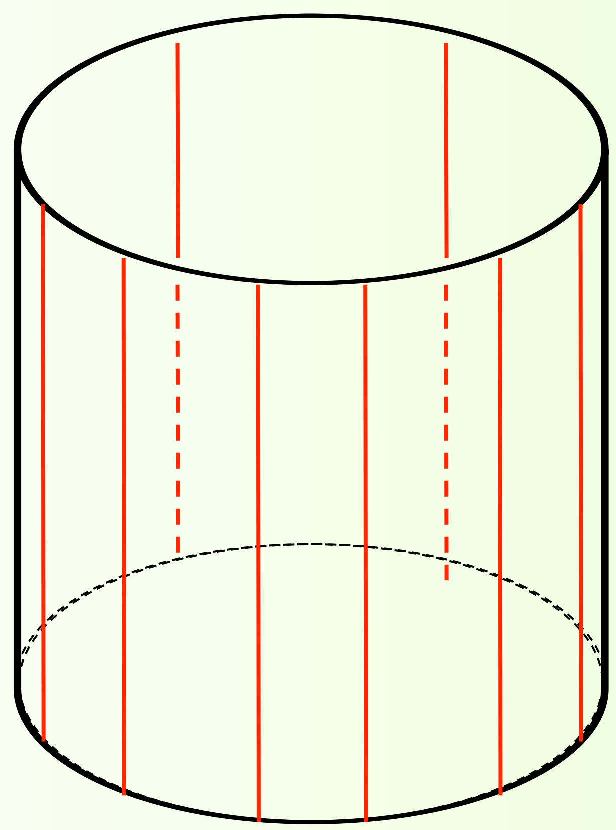

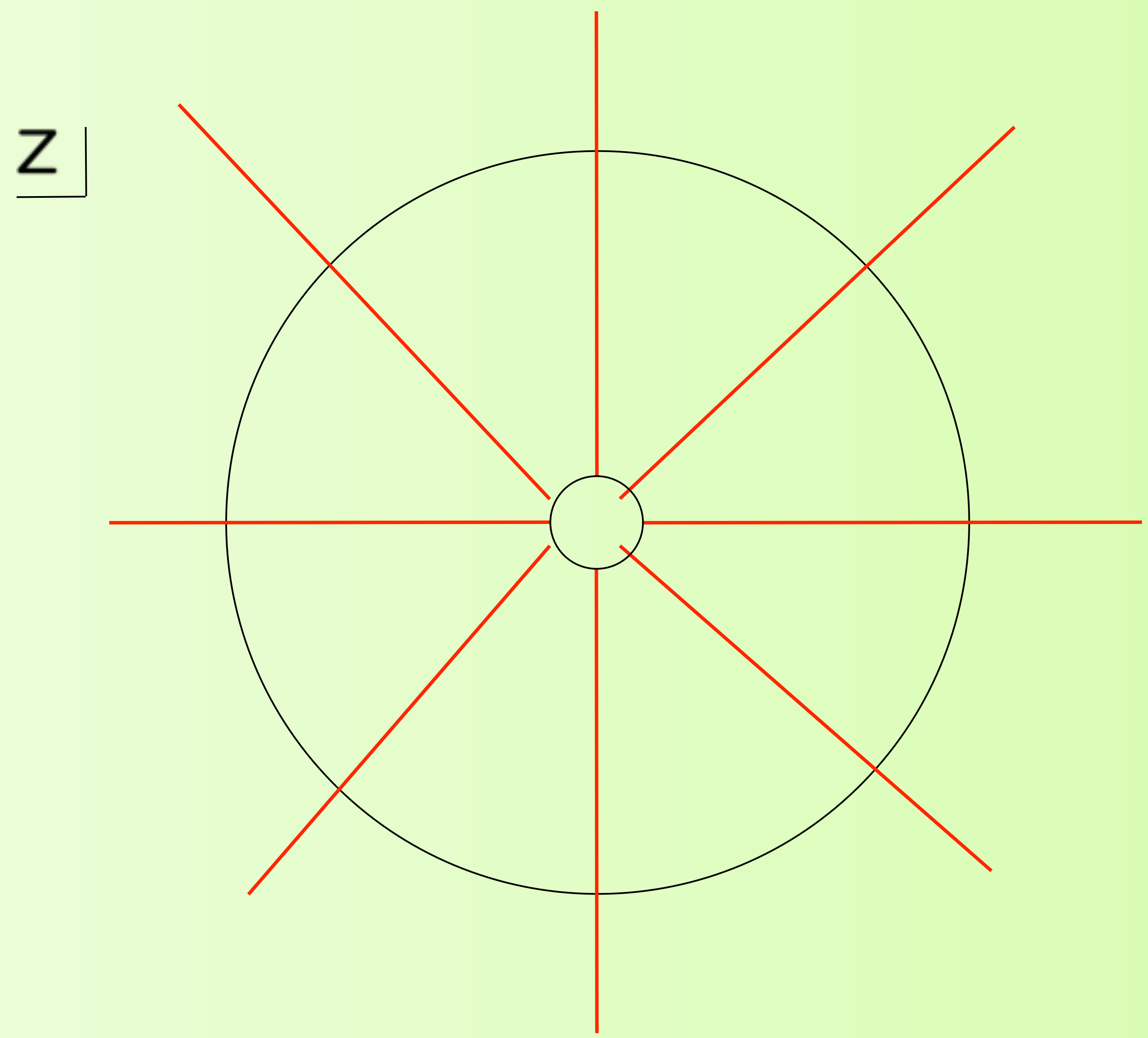

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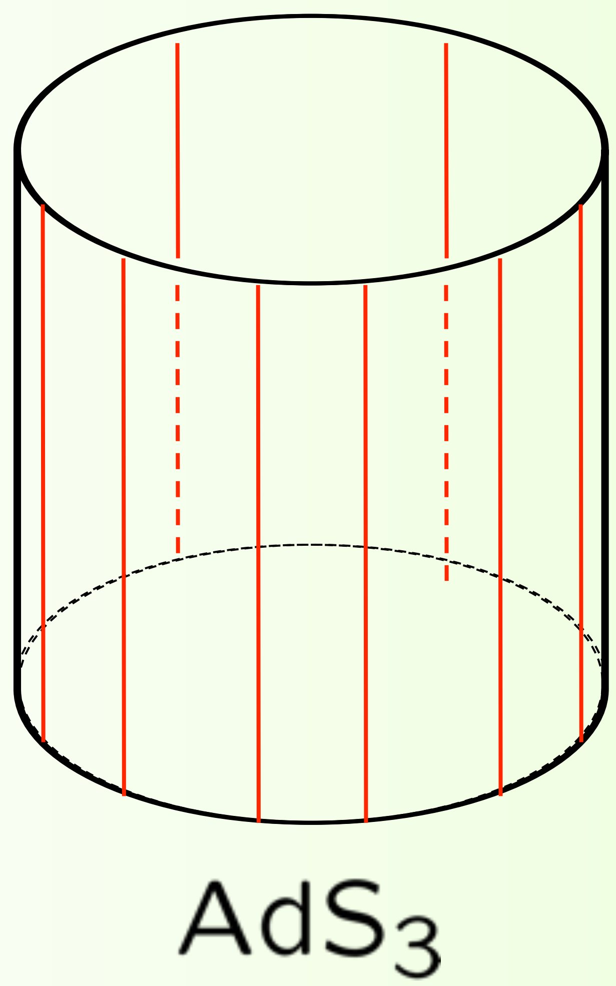

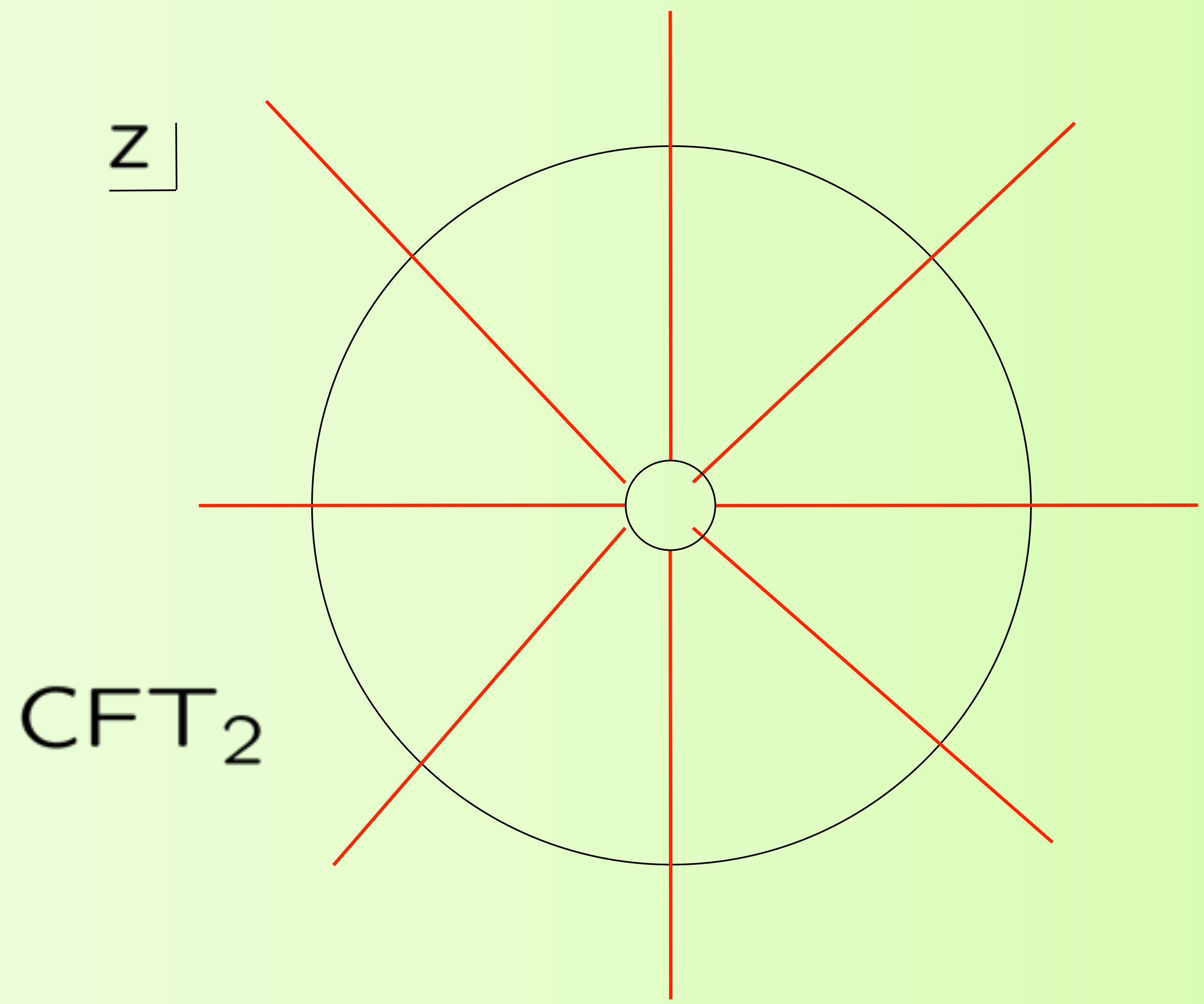

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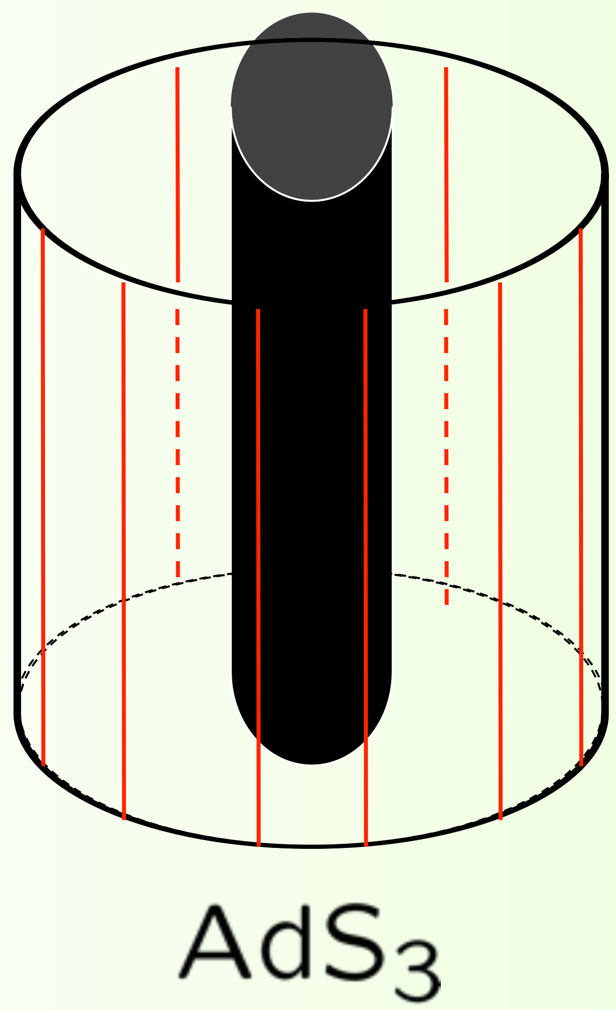

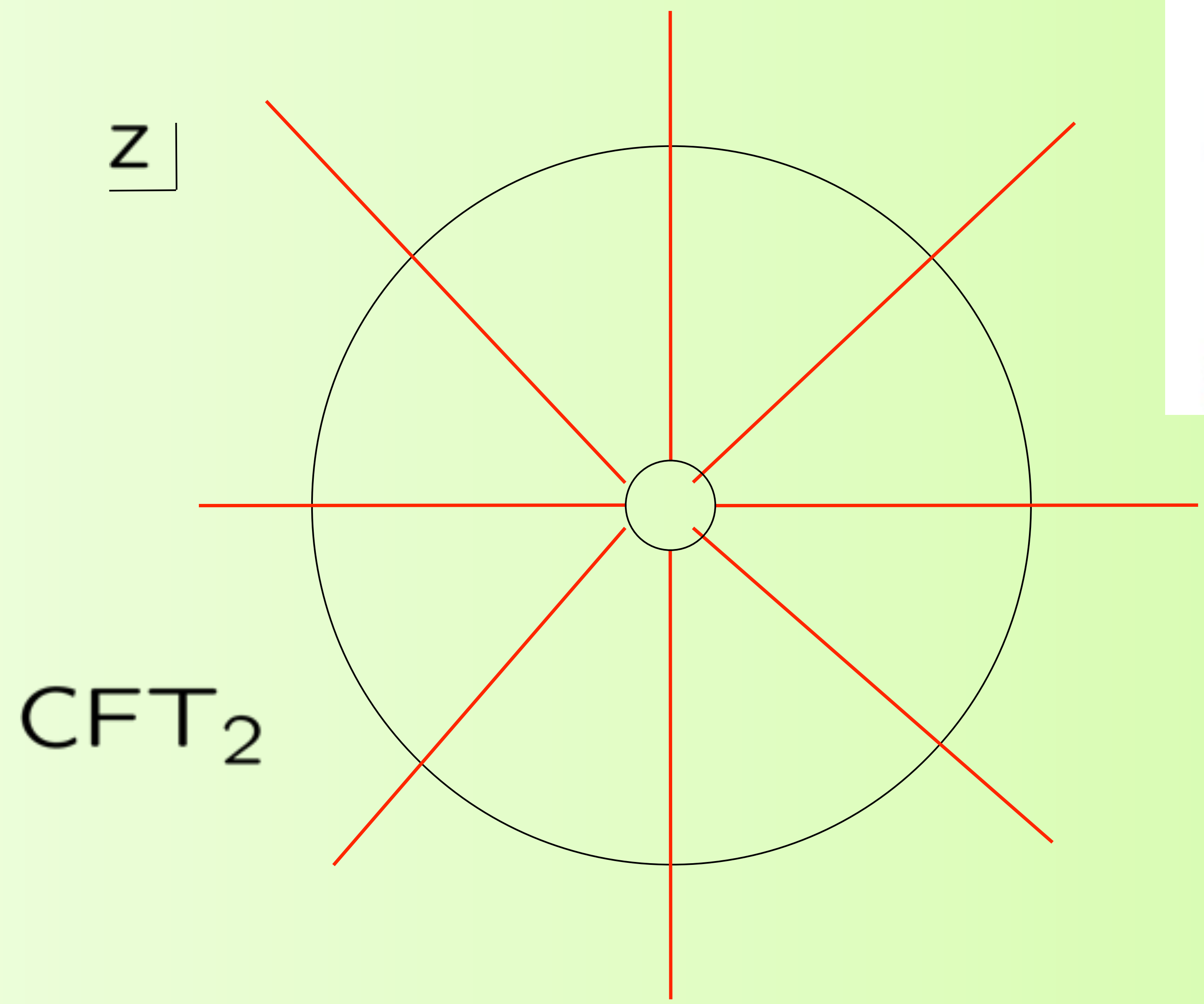

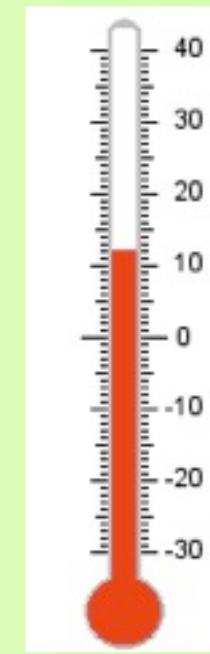

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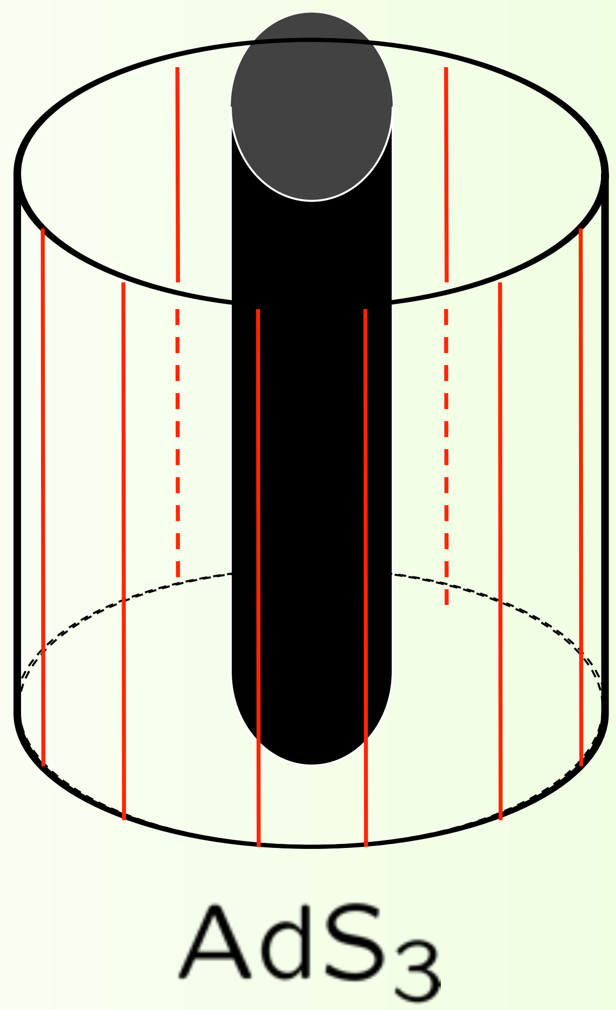

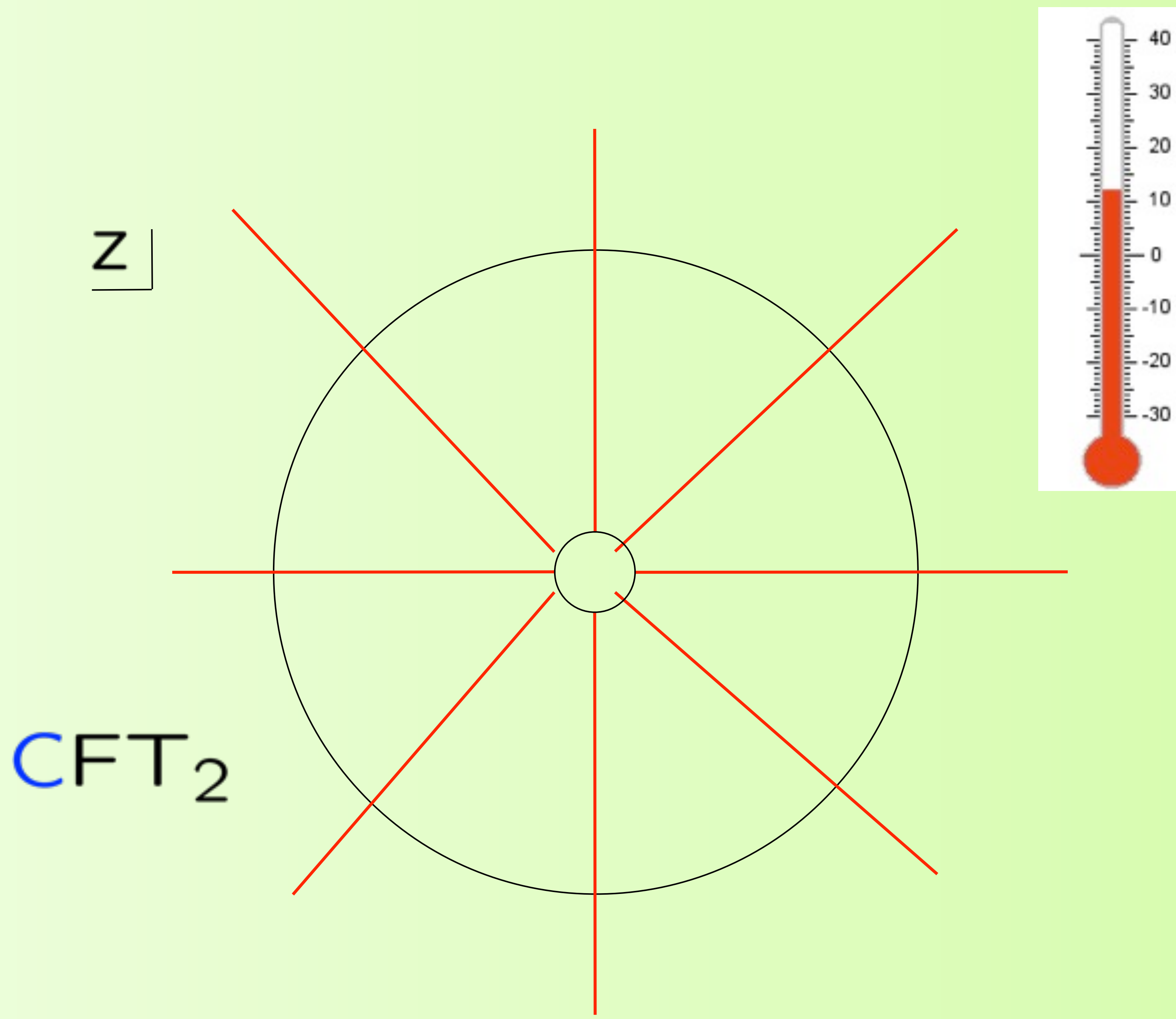

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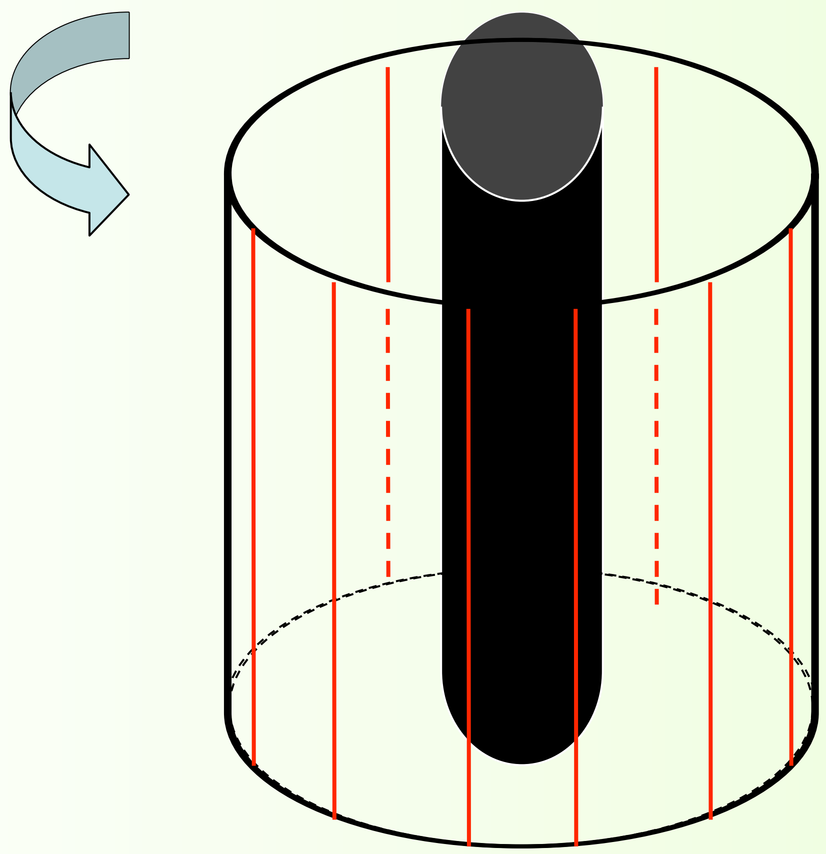

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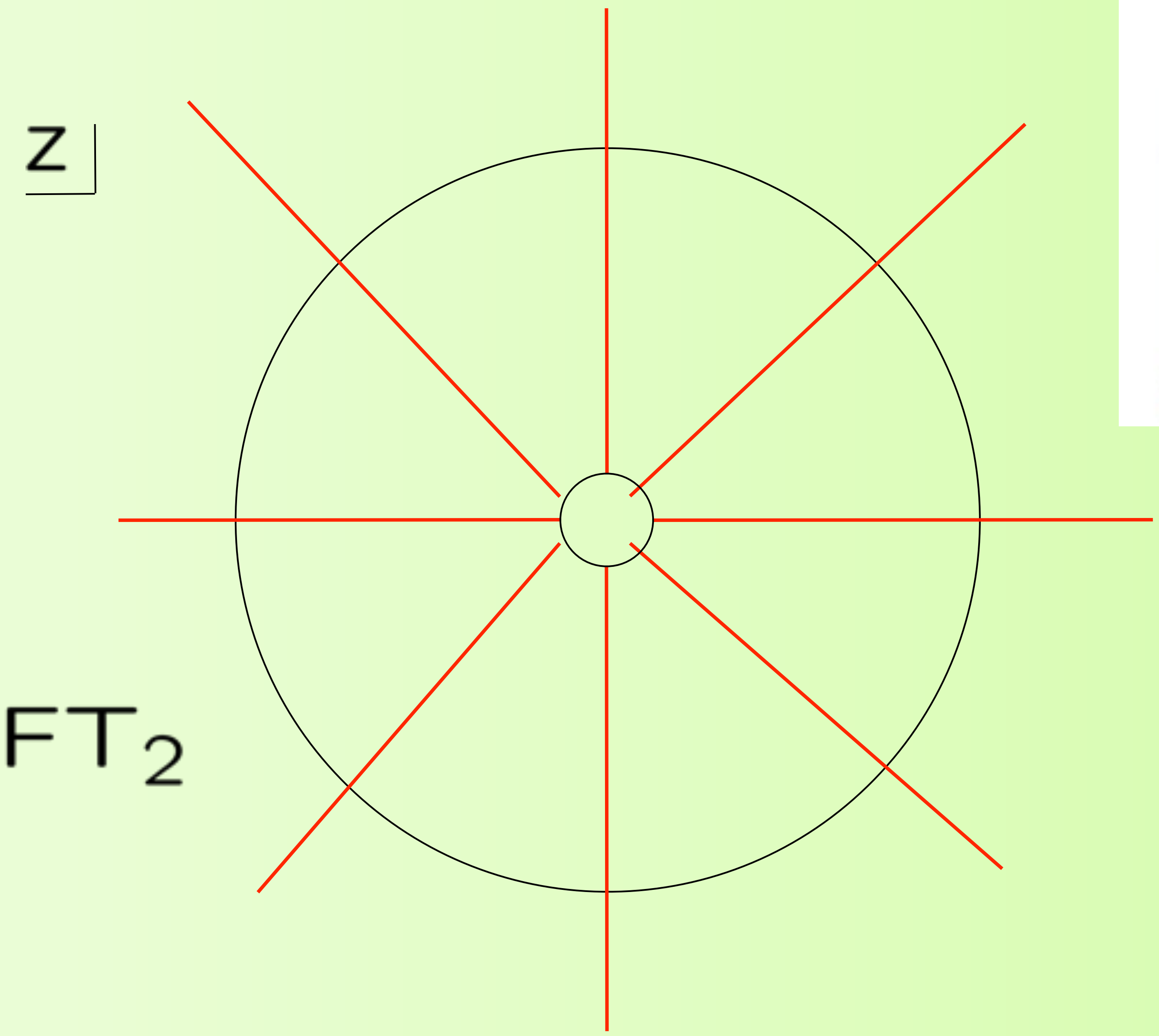
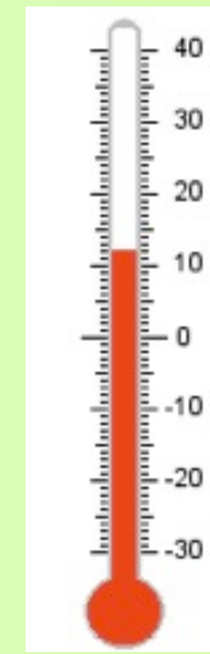
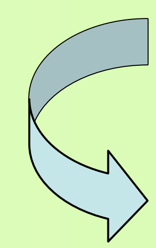


CFT_2



$$L_0, L_m \sim J_3, J_\pm$$



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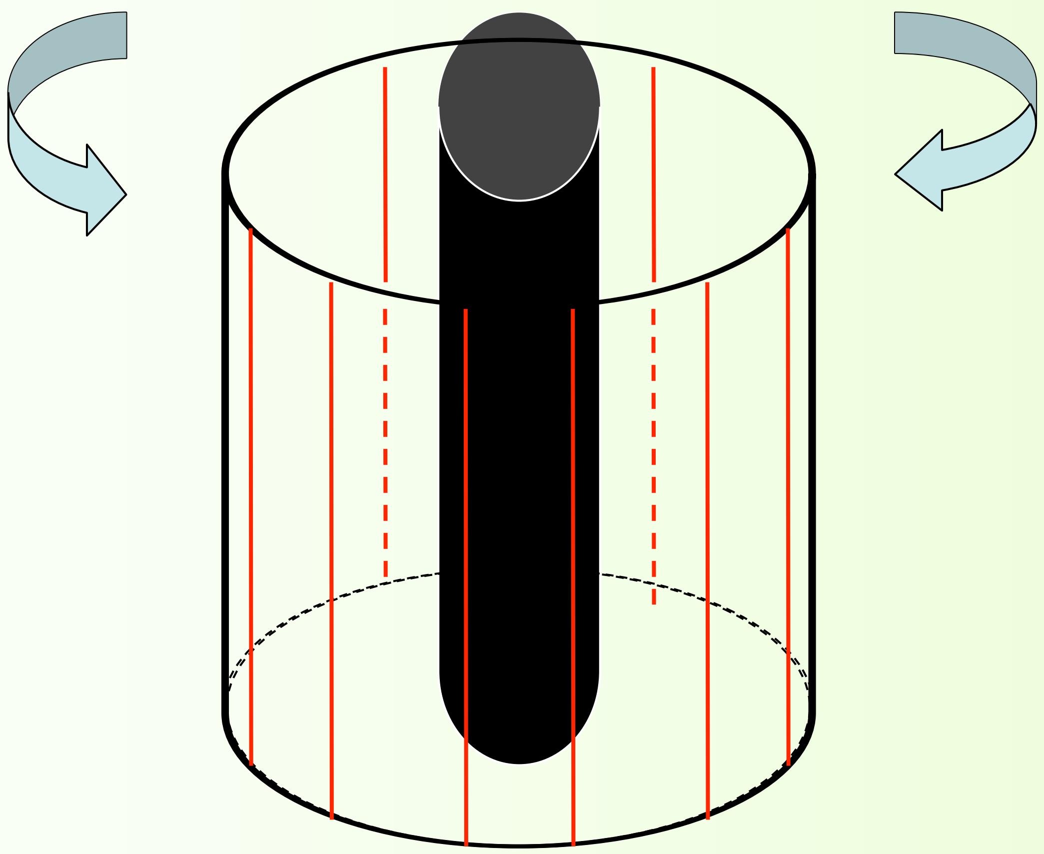

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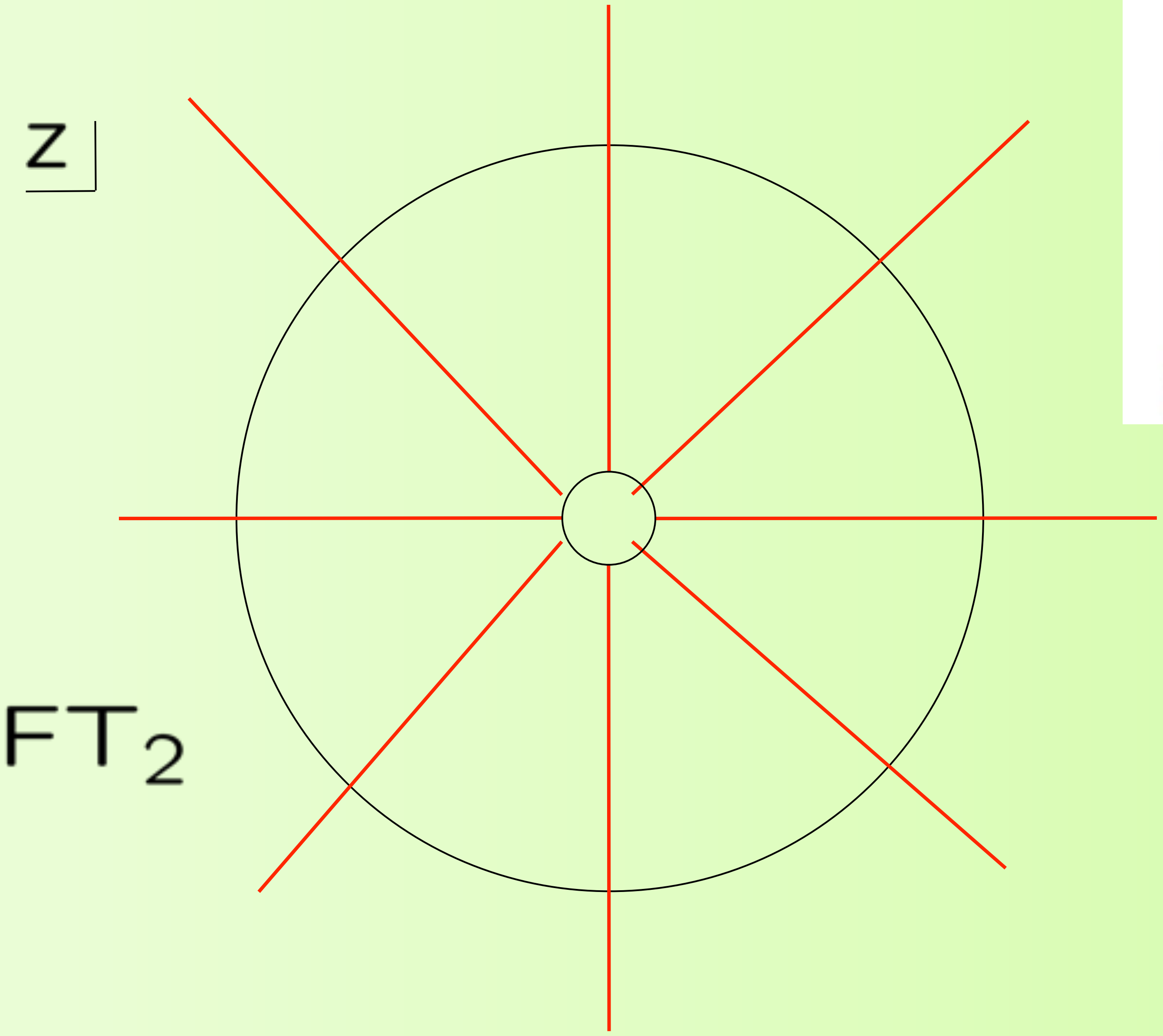
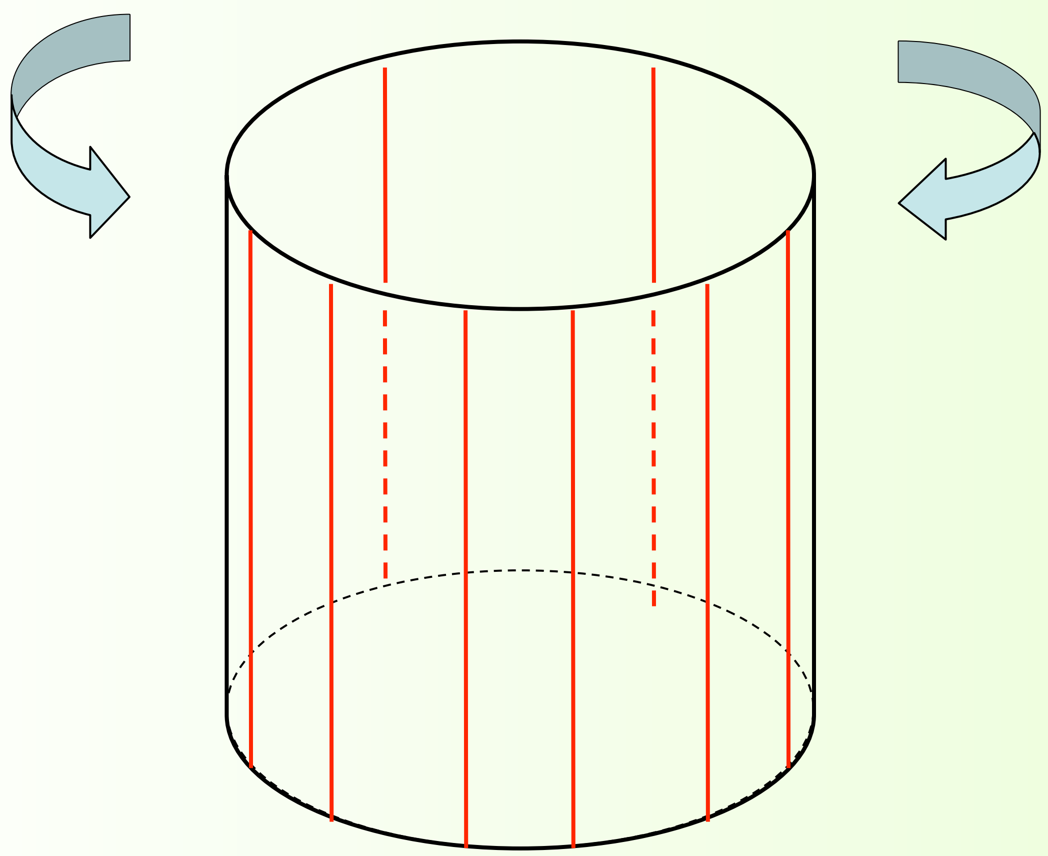


CFT_2

$$L_0, L_m \sim J_3, J_\pm$$

$$\bar{L}_0, \bar{L}_m \sim \tilde{J}_3, \tilde{J}_\pm$$

CFT



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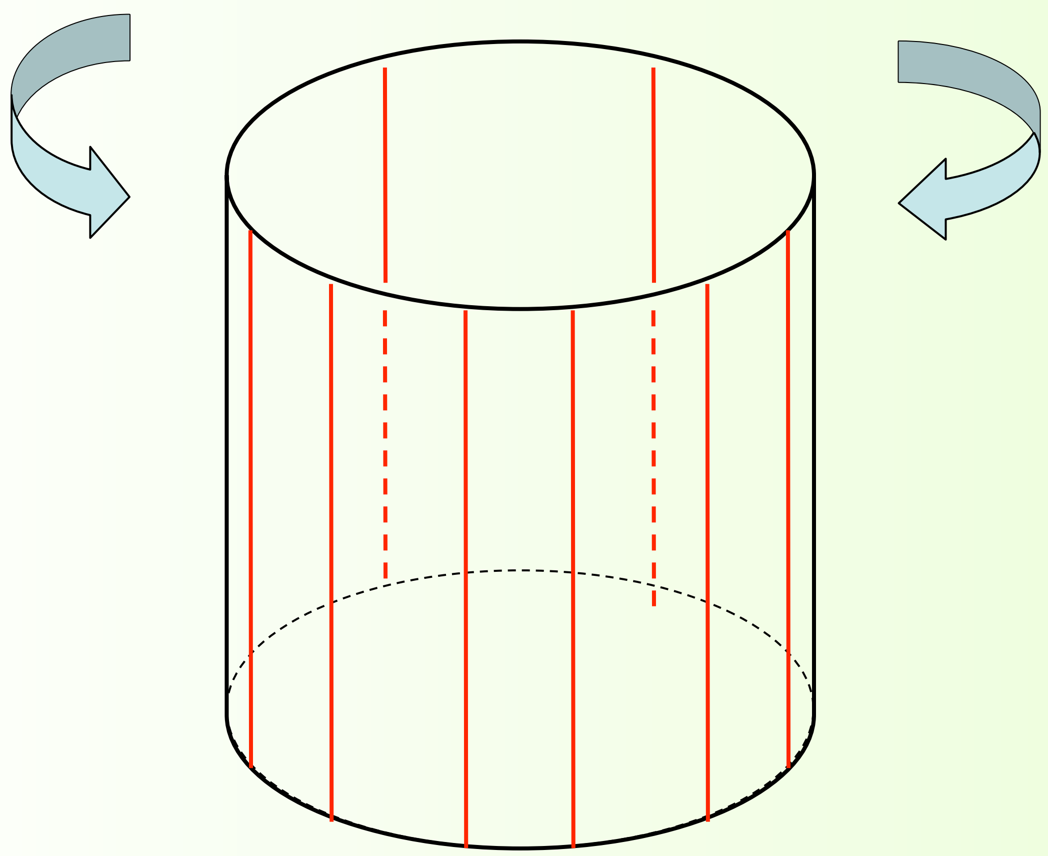
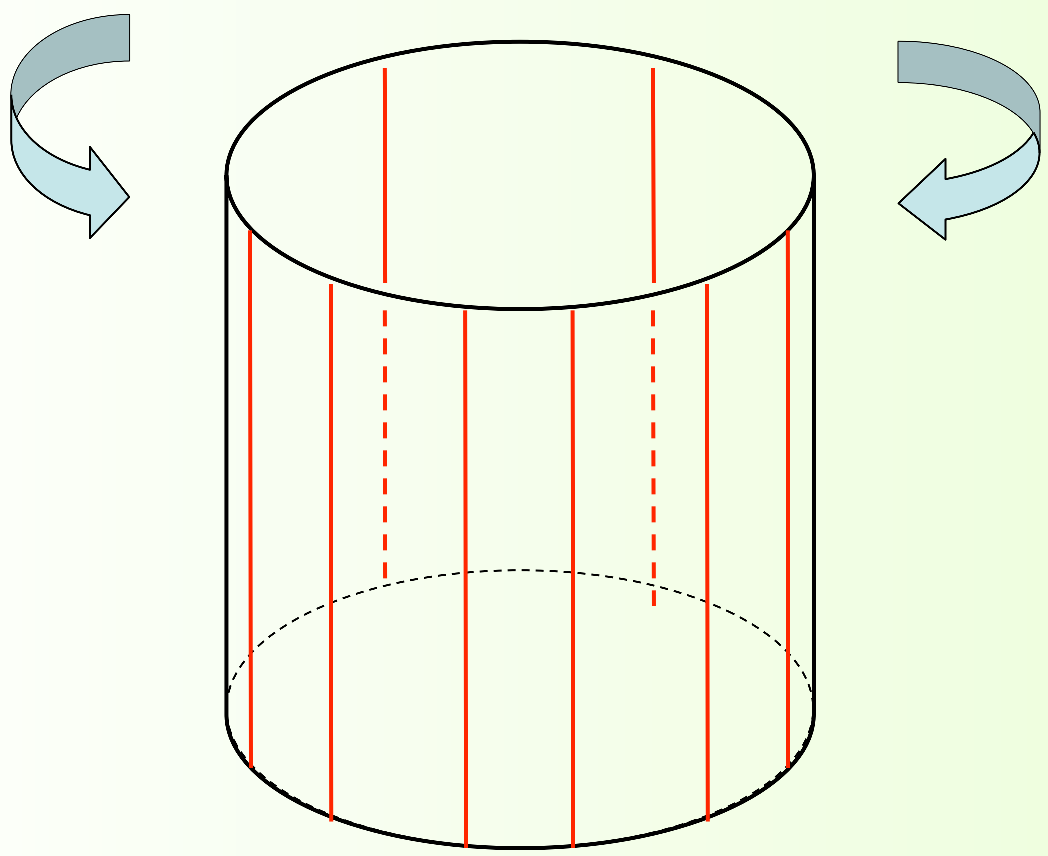


$$L_0|\psi\rangle = h|\psi\rangle$$

$$L_{+n}|\psi\rangle = 0$$

$$L_{-n}|\psi\rangle = \text{new states}$$

CFT



$$J_3|\psi\rangle = m_z|\psi\rangle$$

$$J_+|\psi\rangle = 0$$

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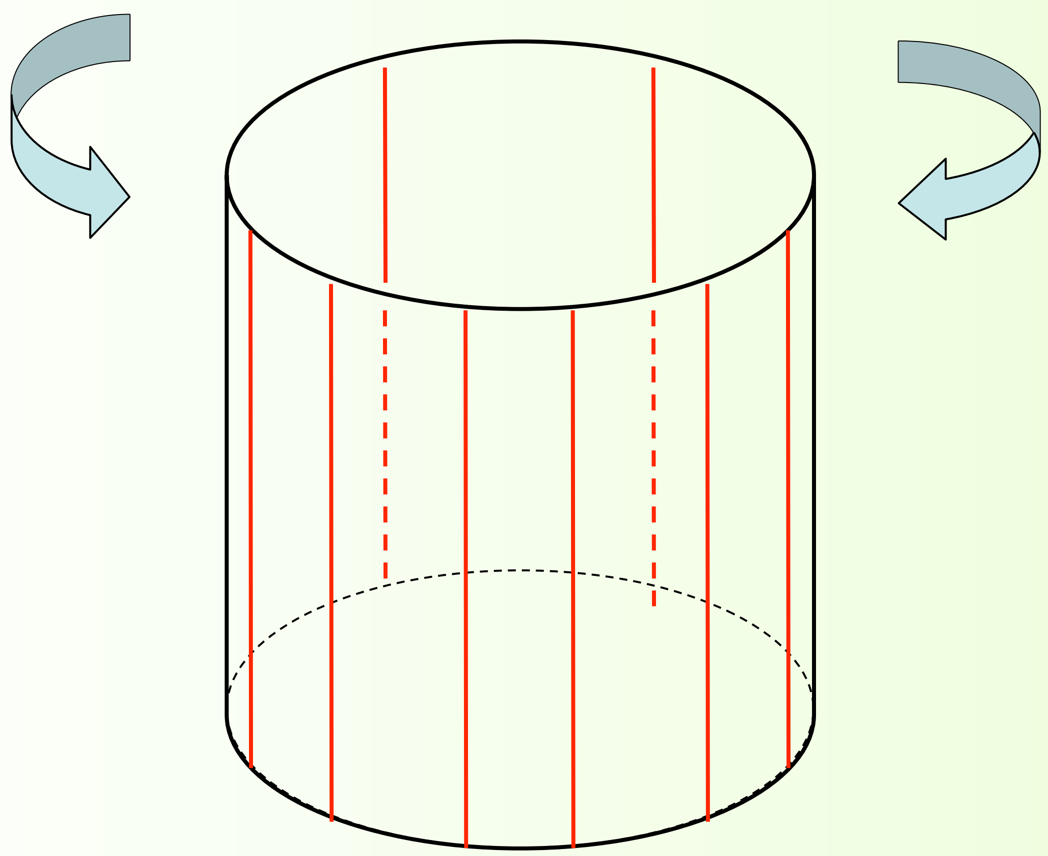
CFT



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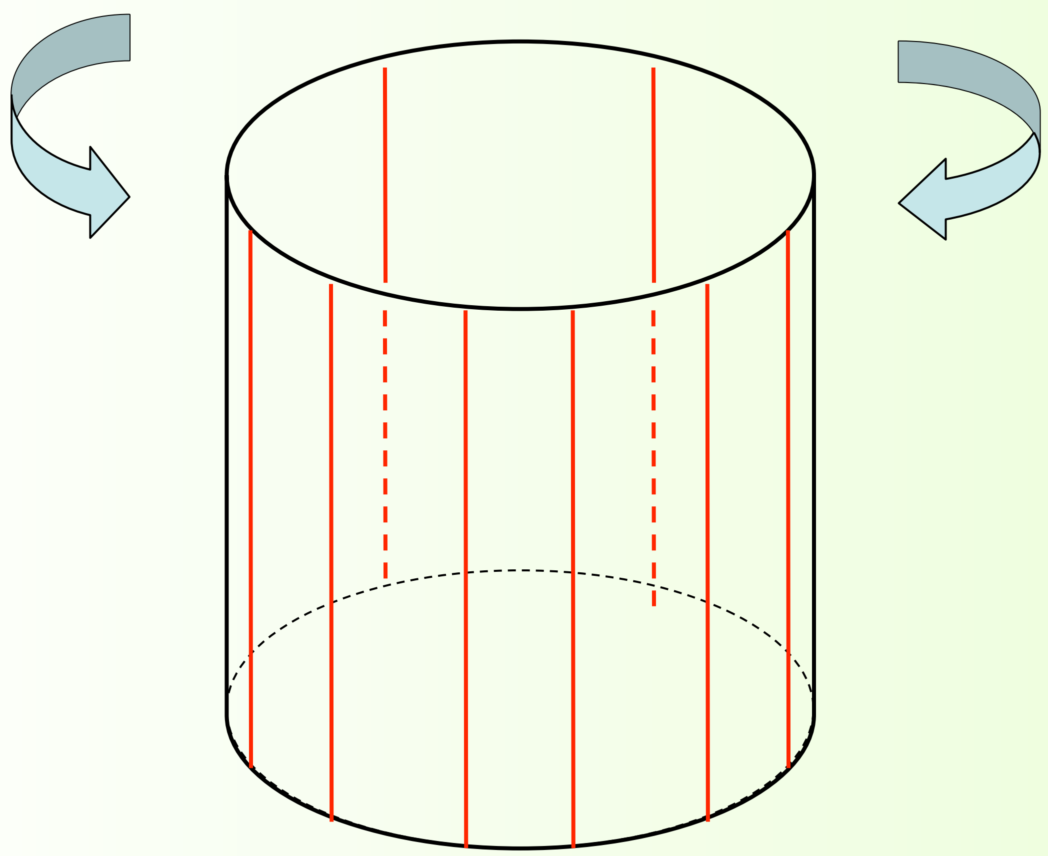

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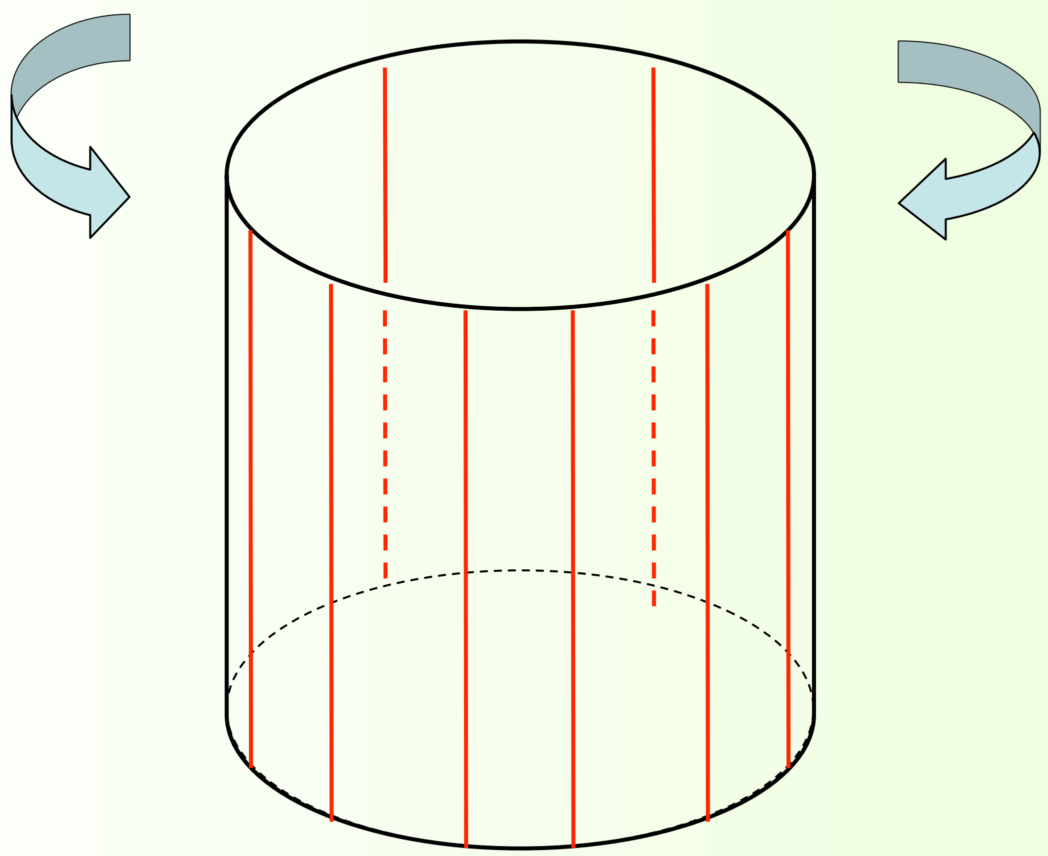
$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c_L}{12}n(n^2-1)\delta_{n+m}$$

CFT



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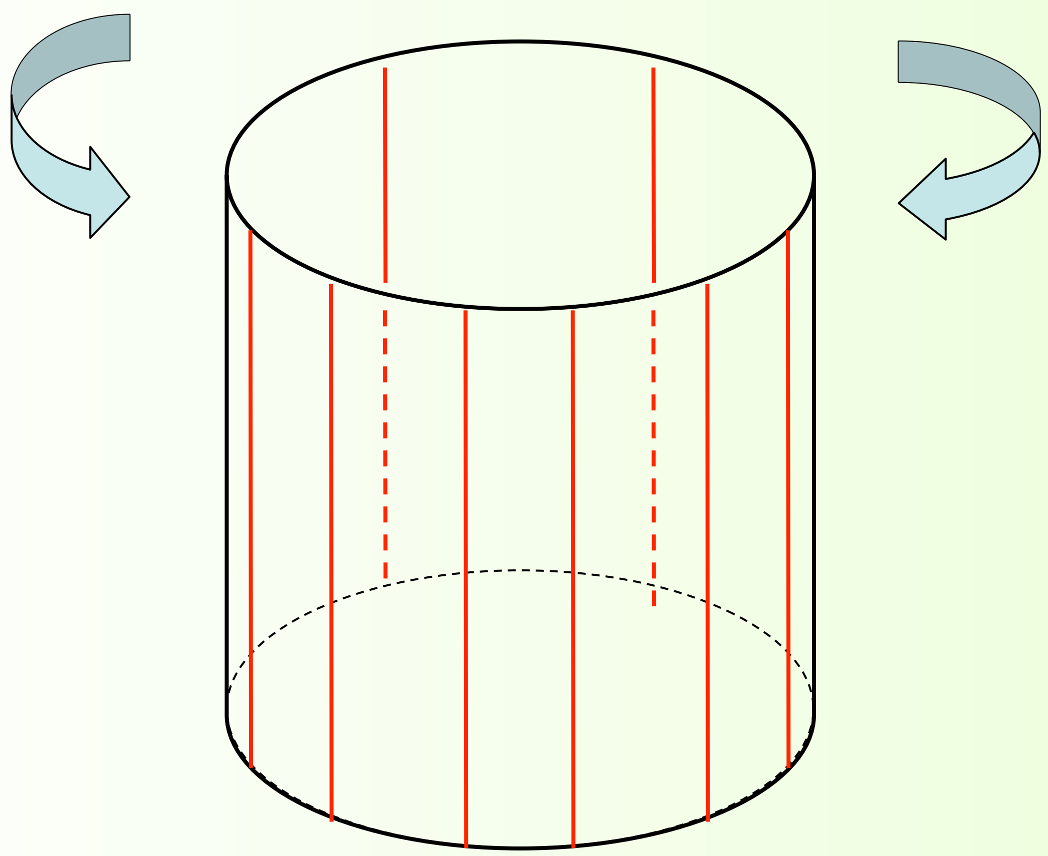

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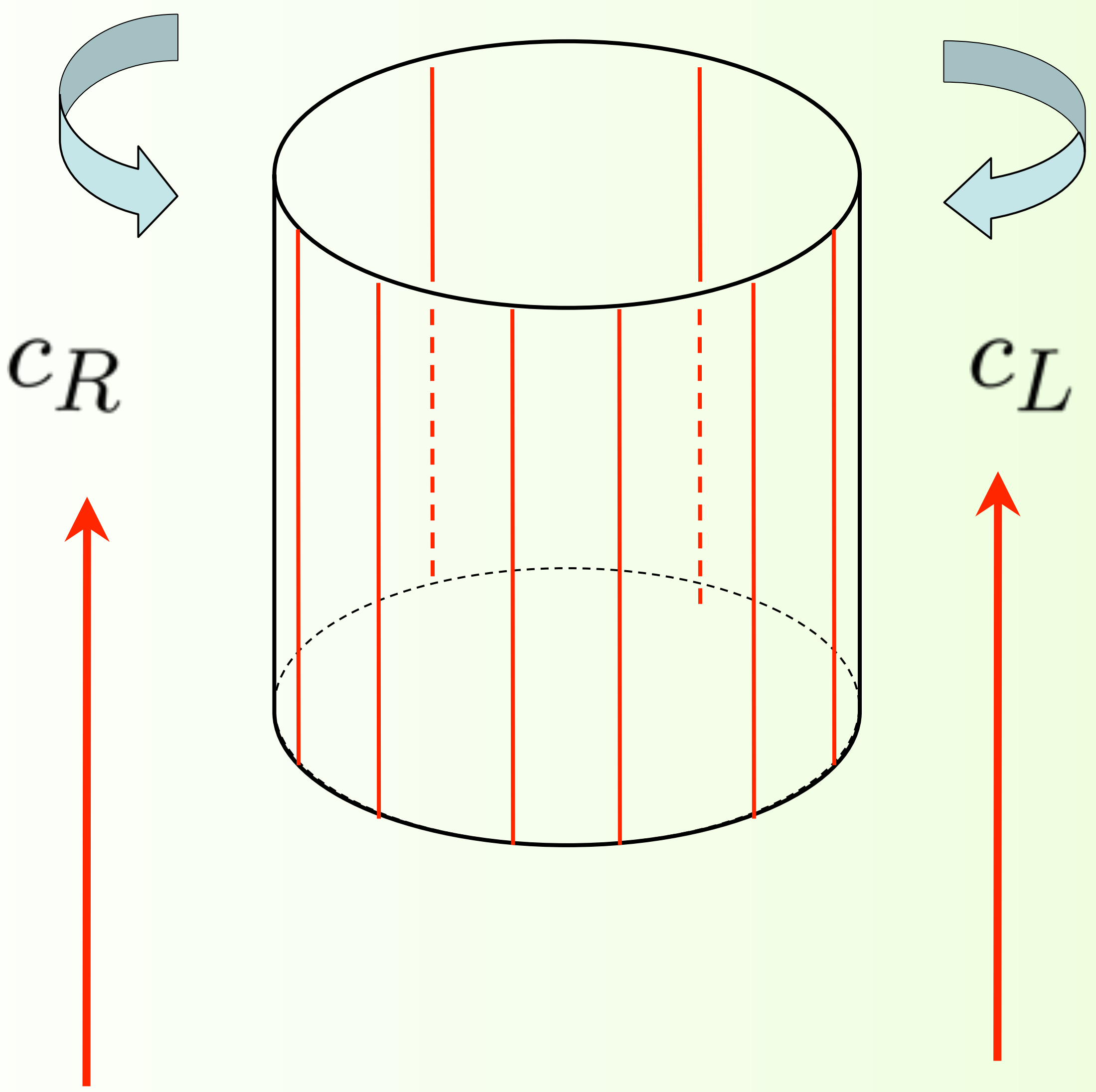

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$\sim$ density of states

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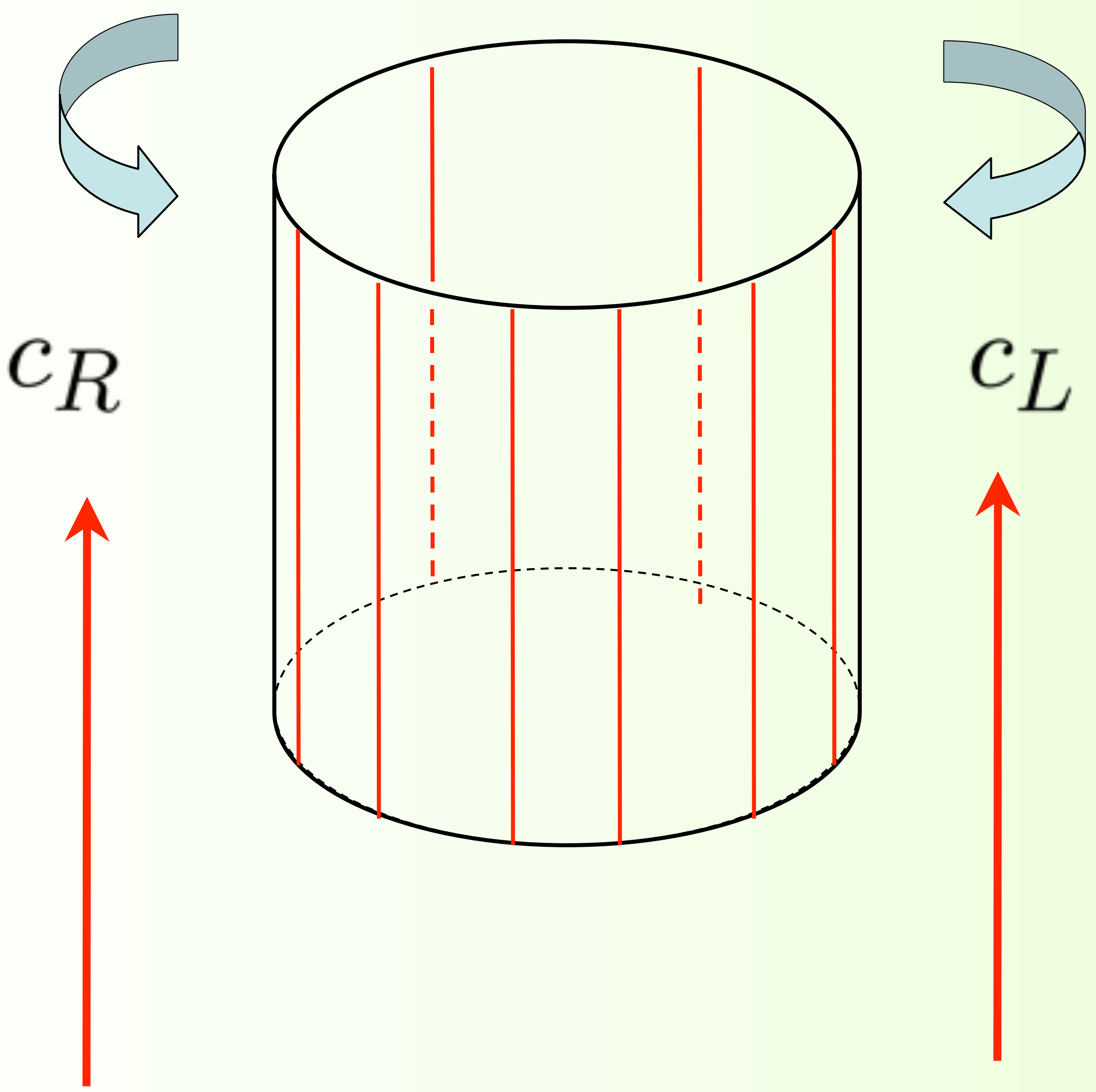

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$\sim$ density of states   Hairiness...

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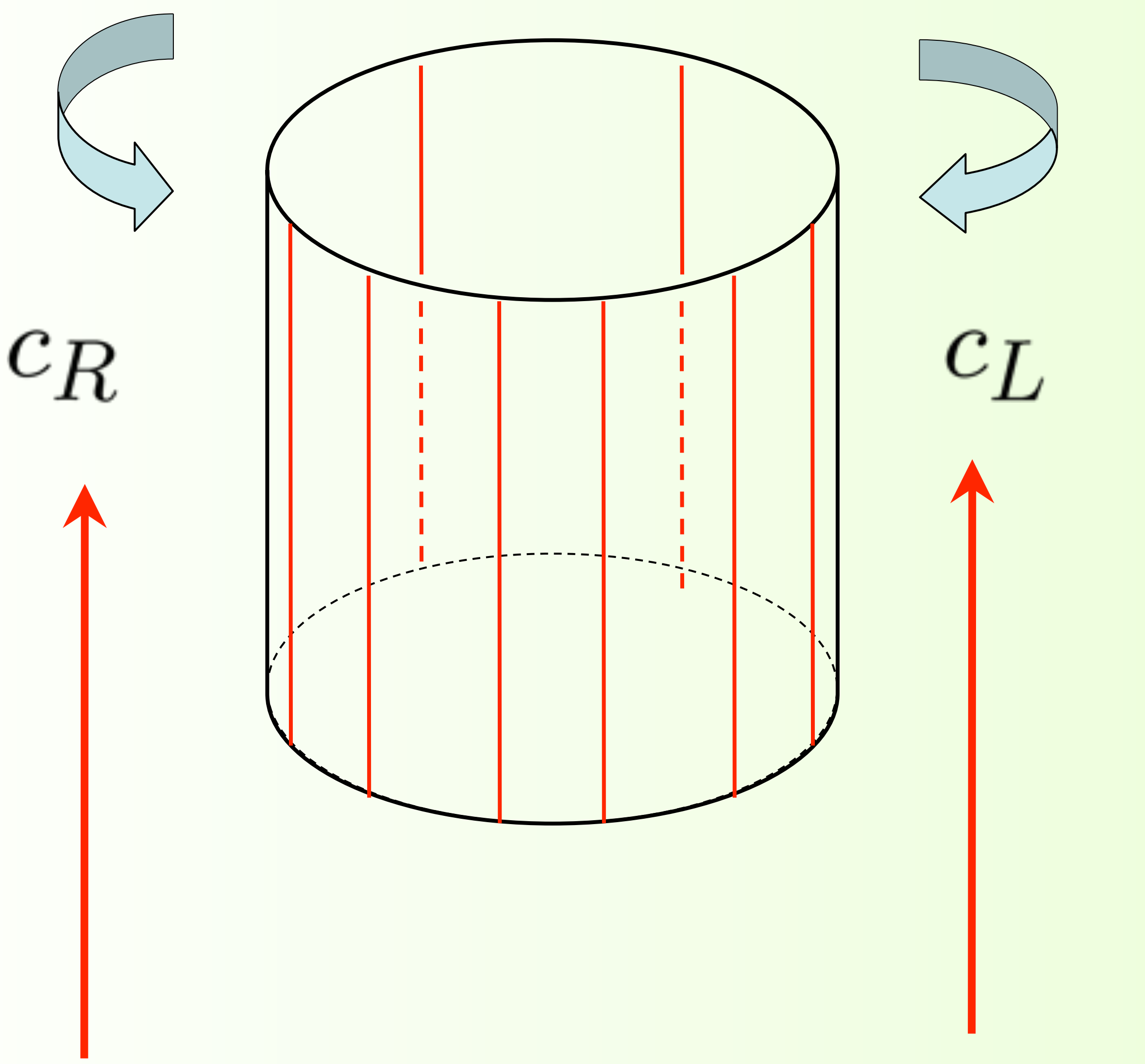

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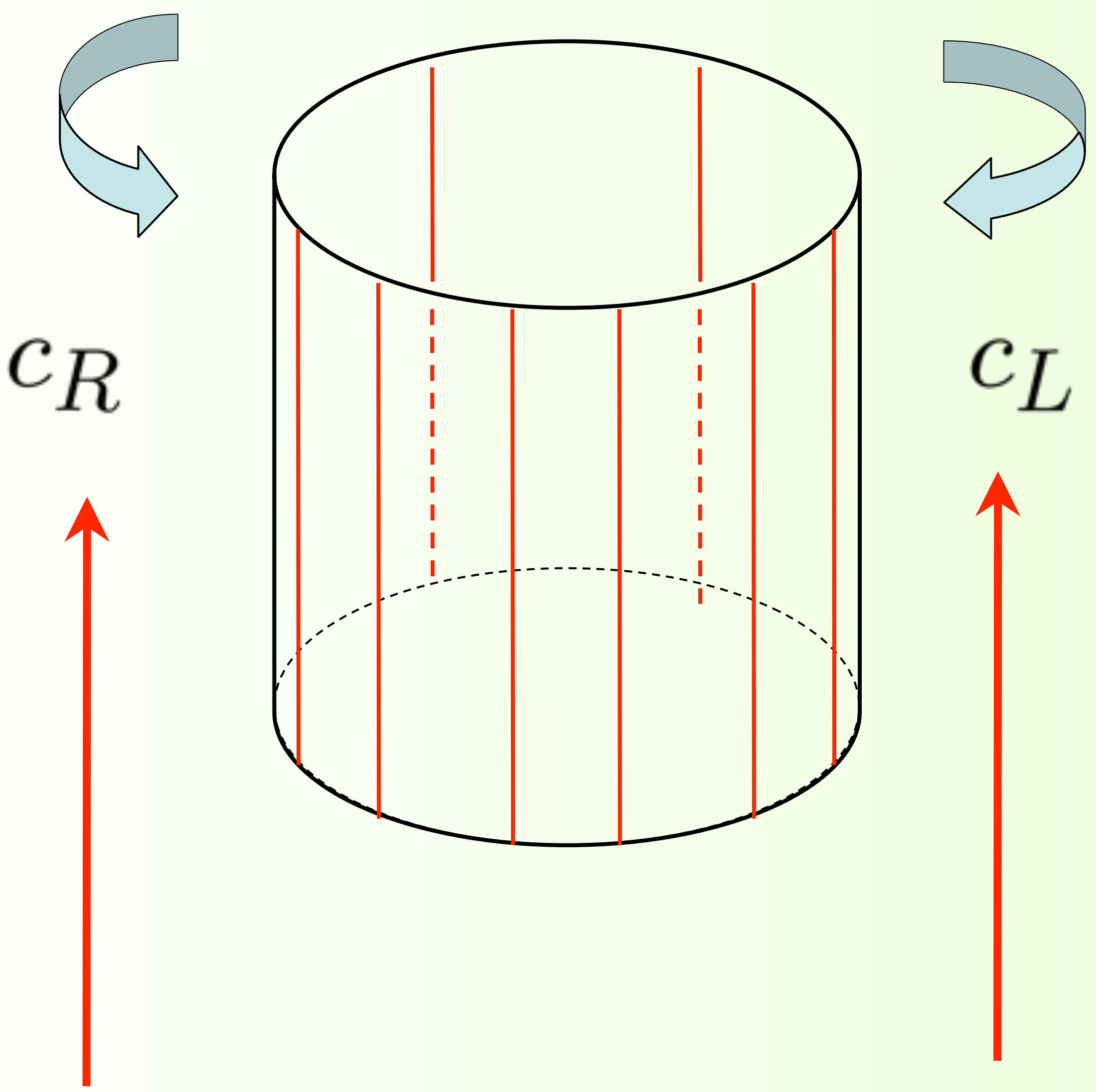

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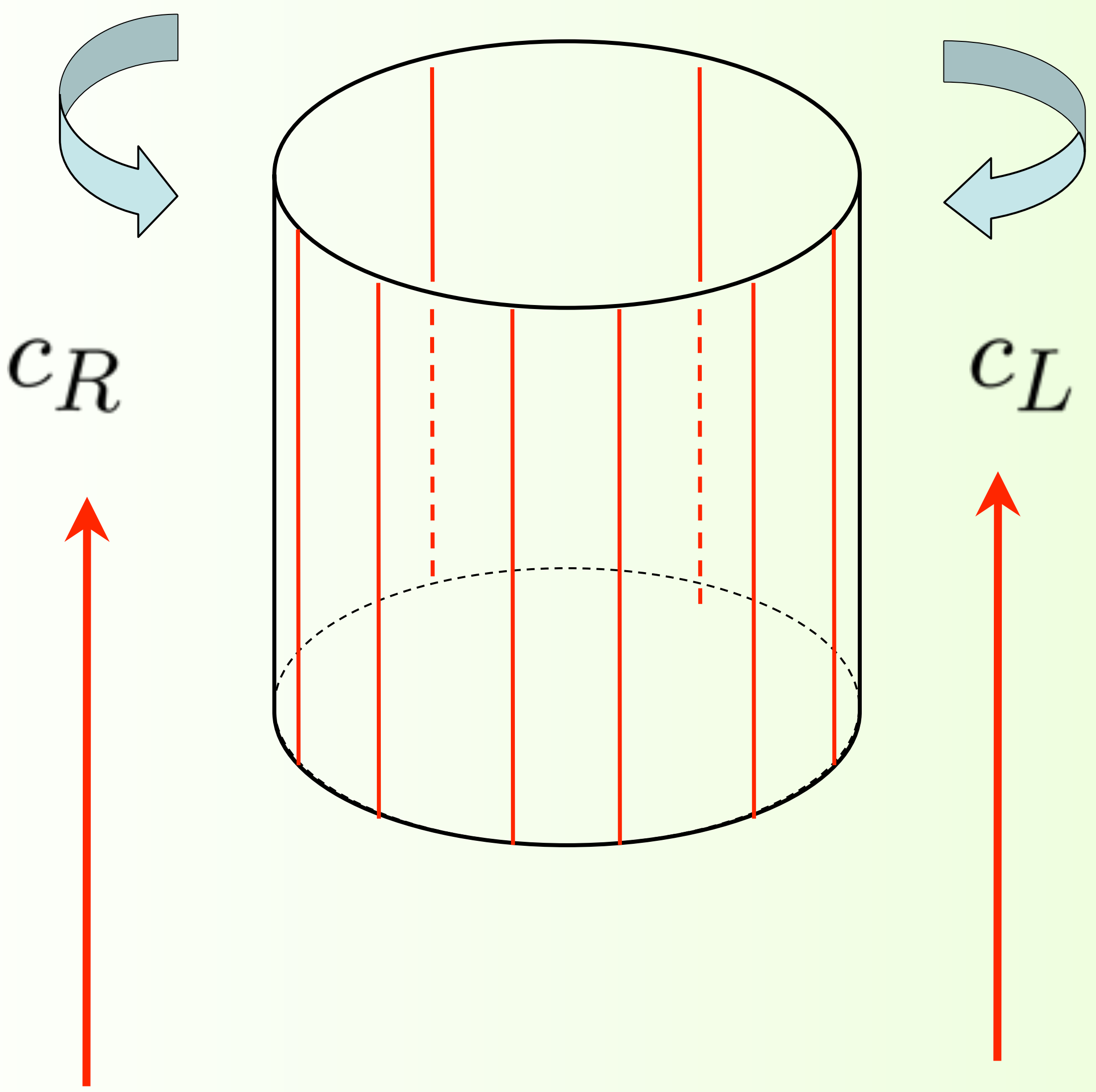
Hairiness...

CFT $\Rightarrow$ quantization!

CFT



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$\sim$ density of states

Hairiness...

CFT $\Rightarrow$ quantization!

Identify the CFT!

3D gravity

$$S = I_{EH} - \frac{1}{\ell^2} I_{CC}$$

HOT!

Achucarra and Townsend

Witten

It is gauge theory with no local dofs!

Brown and Henneaux

Dual to a CFT_2.

Bañados, Teitelboim and Zanelli

Contains BTZ black holes!

AdS/CFT : A Lightening Review

- Maldacena's AdS/CFT propsal
- Strong coupling physics of Gauge Theory = Weak coupling physics of Gravity
- Finite Temperature Gauge Theory = Black Hole Physics
- Applications :
  - Quark Gluon Plasma of Heavy Ion Collision
  - Strongly Correlated Condensed Matter System : QHE, Superconductivity

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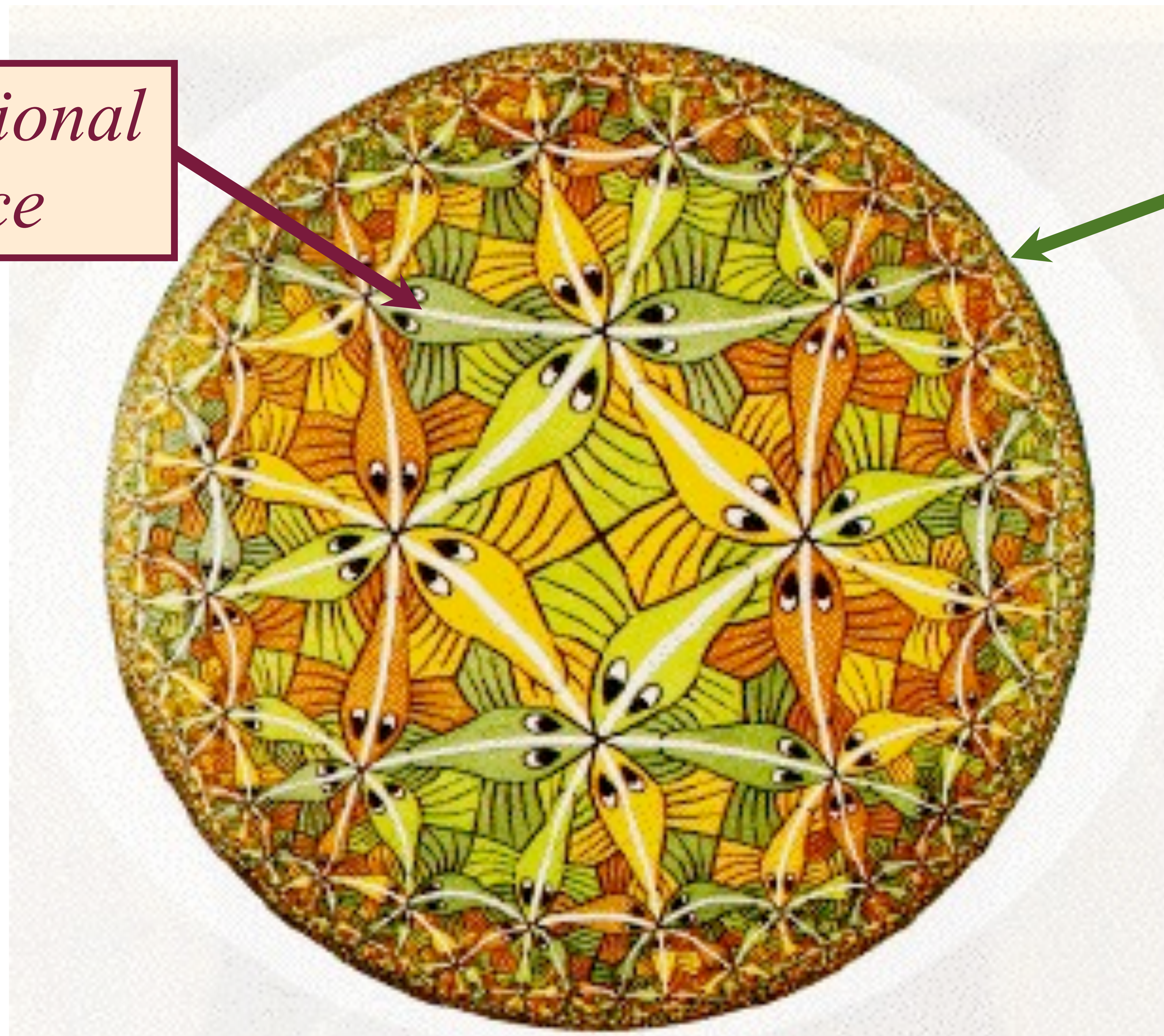
Beyond Maldacena's AdS/CFT

- Conformal Field Theory (CFT) is characterized by c (central charge)
- Conformal Symmetry = Local Scaling Symmetry
- In 2 dim. CFT, c counts the degrees of freedom
- c=1 : boson, c=1/2 : fermion
- Gravity dual of conformal field theory is Gravity in anti-de Sitter space
- AdS = Spacetime with negative cosmological constant
- However we live in de Sitter space (with dark energy)
- Can we go beyond AdS/CFT?
- So far not many examples. SUSY is comfortable with AdS.

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

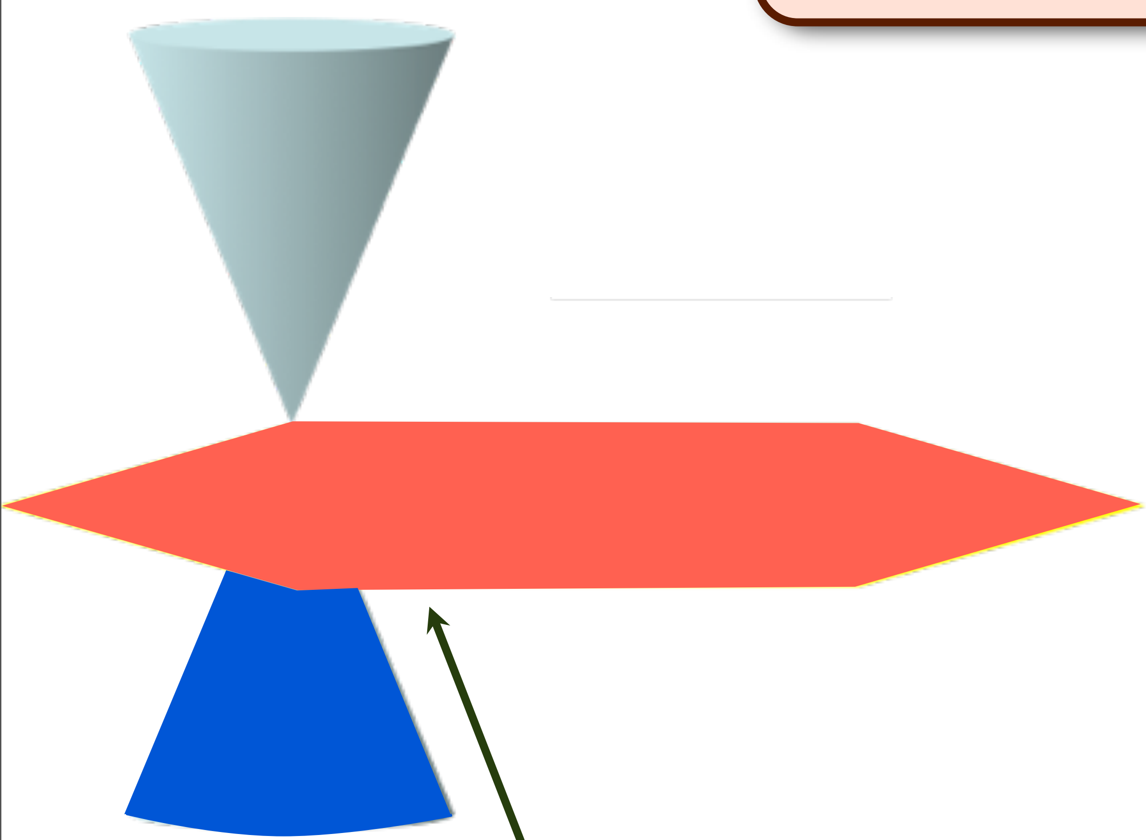
3+1 dimensional AdS space



A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

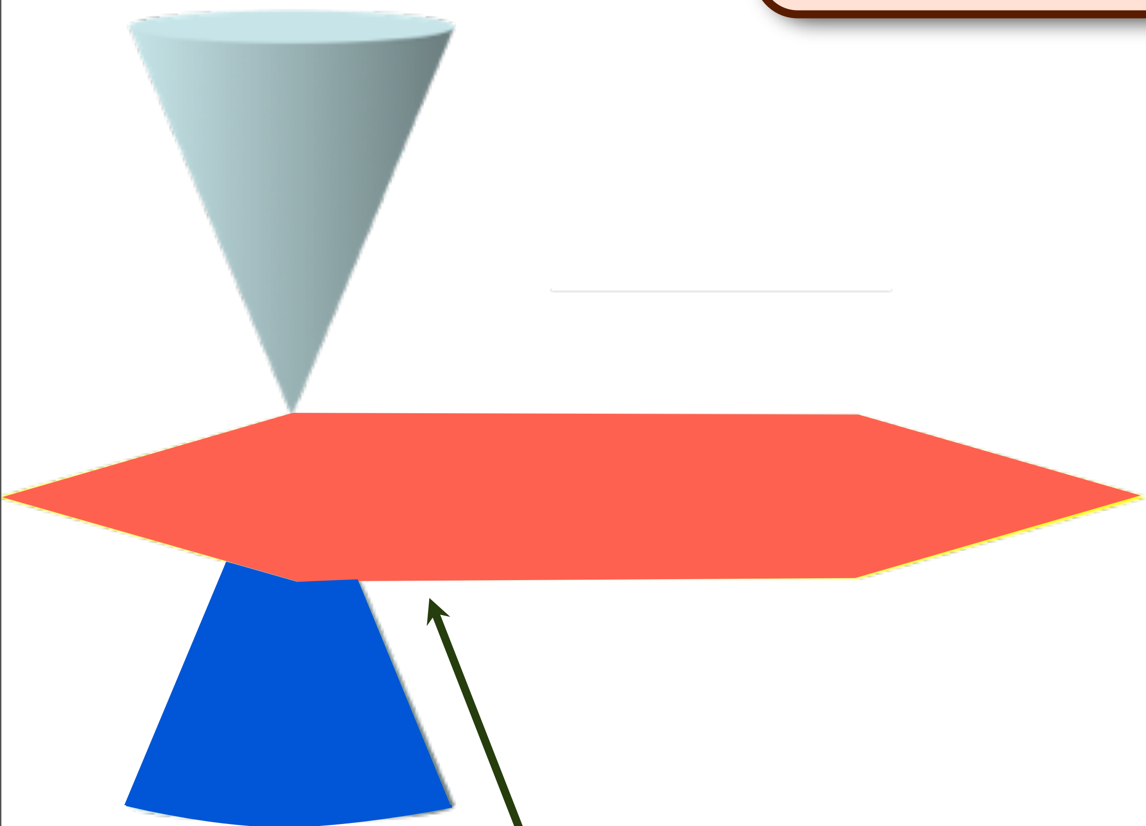
Conformal field theory
in 2+1 dimensions at $T = 0$



e.g.
Graphene
at zero bias

Einstein gravity
on AdS_4

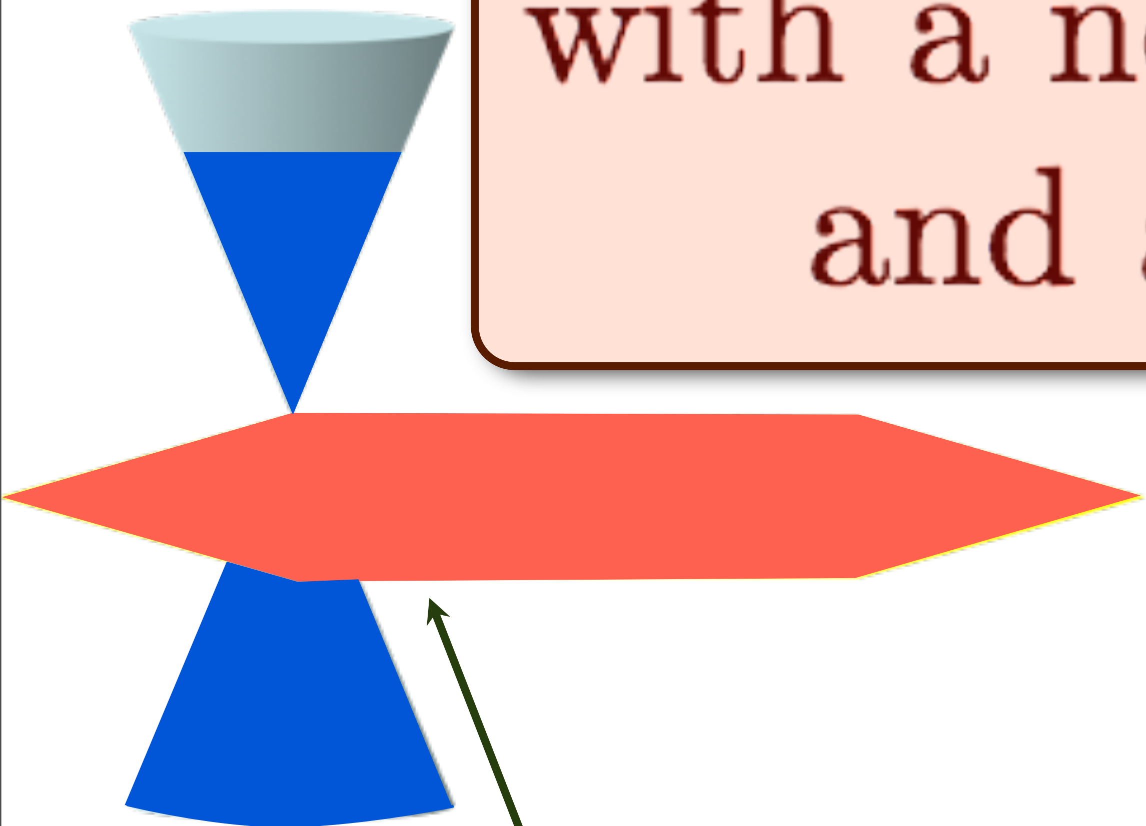
Conformal field theory in 2+1 dimensions at $T > 0$



e.g. Graphene at zero bias

Einstein gravity on AdS_4 with a Schwarzschild black hole

Conformal field theory
in 2+1 dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B



e.g.
Graphene
at non-zero
bias

Einstein gravity on AdS_4
with a Reissner-Nordstrom
black hole carrying electric
and magnetic charges

AdS/CMT

- c=1/2 corresponds to Ising Model at the critical point
- We can very easily calculate correlation functions of physical quantities.
- Compare this with extremely difficult calculation of Onsager.
- Recently there are some activities in String community to use String theory for condense matter physics.
- Quantum Phase Transition from Black Hole Physics
- In this talk we will talk about Black Hole Physics slightly beyond AdS Gravity
- We will also find some possible applications of the dual CFT.

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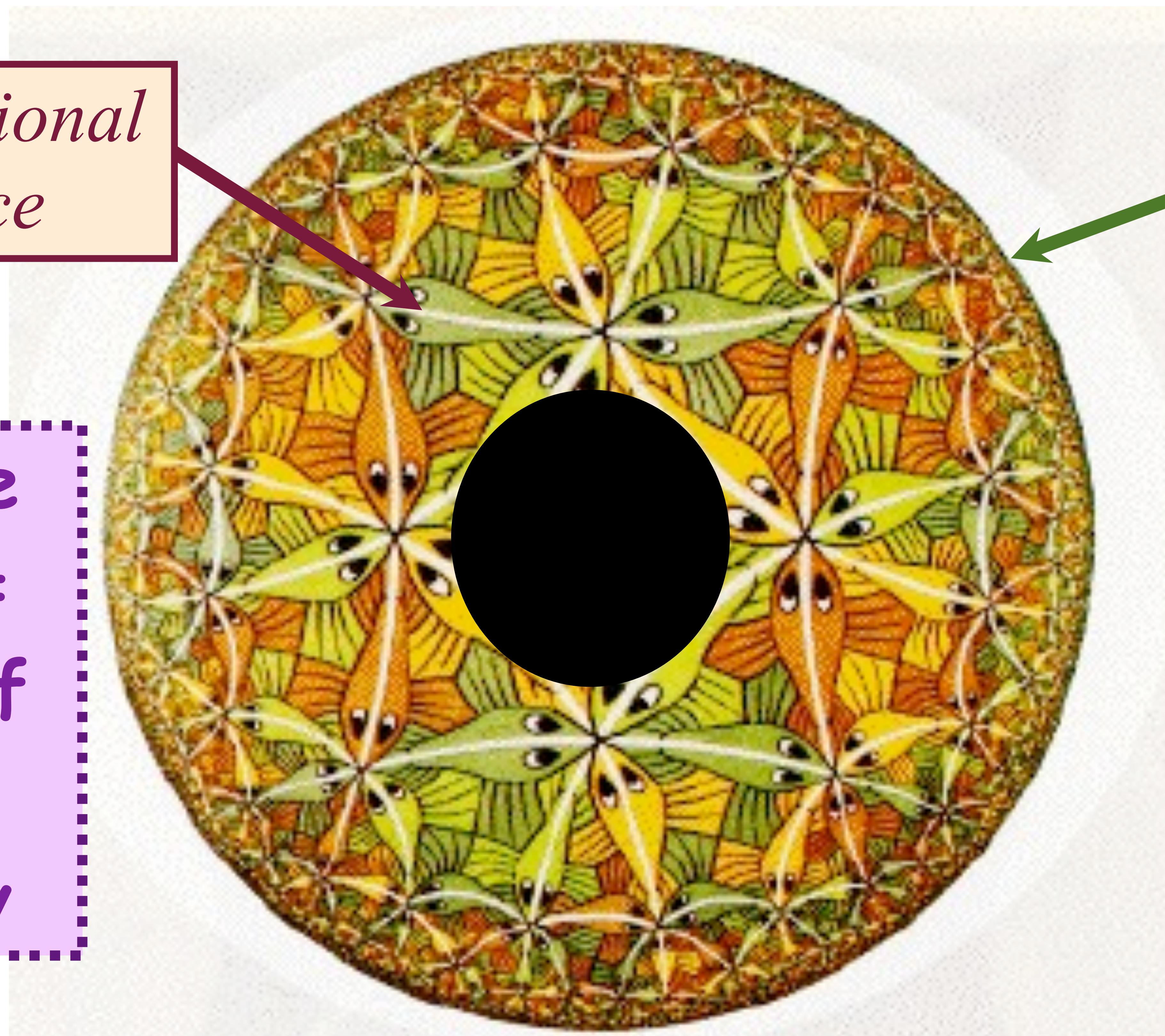
3+1 dimensional AdS space



Quantum criticality in 2+1 dimensions

Black hole entropy = entropy of quantum criticality

Strominger, Vafa

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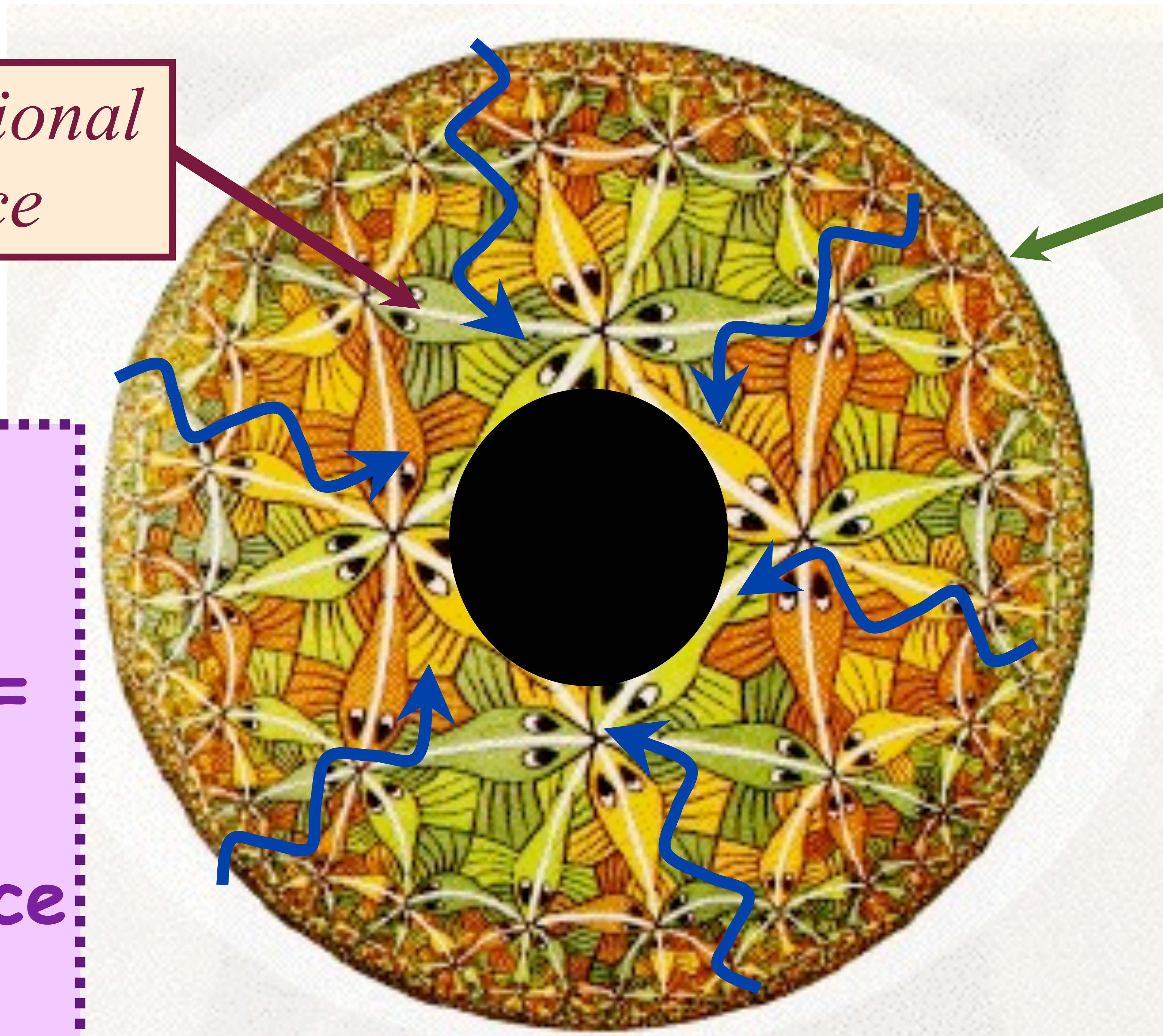
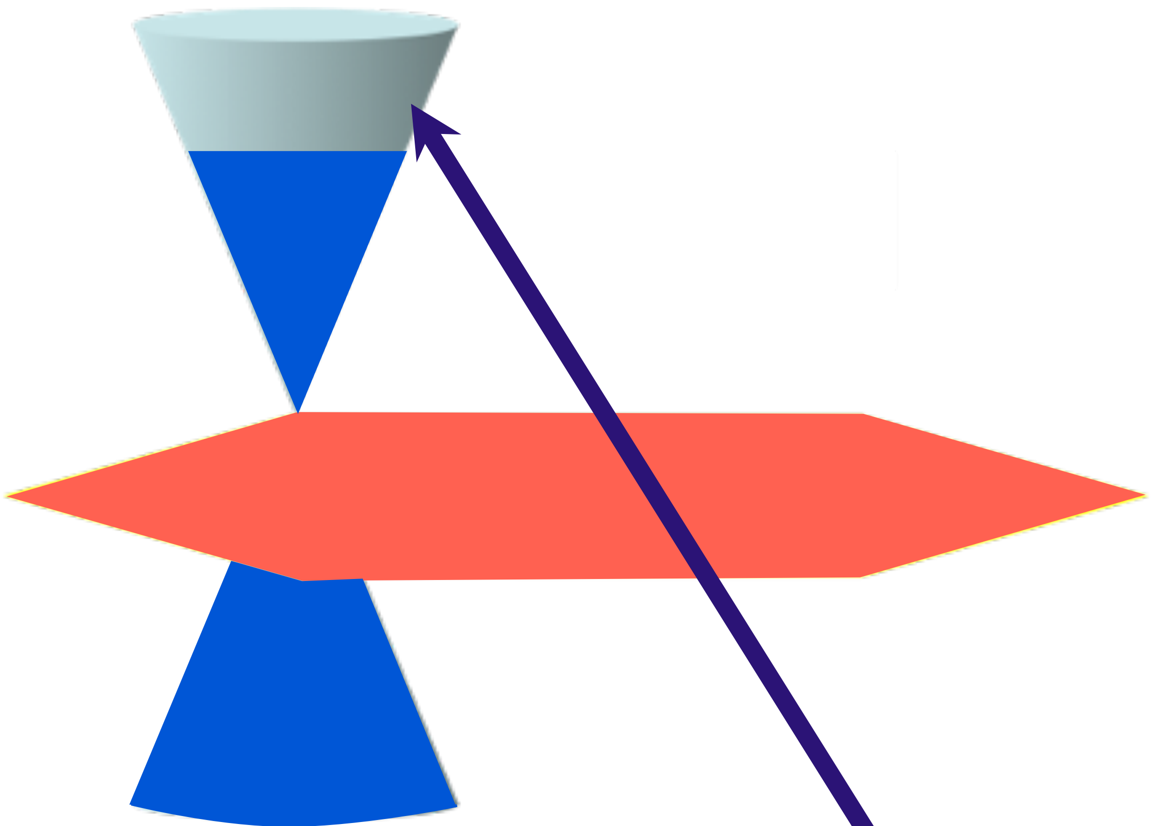
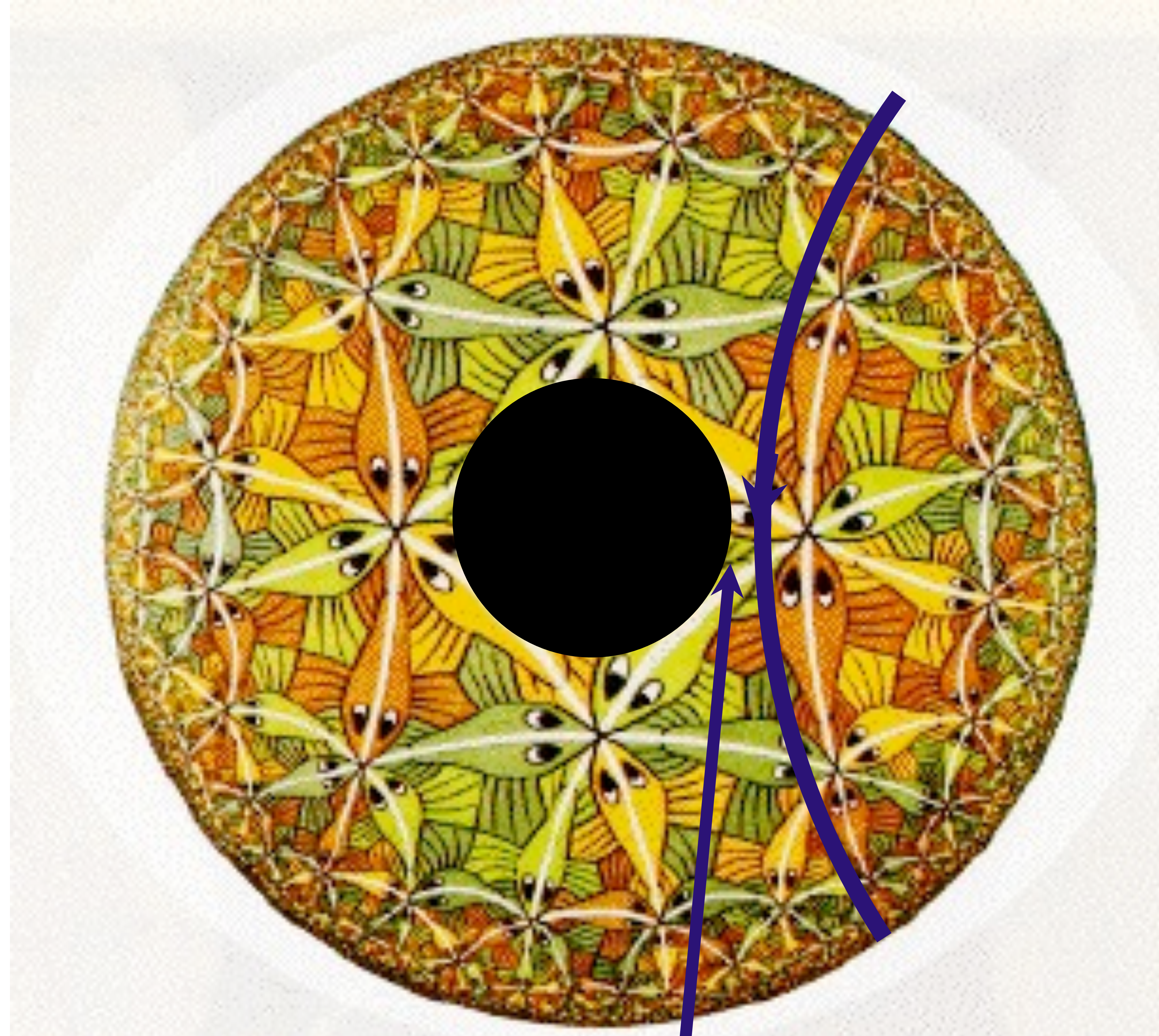


Maldacena, Gubser, Klebanov, Polyakov, Witte





Infrared physics of Fermi surface is linked to the near horizon AdS_2 geometry of Reissner-Nordstrom black hole

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Beyond AdS

- We will study gravity theories beyond Einstein Gravity.
- We will find some solutions which are not AdS asymptotically.
- Moreover the dual CFT can have c = 0.
- Does c=0 mean an empty theory? (No degrees of freedom)
- Actually, c=0 CFT is a Logarithmic CFT (LCFT) !

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Logarithmic CFT

- It was noticed in 1993 (Gurarie) that c=0 actually is not an empty theory.
- Since then people found application of LCFT.
  - Percolation, Self Avoiding Random Walk, Sand Piling Problem
  - Turbulence (one of most challenging theoretical problem).
  - Network and scaling behavior (Social, Internet, etc)
  - Basically we can apply to disorder systems.
  - Perhaps we can apply to biology (protein - gene), medicine etc.

Some Background

1957– Broadbent & Hammersley: Percolation

1972– de Gennes, des Cloizeaux: Polymers

1986– Saleur, Duplantier: Conformal theory of polymers, percolation

1993– Gurarie: Logarithmic operators in CFT

1995– Kausch: Symplectic fermions

1992– Rozansky, Read, Saleur, Schomerus, etc: Supergroup Approach to Log CFT

1996– Rohsiepe, Flohr, Gaberdiel, Kausch, Feigin et al: Algebraic Approach to Log CFT

2006– Pearce, Rasmussen & Zuber: Lattice Approach to Log CFT

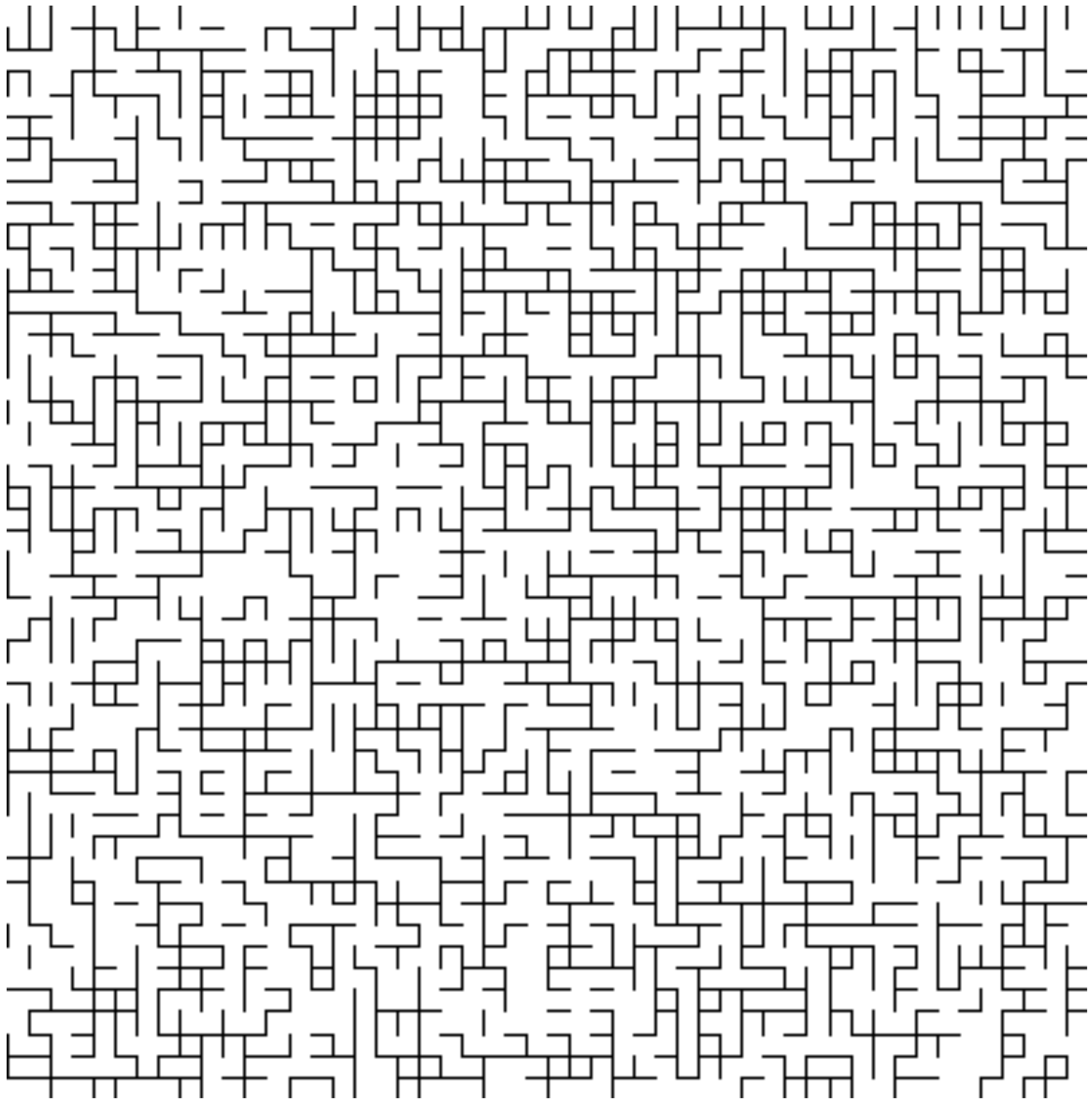
Lattice Approach: For Potts, RSOS models, . . .

local degrees of freedom	$\Rightarrow$	symmetric transfer matrices	$\Rightarrow$	diagonalizable transfer matrices	$\Rightarrow$	no rank ≥ 2 indecomposable representations	$\Rightarrow$	not logarithmic theory

Paradigm Shift:

- Statistical systems with local "point" degrees of freedom yield rational CFTs.
- Polymers and percolation do not have any local degrees of freedom only nonlocal "string" degrees of freedom (polymers, connectivities) and are associated with Logarithmic CFTs . . .

logarithmic theory	$\Rightarrow$	nonlocal degrees of freedom



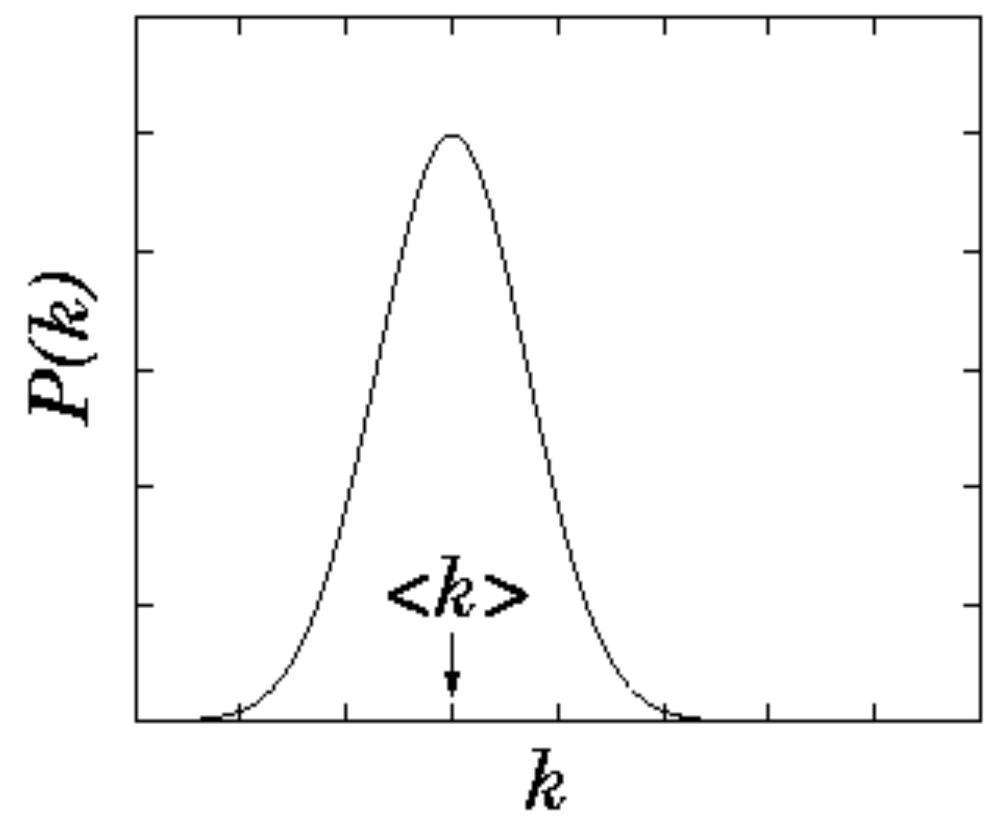
Percolation

복잡계란?

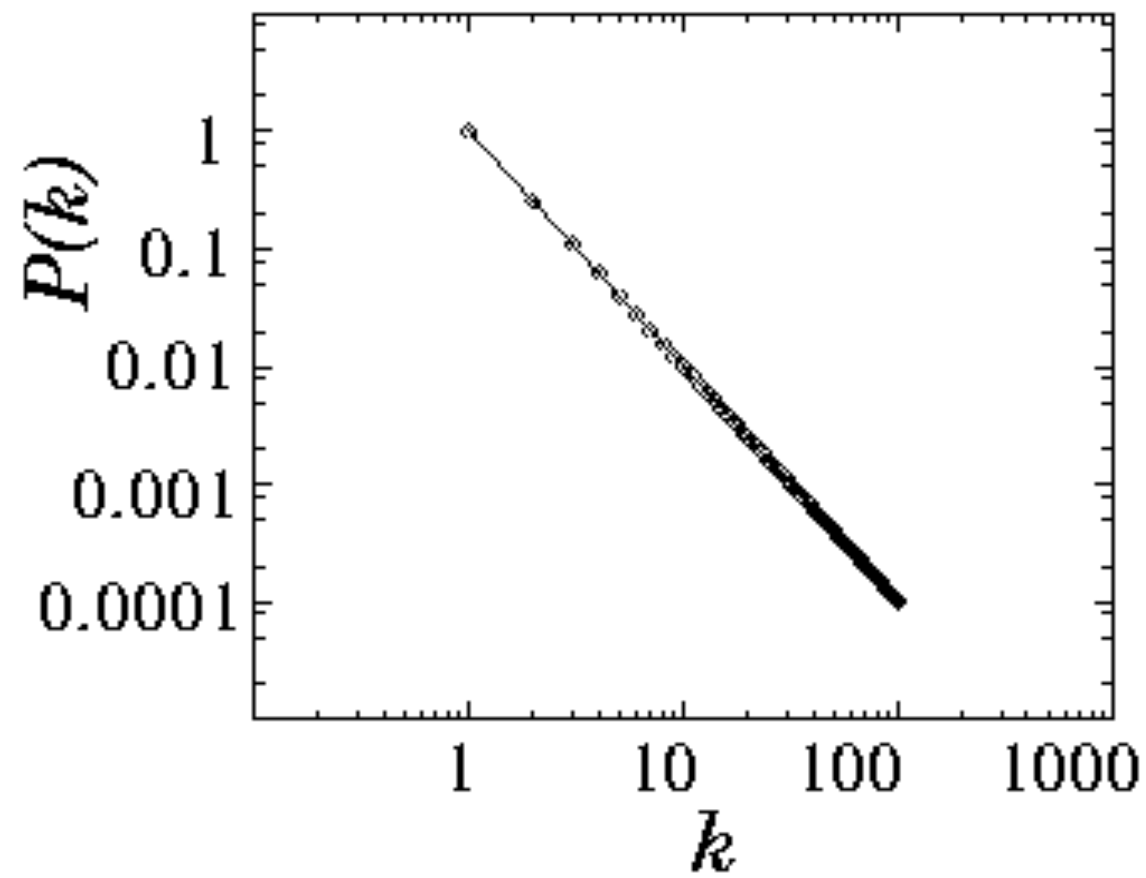
- 컴퓨터 네트워크, Social Network
- 유전자와 단백질
- 여과 - percolation
- 양자홀 효과, 그래핀

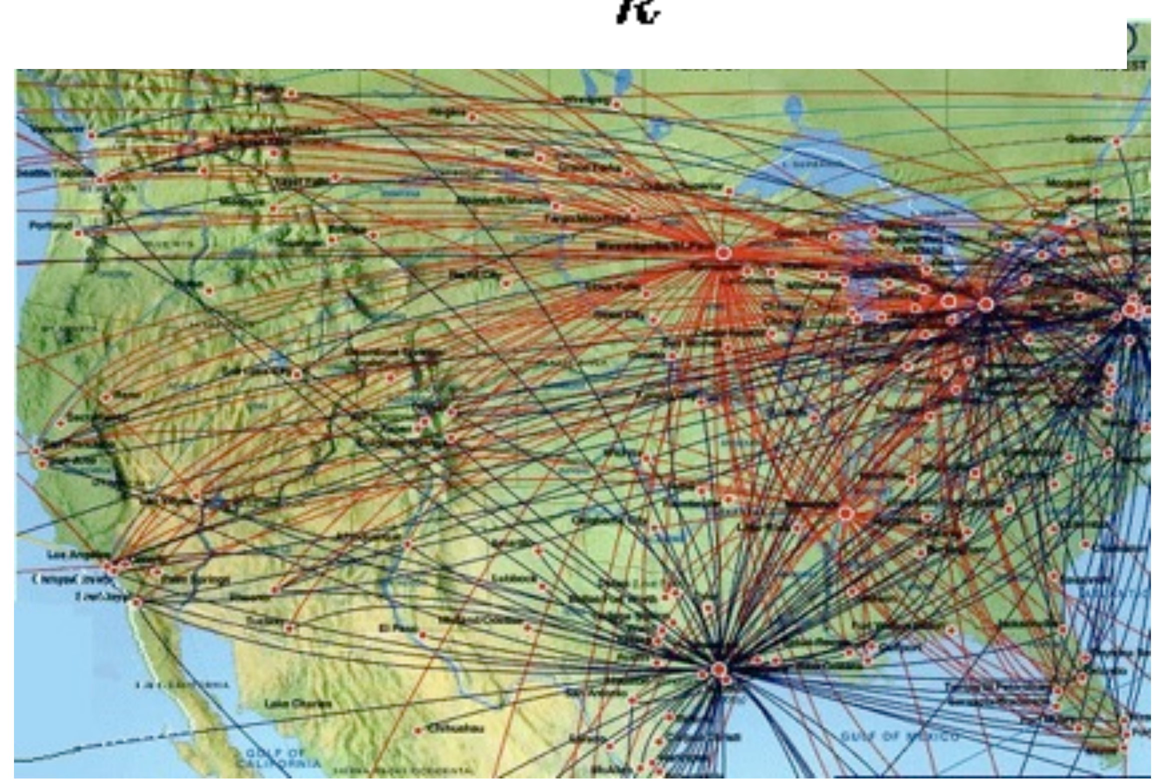
Power-law distribution & Scale-free Network

Poisson distribution



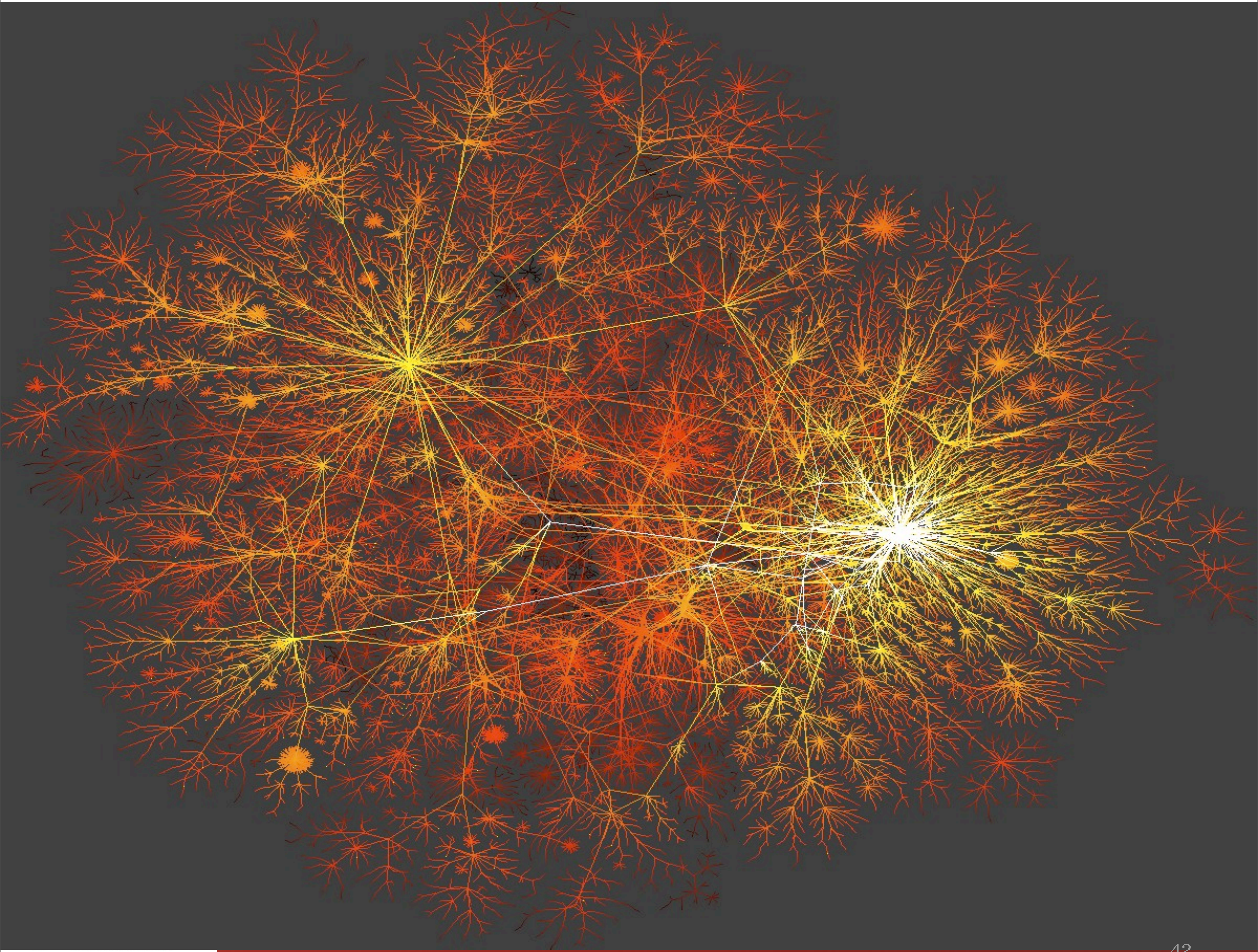
Power-law distribution



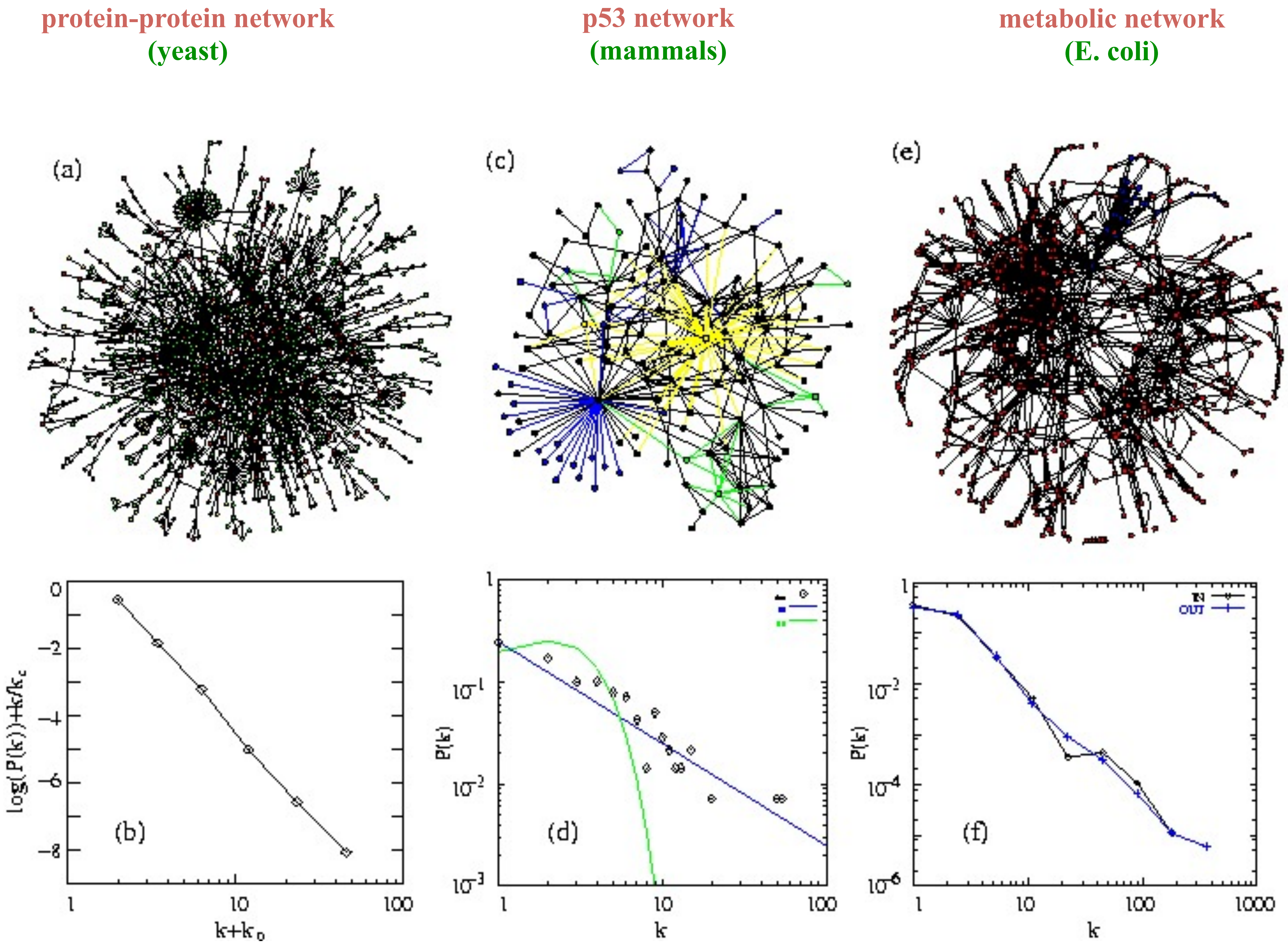


Exponential Network

Scale-free Network



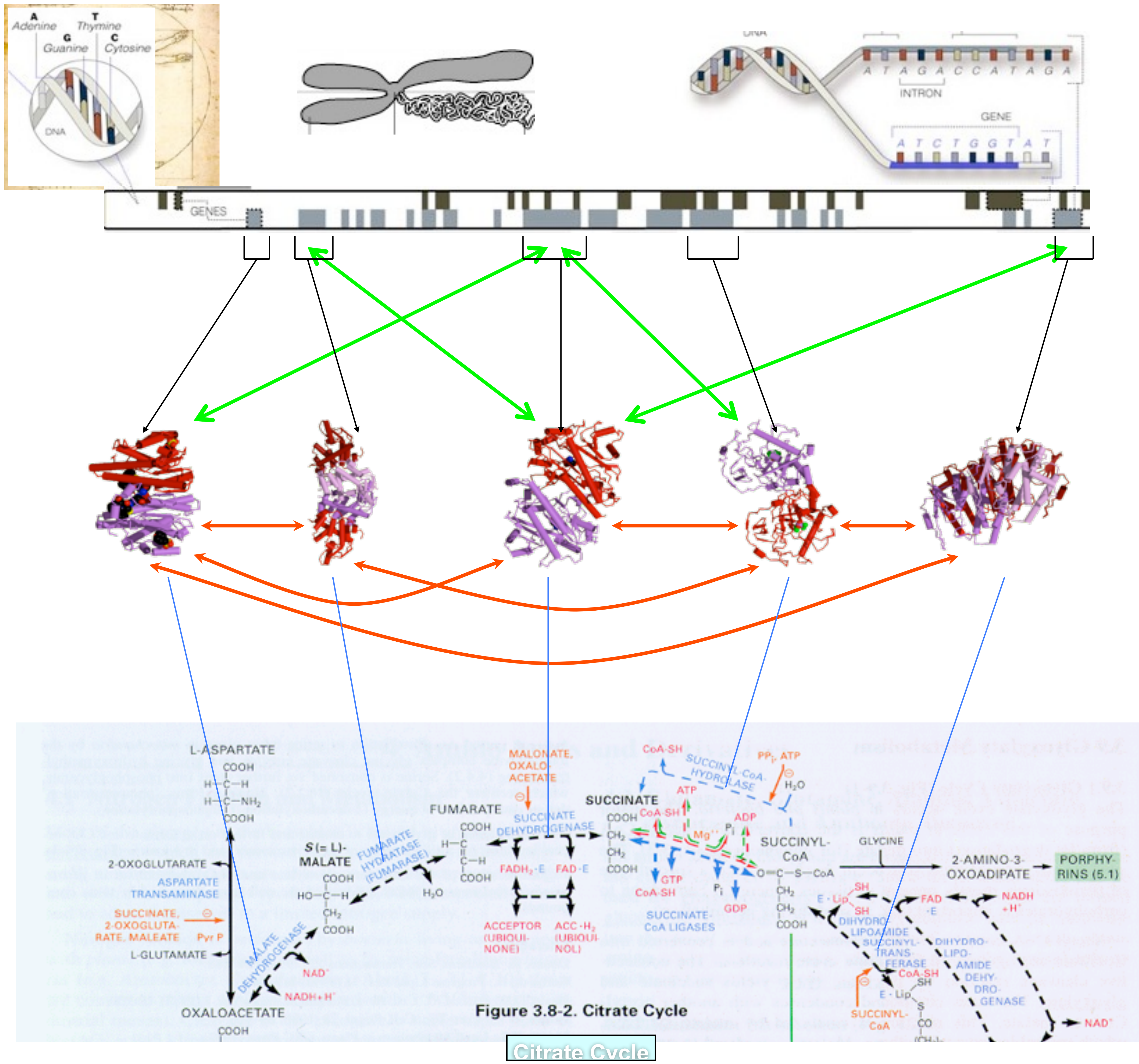
Scale-free Networks in the Life



Jeong et al.
Nature 411, 41 (2001)

Jeong et al.
Nature 407, 651 (2000).

Scale-free Networks in the Body



GENOME

protein-gene interactions

PROTEOME

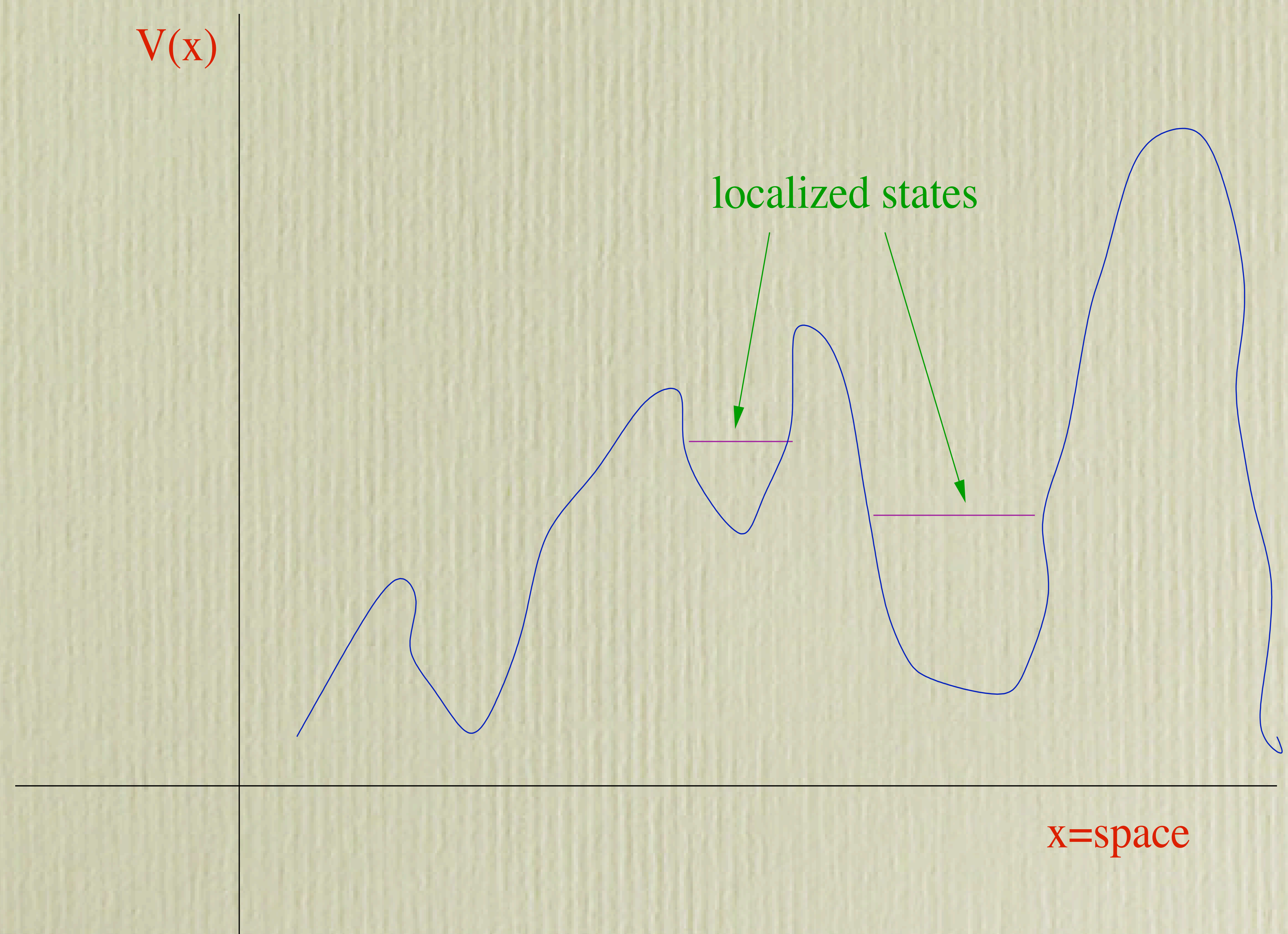
protein-protein interactions

METABOLIOME

Bio-chemical reactions

Anderson localization and the Quantum Hall Transition

Consider electrons moving in a random potential:



Supersymmetric Disorder Averaging

Consider a free hamiltonian in a random potential V(x):

$$H = -\frac{\vec{\nabla}^2}{2m} + V(x)$$

We are interested in disorder averaged Green functions:

$$\overline{< \psi(x)\psi^\dagger(x') >} = \int DV P[V] < \psi(x)\psi^\dagger(x') >_V$$

The problem: properly normalize the Green function at fixed V by Z(V):
The trick: represent Z with bosonic ghosts:

$$\frac{1}{Z(V)} = \int D\beta \; e^{-S(\psi\rightarrow\beta, V)}$$

We can now perform the functional integral over the random potential V:

$$\boxed{\overline{\langle\psi(x)\psi^\dagger(y)\rangle} = \int D\psi D\beta e^{-S_{\text{eff}}} \; \psi(x)\psi^\dagger(y)}$$

S_{eff} is an interacting quantum field theory of fermions and ghosts.

- With is motivation, we will talk about 3 dimensional quantum gravity.
- We will first discuss gravity beyond Einstein Gravity - New Massive Gravity
- Then we will talk about the 'New Type black hole' solution in New Massive Gravity
- We will study some physical properties of the New type black solutions
- This is to introduce to you we basic tools of BH physics
- Black Holes = Hydrogen Atoms of Quantum Gravity

3D Einstein Gravity

- Locally flat outside sources
- No massless graviton in Einstein theory
- We can add higher-derivative terms to have propagating DOF
- We can also study massive gravity
  - Topologically Massive Gravity(parity violating) : Deser, Jackiw, Templeton, Ann. Phys. (1982), (3rd order) Gravitational Chern-Simons
  - New Massive Gravity (parity preserving) : Bergshoeff, Hohm, Townsend , Ann. Phys. (2010) (4th order)

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Topological Massive Gravity [Deser, Jackiw, Templeton]

- Topologically massive gravity is obtained by adding to the Einstein gravity the gravitational CS term,

$$S = \int d^3x \left((R - 2\Lambda) + \frac{1}{2\mu}(\Gamma d\Gamma + \Gamma^3) \right)$$

  where Γ is the 1-form Christoffel symbol.
- This theory admits asymptotically *AdS* solutions, for example the BTZ black hole solves its equations of motion and has perturbative massive modes.
- When $\mu \neq 1$ however, the massive modes have negative energy and the theory is unstable.

The "chiral point", $\mu = 1$.

- When $\mu = 1$ the negative energy modes disappear, the left moving gravitational modes become pure gauge and the theory seems to only contain a purely right moving sector.
- This led to the conjecture that the theory is stable and consistent when $\mu = 1$ and dual to a 2*d* chiral CFT [Li, Song, Strominger (2008)].
- This created a lot of controversy as other authors found non-chiral modes and instabilities at the chiral point [Carlip etal], [Grumiller, Johansson], [Giribet etal] ...

Log Solutions

- Log solution make this theory more interesting.
- Two more solutions broken Brown-Henneaux boundary conditions appears.

$$\psi_{\mu\nu}^{\mathrm{newL}} \equiv \lim_{m^2\ell^2 \to 1/2} \frac{\psi_{\mu\nu}^{ML}(m^2\ell^2) - \psi_{\mu\nu}^{mL}}{m^2\ell^2 - 1/2} = y(\tau,\rho)\psi_{\mu\nu}^{mL}$$

$$\psi_{\mu\nu}^{\mathrm{newR}} \equiv \lim_{m^2\ell^2 \to 1/2} \frac{\psi_{\mu\nu}^{MR}(m^2\ell^2) - \psi_{\mu\nu}^{mR}}{m^2\ell^2 - 1/2} = y(\tau,\rho)\psi_{\mu\nu}^{mR}$$

$$y(\tau,\rho) = (-i\tau - \ln\cosh\rho)/2$$

- We could relax the boundary condition to include these interesting modes.

We now move to apply this methodology to the topologically massive gravity. We first discuss the theory at the "chiral point". There is one new element compared to previous discussions:

- The field equations are third order in derivatives, so there are two independent boundary data: one can fix the metric and the extrinsic curvature.
- The boundary metric $g_{(0)}$ is the source for the energy momentum tensor T_{ij}.
- The boundary field $b_{(0)ij}$ parametrizing the extrinsic curvature is a source for a new operator t_{ij}.

- Topologically massive gravity at the "chiral point" is dual to Logarithmic CFT and therefore it is not unitary.
- One may try to restrict to the right-moving sector of the theory, which could yield a consistent chiral theory.

  This requires $\langle t\bar{T}\bar{T}\rangle = 0$, which holds for certain LCFTs. It would be interesting to compute this 3-point function holographically for TMG at $\mu = 1$.

Results: 2-point functions

From the most general solution of the linearized equations of motion we extracted the following non-zero 2-point functions:

$$
\begin{aligned}
\langle t_{zz}(z,\bar{z})t_{zz}(0)\rangle &= \frac{-(-3/G_N)\log|z|^2}{z^4}, \\
\langle t_{zz}(z,\bar{z})T_{zz}(0)\rangle &= \frac{(-3/G_N)}{2z^4}, \\
\langle T_{\bar{z}\bar{z}}(z,\bar{z})T_{\bar{z}\bar{z}}(0)\rangle &= \frac{(3/G_N)}{2\bar{z}^4},
\end{aligned}
$$

These are precisely the non-zero 2-point functions of a Logarithmic CFT with

$$
c_L = 0, \qquad c_R = \frac{3}{G_N}, \qquad b = -\frac{3}{G_N}.
$$

New Massive Gravity

Simplest version of NMG of which action is given by

$$S = \frac{\eta}{2\kappa^2}\int d^3x\sqrt{-g}\left[\sigma R + \frac{2}{l^2} + \frac{1}{m^2}K\right]$$

where η and σ take 1 or -1, and K is defined by

$$K = R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2\,.$$

The EOM of NMG $\mathcal{E}_{\mu\nu} = \eta\left[\sigma G_{\mu\nu} - \frac{1}{l^2}g_{\mu\nu} + \frac{1}{2m^2}K_{\mu\nu}\right] = 0\,,$

where

$$K_{\mu\nu} = g_{\mu\nu}\left(3R_{\alpha\beta}R^{\alpha\beta} - \frac{13}{8}R^2\right) + \frac{9}{2}RR_{\mu\nu} - 8R_{\mu\alpha}R^{\alpha}_{\nu}$$
$$+\frac{1}{2}\left(4\nabla^2 R_{\mu\nu} - \nabla_\mu\nabla_\nu R - g_{\mu\nu}\nabla^2 R\right)$$

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Auxiliary Field

We can introduce auxiliary field f.

$$S[g,f] = \frac{1}{\kappa^2}\int d^3x\sqrt{|g|}\left[-2\lambda m^2 + \sigma R + f^{\mu\nu}G_{\mu\nu} - \frac{1}{4}m^2(f^{\mu\nu}f_{\mu\nu} - f^2)\right]$$

The auxiliary field $f_{\mu\nu}$ can be eliminated by

$$f_{\mu\nu} = \frac{2}{m^2}\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu}\right)$$

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Fierz-Pauli

Consider the Fierz-Pauli equation for symmetric field $\phi_{\mu\nu}$

$$(\Box - m^2)\phi_{\mu\nu} = 0, \quad \eta^{\mu\nu}\phi_{\mu\nu} = 0, \; \partial^{\nu}\phi_{\nu\mu} = 0,$$

Number of propagating modes:

$$\mathcal{N} = \frac{1}{2}D(D+1) - 1 - D.$$

$\mathcal{N} = 2$ in 3D. Only in 3D it has non-linear generalization. => New Massive Gravity

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Renormalizability

$$S[g] = \int d^d x \sqrt{|g|}[\sigma R + aR^{\mu\nu} R_{\mu\nu} + bR^2], \; d = 3, 4, \; \sigma = \pm 1.$$

- Gauss-Bonnet term : total derivative in D = 4, zero in D = 3
- $D = 4$, $a = 0$, by Bicknell's theorem (1974), unitary but non-renormalizable.
- $D = 4$. non-unitary if $a \neq 0$, but if $a \neq 3b$ then renormalizable. (Stelle, 1977)
- $D = 3$ and $\sigma = 1$. Unitary but non-renormalizable for $a = 0$. (Scalar massive gravity)
- $D = 3$ and $\sigma = -1$. Unitary for $3a = 8b$. (New Massive gravity) Since $D = 4$ theory is renormalizable and $a(a - 3b) \neq 0$, we expect super-renormalizable. (Oda 2009, Deser 2009)

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Scalar Massive Gravity

$$S = \int d^D x \sqrt{|g|}[R - \frac{1}{2m^2} R^2]$$

is equivalent to

$$S = \int d^D x \sqrt{|g|}[R - fR + \frac{1}{2} m^2 f^2],$$

where f is an auxiliary scalar field.

Maximally symmetric solutions

Consider the metric ansatz of the following form:

$$ds^2 = -ab^2 dt^2 + a^{-1} dr^2 + \rho^2 d\theta^2$$

for some functions (a, b, ρ) of r.

The Einstein's tensor is

$$G = -\frac{ab^2}{2\rho}[\rho' a' + 2a\rho'']dt^2 + \frac{\rho'(ba' + 2ab')}{2ab\rho}dr^2 + \frac{\rho^2(ba' + 2ab')'}{2b}d\theta^2$$

We also have static ansatz for the auxiliary field. From EOM, it has to be diagonal.

For convenience sake we eliminate f^{rr} and f^{tt} and introduce one new function

$$c(r) = m^2 r^3 f^{\theta\theta}(r)$$

Wednesday, March 30,

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Energy Functional

We need to minimize the energy functional

$$E[a,b,c,\rho] = \int dr \left\{ 2\lambda m^2 b\rho - \sigma a(\rho' b' - \rho'' b) + \frac{1}{2m^2\rho}\left[(ba' + 2ab')\left(\left[a(\rho')^2\right]' + \rho c'\right) + bc^2 + 2(ab\rho')'c\right]\right\}$$

Gauge transformations

$$\delta_\xi \rho = \xi\rho', \quad \delta_\xi b = (\xi b)', \quad \delta_\xi a = \xi a' - 2\xi' a, \quad \delta_\xi c = \xi c'$$

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Equations of Motion

First fix the gauge invariance by setting $\rho(r) = r$.

$$c = \frac{r}{2b}(ba' + 2ab')' - \frac{(ab)'}{b}$$

$$r(2m^2\sigma b' + bc'' - b'c') + ba'' - b'a' + 2bc' = 0$$

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Black Hole Solutions of NMG

Substituting into EOM, we get

$$\alpha = 0, \quad a_1(a_2 - 4m^2\sigma) = 0, \quad a_2 = 4m^2\left[\sigma \pm \sqrt{1+\lambda}\right]$$

- For $\lambda < -1$, no solution.
- For $\lambda > -1$ we must have $a_1 = 0$.
  - In this case when $a_0 = 1$ we get (A)dS vacua.
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Properties of New Black Hole

Static new type black holes are given by

$$ds^2 = L^2\left[-(r^2+br+c)dt^2 + \frac{dr^2}{(r^2+br+c)} + r^2 d\phi^2\right],$$

which have horizons at $r_\pm = (-b \pm \sqrt{b^2-4c})/2$.

The surface gravity of this black holes at the outer and inner horizons is given by

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Proposals to extend NMG to even higher curvature terms.

- R^3-terms : consistency with holographic c-theorem (Sinha)
- Born-Infeld type extension : Identical to R^3 to that order
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Consider the action

$$S = \frac{\eta}{2\kappa^2}\int d^3x\sqrt{-g}\left[\sigma R + \frac{2}{l^2} + \frac{1}{m^2}K + \frac{\xi}{12\mu^4}K'\right]$$

$$K = R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^3$$

$$K' = 17R^3 - 72R_{\mu\nu}R^{\mu\nu}R + 64R_\mu^\nu R_\nu^\rho R_\rho^\mu .$$

There are interesting relations between K and K'.

$$g_{\mu\nu}\frac{\partial K}{\partial R_{\mu\nu}} = -\frac{1}{4}R , \qquad g_{\mu\nu}\frac{\partial K'}{\partial R_{\mu\nu}} = -24K .$$

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EOM that we would like to solve is the following:

$$\sigma G_{\mu\nu} - \frac{1}{l^2} g_{\mu\nu} + \frac{1}{2m^2} K_{\mu\nu} - \frac{\xi}{12\mu^4} K'_{\mu\nu} = 0\,,$$

where

$$\begin{aligned}
K_{\mu\nu} &= g_{\mu\nu}\Big(3R_{\alpha\beta}R^{\alpha\beta} - \frac{13}{8}R^2\Big) + \frac{9}{2}RR_{\mu\nu} - 8R_{\mu\alpha}R^{\alpha}_{\nu} + \frac{1}{2}\Big(4\nabla^2 R_{\mu\nu} - \nabla_\mu\nabla_\nu R - g_{\mu\nu}\nabla^2 R\Big)\,,\\
K'_{\mu\nu} &= 17\Big[-3R^2R_{\mu\nu} + 3\nabla_\mu\nabla_\nu R^2 + \frac{1}{2}g_{\mu\nu}R^3 - 3g_{\mu\nu}\nabla^2R^2\Big]\\
&\quad -72\Big[-2RR_{\mu\alpha}R^{\alpha}_{\nu} - R_{\alpha\beta}R^{\alpha\beta}R_{\mu\nu} - \nabla^2(RR_{\mu\nu}) + \nabla_\mu\nabla_\nu(R_{\alpha\beta}R^{\alpha\beta}) + 2\nabla_\alpha\nabla_{(\mu}(R^{\alpha}_{\nu)}R)\\
&\quad +\frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}R - g_{\mu\nu}\nabla_\alpha\nabla_\beta(RR^{\alpha\beta}) - g_{\mu\nu}\nabla^2(R_{\alpha\beta}R^{\alpha\beta})\Big]\\
&\quad +64\Big[-3R^{\rho}_{\mu}R^{\sigma}_{\rho}R_{\sigma\nu} - \frac{3}{2}\nabla^2(R_{\mu\alpha}R^{\alpha}_{\nu}) + 3\nabla_\alpha\nabla_{(\mu}(R^{\beta}_{\nu)}R^{\alpha}_{\beta})\\
&\quad +\frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - \frac{3}{2}g_{\mu\nu}\nabla_\alpha\nabla_\beta(R^{\alpha\rho}R^{\rho\beta})\Big]\,.
\end{aligned}$$

The Born-Infeld Extension of New Massive Gravity

The Born-Infeld extension of the new massive gravity (BI-NMG)

$$S = -\eta \frac{2m^2}{\kappa^2} \int d^3x \sqrt{-g} \left[\sqrt{\det\left(\delta^{\mu}_{\nu} + \frac{\sigma}{m^2} G^{\mu}_{\nu}\right)} - 1 - \frac{1}{2m^2 l^2} \right] ,$$

where $G_{\mu\nu}$ denotes the Einstein tensor, and η, $\sigma = \pm 1$.

Equations of motion for the BI-NMG with $\sigma^2 = 1$ are given by

$$
\begin{aligned}
0 \;=\;& \sqrt{\det \mathcal{A}}\Big[2\mathcal{B}^{\alpha}{}_{(\mu}R_{\nu)\alpha} - \mathcal{B}R_{\mu\nu}\Big] - 2\sigma m^2 g_{\mu\nu}\Big(\sqrt{\det \mathcal{A}} - 1 - \frac{1}{2m^2 l^2}\Big) \\
& + g_{\mu\nu}\Big[\nabla_\alpha\nabla_\beta\Big(\sqrt{\det \mathcal{A}}\,\mathcal{B}^{\alpha\beta}\Big) - \nabla^2\Big(\sqrt{\det \mathcal{A}}\,\mathcal{B}\Big)\Big] \\
& + \nabla_\mu\nabla_\nu\Big(\sqrt{\det \mathcal{A}}\,\mathcal{B}\Big) + \nabla^2\Big(\sqrt{\det \mathcal{A}}\,\mathcal{B}_{\mu\nu}\Big) - \nabla^\alpha\nabla_\mu\Big(\sqrt{\det \mathcal{A}}\,\mathcal{B}_{\nu\alpha}\Big) \\
& - \nabla^\alpha\nabla_\nu\Big(\sqrt{\det \mathcal{A}}\,\mathcal{B}_{\mu\alpha}\Big)\,,
\end{aligned}
$$

where $\mathcal{A}$ and $\mathcal{B}$ are defined by

$$
\mathcal{A}^{\mu}{}_{\nu} \equiv \delta^{\mu}{}_{\nu} + \frac{\sigma}{m^2}G^{\mu}{}_{\nu}\,, \qquad \mathcal{B}^{\mu}{}_{\nu} \equiv (\mathcal{A}^{-1})^{\mu}{}_{\nu}\,, \qquad \mathcal{B} \equiv \mathcal{B}^{\mu}{}_{\mu}\,.
$$

Black Hole Solutions

Use dimensional reduction procedure of Clement and find solutions with stationary circular symmetry.

Take a metric ansatz with two Killing vectors ∂_t and ∂_ϕ as

$$ds^2 = \lambda_{ab}(\rho)dx^a dx^b + \zeta(\rho)^{-2}R(\rho)^{-2}d\rho^2\,,$$

$x^0 = t, x^1 = \phi$, $R(\rho)^2 = -\det\lambda$, $\zeta(\rho)$ is the scale factor for arbitrary reparametrizations of ρ.

$SL(2,R) \approx SO(2,1)$Lorentz Group suggests the parametrization of the matrix λ

$$\lambda = \begin{pmatrix} T(\rho)+X(\rho) & Y(\rho) \\ Y(\rho) & T(\rho)-X(\rho) \end{pmatrix}$$

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For the chosen metric ansatz, we can obtain the Ricci tensor components

$$\mathcal{R}^a{}_b = -\frac{\zeta}{2}\Big((\zeta R R')'\mathbf{1} + (\zeta\ell)'\Big)^a{}_b, \quad \mathcal{R}^\rho{}_\rho = -\zeta(\zeta R R')' + \frac{1}{2}\zeta^2(\vec{X}'^2),$$

where ℓ represents the matrix defined in terms of the components of the vector $\vec{L} = (L^T, L^X, L^Y) \equiv \vec{X} \wedge \vec{X}'$

$$\ell = \begin{pmatrix} -L^Y & -L^T + L^X \\ L^T + L^X & L^Y \end{pmatrix}.$$

$$\vec{X}\cdot\vec{Y} = \eta_{ij}X^iY^j, \quad (\vec{X}\wedge\vec{Y})^i = \eta^{ij}\epsilon_{jkl}X^kY^l,$$

with $\epsilon_{012} = +1$ for the wedge product.

$$
\begin{aligned}
\mathcal{R} &= \sigma\zeta^2\left\{-2(RR')' + \frac{1}{2}(\vec{X}'^2)\right\} - 2\sigma\zeta\zeta' RR' ,\\
K &= \zeta^4\left\{\frac{1}{2}(\vec{L}'^2) - \frac{1}{4}(RR')'(\vec{X}'^2) + \frac{5}{32}(\vec{X}'^2)^2\right\}\\
&\quad +\zeta\zeta'\left\{(\vec{L}\cdot\vec{L}') - \frac{1}{4}RR'(\vec{X}'^2)\right\} + \frac{1}{2}\zeta^2\zeta'^2(\vec{L}^2) ,\\
K' &= \zeta^6\left\{-\frac{3}{2}(RR')'(\vec{X}'^2)^2 + \frac{9}{8}(\vec{X}'^2)^3 + 24(RR')'(\vec{L}'^2) - 18(\vec{X}'^2)(\vec{L}'^2)\right\}\\
&\quad +\zeta^5\zeta'\left\{-\frac{3}{2}(RR')'(\vec{X}'^2)^2 + 24(RR')(\vec{L}'^2) + 48(RR')'(\vec{L}\cdot\vec{L}')\right.\\
&\quad \left.-36(\vec{X}'^2)(\vec{L}\cdot\vec{L}')\right\} + \zeta^4\zeta'^2\left\{24(RR')'(\vec{L}^2) - 18(\vec{X}'^2)(\vec{L}\cdot\vec{L}')\right\}\\
&\quad +24\zeta^3\zeta'^3(RR')(\vec{L}^2) .
\end{aligned}
$$

Equation of motion for R^3 case

$$0=\left[\xi\left(-\frac{33}{8}b^4+30b^2z+24z^2\right)-\frac{\mu^4}{m^2\zeta^2}\left(\frac{17}{4}b^2+2z\right)-2\frac{\mu^4}{\zeta^4}\sigma\right]\vec{\alpha}\,,$$

$$0=\frac{\xi}{12\mu^4}\left\{-\frac{3}{8}b^6-216b^2z^2+36b^4z-384z^3\right\}$$

$$+\frac{1}{\zeta^2m^2}\left\{2z^2-\frac{1}{32}b^4+3b^2z\right\}+\frac{\sigma}{\zeta^4}\frac{b^2}{2}-\frac{2}{\ell^2\zeta^6}\,,$$

BTZ Black Hole

If we consider the condition $\vec{\alpha} = 0$, then the vector ansatz reduces to

$$\vec{X} = \vec{\beta}\rho + \vec{\gamma}\,.$$

For $\vec{\beta} = [-\frac{1-\ell^2}{\ell^2}, -\frac{1+\ell^2}{\ell^2}, 0]$ and $\vec{\gamma} = [\frac{M(1+\ell^2)}{4}, \frac{M(1-\ell^2)}{4}, -\frac{J}{2}]$, we get rotating BTZ BH

$$ds^2 = \left(-\frac{2}{\ell^2}\rho + \frac{M}{2}\right)dt^2 - Jdtd\phi + \left(2\rho + \frac{M\ell^2}{2}\right)d\phi^2 + \frac{d\rho^2}{\left(\frac{4}{\ell^2}\rho^2 - \frac{M^2\ell^2 - J^2}{4}\right)}\,,$$

for $\zeta = 1$ and $b^2 = 4/\ell^2$.

Hawking Temperature

$$\kappa = \Big(-\frac{1}{2}\nabla_\mu \xi_\nu \nabla^\mu \xi^\nu \Big)^{1/2}\Big|_{r=r_+} = \frac{1}{L}\frac{\partial_r N}{\sqrt{g_{rr}}}\Big|_{r=r_+},$$

ξ : null Killing vector at the Horizon with the normalization as $\xi^2 \to -r^2$ for $r \to \infty$.

$$T_H = \frac{r_+}{2\pi L}\Big(1 - \frac{r_-^2}{r_+^2}\Big).$$

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R^3-NMG case

$\vec{\alpha} = 0$ gives the BTZ black hole solution. This solution has to satisfy, with $b^2 = 4/\ell^2$ and $\zeta = \ell/L$, L^2,

$$\sigma - \frac{L^2}{l^2} - \frac{1}{4m^2L^2} - \frac{\xi}{\mu^4 L^4} = 0\,.$$

The entropy is $S_{BH} = \frac{A_H}{4G}\eta\left(\sigma + \frac{1}{2m^2L^2} + \frac{\xi}{\mu^4L^4}\right)$, $A_H \equiv 2\pi L r_+$, where L^2 is given by the solution above condition. The mass and angular momentum

$$M = \frac{r_+^2 + r_-^2}{8G}\eta\left(\sigma + \frac{1}{2m^2L^2} + \frac{\xi}{\mu^4L^4}\right), J = \frac{Lr_+r_-}{4G}\eta\left(\sigma + \frac{1}{2m^2L^2} + \frac{\xi}{\mu^4L^4}\right).$$

The central charge for of the dual CFT as

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Warped AdS Black Hole Solutions

Put $\vec{\beta}^2 = b^2 = 1, (\vec{\beta}\cdot\vec{\gamma}) = 0$, then we have
$R^2 = (1-2z)\rho^2 + \vec{\gamma}^2 = \beta^2(\rho^2 - \rho_0^2)$, $(\vec{\gamma}^2 = -\beta^2\rho_0^2, (1-2z) = \beta^2)$.
In addition, we can choose vectors as follows
$(u = \beta^2\rho_0^2/4z + \omega^2 z)$

$$\vec{\alpha} = (1/2, -1/2, 0), \quad \vec{\beta} = (\omega, -\omega, -1), \quad \vec{\gamma} = (z+u, z-u, -2\omega z) \quad ,$$

which lead to the metric form

$$ds^2 = -\frac{\beta^2(\rho^2-\rho_0^2)}{\Delta^2}dt^2 + \frac{d\rho^2}{\zeta^2\beta^2(\rho^2-\rho_0^2)} + \Delta^2\left(d\phi - \frac{\rho + (1-\beta^2)\omega}{\Delta^2}dt\right)^2 ,$$

where Δ^2 is defined by

$$\Delta^2 = \rho^2 + 2\omega\rho + 2u = \rho^2 + 2\omega\rho + (1-\beta^2)\omega^2 + \frac{\beta^2\rho_0^2}{1-\beta^2} .$$

By a suitable coordinate transformation with parameters relations

$$\rho_0 = \frac{1}{2}(r_+ - r_-)\,,\ (1-\beta^2)\omega = \frac{1}{2}\Big(r_+ + r_- - 2\beta\sqrt{r_+ r_-}\Big)\,,\ \beta^2 = \frac{\nu^2+3}{4\nu^2}\,,$$

we get

$$ds^2 = L^2\Big[-N(r)^2 dt^2 + R(r)^2(d\theta + N^\theta(r)dt)^2 + \frac{dr^2}{R(r)^2 N(r)^2}\Big]\,,$$

where

$$\begin{aligned} N(r)^2 &= \frac{(\nu^2+3)(r-r_+)(r-r_-)}{R(r)^2}\,, \\ N^\theta(r) &= \frac{2\nu r - \sqrt{r_+ r_-(\nu^2+3)}}{2R(r)^2}\,, \\ R(r)^2 &= \frac{r}{4}\Big(3(\nu^2-1)r + (\nu^2+3)(r_+ + r_-) - 4\nu\sqrt{r_+ r_-(\nu^2+3)}\Big)\,. \end{aligned}$$

If we take the values $b^2 = 1, z = (1-\beta^2)/2$ and $\zeta^2 = 3/(4\beta^2-1)\cdot 4/L^2$, then we obtain warped black holes solution and L and β has to satisfy (from EOM)

$$
\begin{aligned}
0 &= \sigma - \frac{3(4\beta^2-21)}{2(4\beta^2-1)}\frac{1}{m^2L^2} - \xi\frac{27(4\beta^2-3)(4\beta^2-15)}{(4\beta^4-1)^2}\frac{1}{\mu^4L^4}\,, \\
0 &= \sigma - \frac{4\beta^2-1}{3}\frac{L^2}{l^2} + \frac{3(16\beta^4-80\beta^2+63)}{4\beta^2-1}\frac{1}{4m^2L^2} + \frac{9(4\beta^2-3)(32\beta^4-108\beta^2+75)}{(4\beta^4-1)^2}\frac{\xi}{\mu^4L^4}\,.
\end{aligned}
$$

Note that L can be solved as

$$
L^2 = \frac{\sigma}{(4\beta^2-1)}\frac{3}{4m^2\mu^2}\left\{(4\beta^2-21)\mu^2 \pm \sqrt{(4\beta^2-21)^2\mu^4 + 48\xi\sigma(4\beta^2-3)(4\beta^2-15)m^4}\right\}.
$$

Using Wald's formula, one obtains

$$S_{BH} = \frac{A_H}{4G}\eta\left[\sigma + \frac{3(5-4\beta^2)}{2(4\beta^2-1)m^2L^2} - \xi\frac{9(3-4\beta^2)(13-12\beta^2)}{(4\beta^4-1)^2\mu^4L^4}\right],$$

$$A_H \equiv 2\pi L R(r_+) = 2\pi L\sqrt{\frac{3}{4\beta^2-1}}\left(r_+ - \beta\sqrt{r_+r_-}\right),$$

where β and L^2 are given by the solutions of the above equation.

The central charges for the warped AdS black holes by the Cardy formula

$$S = \frac{\pi^2 L}{3}(c_L T_L + c_R T_R) = \frac{\pi^2 L}{3}c(T_L + T_R),$$

which leads to

$$c = \frac{\eta L}{2G}\frac{\sqrt{3(4\beta^2-1)}}{\beta^2}\left[\sigma + \frac{3(5-4\beta^2)}{2(4\beta^2-1)m^2L^2} - \xi\frac{9(3-4\beta^2)(13-12\beta^2)}{(4\beta^4-1)^2\mu^4L^4}\right].$$

Different from what can be derived from the central charge function formalism.

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Different from what can be derived from the central charge function formalism.

Central Charge Function Formalism

$$c \equiv \frac{L}{2G} g_{\mu\nu} \frac{\partial \mathcal{L}}{\partial R_{\mu\nu}} \, .$$

gives

$$c = \frac{3L}{2G}\eta\left[\sigma + \frac{1}{2m^2L^2} - \xi\frac{3(4\beta^2-3)(4\beta^2-7)}{(4\beta^4-1)^2\mu^4L^4}\right],$$

which gives the different answer. This is not unexpected since the central charge function formalism is developed for the *AdS* space not warped *AdS* space.

New Type Black Hole

$$ds^2 = L^2\left[-(r^2+br+c)dt^2 + \frac{dr^2}{r^2+br+c} + r^2 d\phi^2\right].$$

When b is zero, this reduces the BTZ case of $r_- = 0$.
To satisfy the EOM's, we must have

$$l^2 = \frac{(12\sigma\xi m^4 + \mu^4)L^2 + 6\xi m^2}{m^2(8\xi m^2 + \mu^4 L^2)},$$

and L^2 is given by

$$L^2 = \frac{\sigma}{4m^2\mu^2}\left(\mu^2 \pm \sqrt{\mu^4 + 48\sigma\xi m^4}\right).$$

Note that in the NMG limit (or $\xi \to 0$) these conditions become
$l^2 = \frac{1}{m^2}$, $L^2 = \frac{1}{2m^2}$, $\sigma = 1$.

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For the $b \neq 0$ case, the black hole horizon is at $r_\pm = (-b \pm \sqrt{b^2 - 4c})/2$ and the entropy and the temperature of the black holes are given by

$$\begin{aligned} S_{BH} &= \frac{\pi L}{2G} \frac{\sqrt{b^2 - 4c}}{3} \eta \left[2\sigma + \frac{1}{2m^2 L^2} \right], \\ T_H &= \frac{\sqrt{b^2 - 4c}}{4\pi L} = \frac{r_+ - r_-}{4\pi L}, \end{aligned}$$

ADT formalism - Mass and Angular Momenta

Around a background metric $\bar{g}_{\mu\nu}$ we expand an arbitrary metric as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \, .$$

The background metric : the solution of equations of motion (EOM) $\mathcal{E}_{\mu\nu}(\bar{g}) = 0$.

The general covariance of gravity $\rightarrow$ the Bianchi identity for an arbitrary metric $g_{\mu\nu}$:

$$\nabla_{\mu}\mathcal{E}^{\mu\nu} = 0 \, .$$

We have the linearized Bianchi identity on the linearized EOM expression, $\delta\mathcal{E}^{\mu\nu}$, through EOM for the background metric $\bar{g}$, as $\bar{\nabla}_{\mu}\delta\mathcal{E}^{\mu\nu} = 0$.

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The ADT currents : the contraction of the linearized EOM expression, $\delta\mathcal{E}$ and a Killing vector ξ as $J^{\mu} \equiv \delta\mathcal{E}^{\mu\nu}\xi_{\nu}$.

Conserved covariantly by the linearized Bianchi identity of $\delta\mathcal{E}^{\mu\nu}$ and the Killing property of ξ_{ν}.

Under EOM of the background metric, antisymmetric tensor potential, $Q^{\mu\nu}$ for this current can defined by

$$J^{\mu} = \nabla_{\nu}Q^{\mu\nu} ,$$

which guarantees current conservation by the antisymmetric property of the potential $Q^{\mu\nu}$. To define the potential, Q without EOM, we have $Q^{\mu\nu}$

$$J^{\mu} = \delta\mathcal{E}^{\mu\nu}\xi_{\nu} \equiv -\mathcal{E}^{\mu\alpha}h_{\alpha\nu}\xi^{\nu} + \frac{1}{2}\xi^{\mu}\mathcal{E}^{\alpha\beta}h_{\alpha\beta} - \frac{1}{2}h\mathcal{E}^{\mu}_{\nu}\xi^{\nu} + \nabla_{\nu}Q^{\mu\nu} ,$$

which reduces the previous definition of the potential when EOM is imposed.

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ADT Charges

$$Q(\xi) = \frac{1}{2\kappa^2} \int d\Sigma_{\mu\nu} Q^{\mu\nu}(\xi) \,,$$

where $2\kappa^2 = 16\pi G$ is the Newton's constant. Mass and angular momentum are

$$M = \frac{1}{4G} \sqrt{-\det g}\, Q^{rt}(\xi_T)\Big|_{r\to\infty} \,, \qquad J = \frac{1}{4G} \sqrt{-\det g}\, Q^{rt}(\xi_R)\Big|_{r\to\infty} \,,$$

$$\xi_T = \frac{1}{L}\frac{\partial}{\partial t} \,, \qquad \xi_R = \frac{\partial}{\partial \phi} \,,$$

EOM for New Massive Gravity

$$\mathcal{E}_{\mu\nu} = \eta\left[\sigma G_{\mu\nu} - \frac{1}{l^2} g_{\mu\nu} + \frac{1}{2m^2} K_{\mu\nu}\right] = 0\,,$$

where

$$K_{\mu\nu} = g_{\mu\nu}\left(3R_{\alpha\beta}R^{\alpha\beta} - \frac{13}{8}R^2\right) + \frac{9}{2}RR_{\mu\nu} - 8R_{\mu\alpha}R^{\alpha}_{\nu} + \frac{1}{2}\left(4\nabla^2 R_{\mu\nu} - \nabla_\mu\nabla_\nu R - g_{\mu\nu}\nabla^2 R\right).$$

Clement's superangular momentum

Let us take the three dimensional metric ansatz as

$$ds^2 = \lambda_{ab}(\rho)dx^a dx^b + \frac{d\rho^2}{\zeta^2 U^2(\rho)}\,, \qquad x^a = (t, \phi).$$

where all variables are functions only of the radial coordinate ρ. Explicitly, one may parameterize generic symmetric two by two matrix λ as

$$\lambda_{ab} = \begin{pmatrix} X^0 + X^1 & X^2 \\ X^2 & X^0 - X^1 \end{pmatrix}\,,$$

define the inner and cross product of $SO(1,2)$ vectors in the standard way as

$$\mathbf{A}\cdot\mathbf{B}=\eta_{ij}A^iB^j\,,\qquad (\mathbf{A}\times\mathbf{B})^i=\eta^{im}\epsilon_{mjk}A^jB^k\,,\qquad (\epsilon_{012}=1)$$

and note that the product of two matrices dual to vectors **A** and **B** is given by

$$\langle\mathbf{A}\rangle\langle\mathbf{B}\rangle=(\mathbf{A}\cdot\mathbf{B})\,\mathbf{1}+\langle\mathbf{A}\times\mathbf{B}\rangle\,.$$

L and Σ are defined by

$$\mathbf{L}\equiv\mathbf{X}\times\mathbf{X}'\,,\qquad \Sigma\equiv\mathbf{X}\times\delta\mathbf{X}\,.$$

$$\eta\left[\sigma Q_R^{\rho t} + \frac{1}{m^2} Q_K^{\rho t}\right]_{\xi_T} = \frac{1}{L}\left[-\frac{\zeta^2}{2}\,\delta J^2 + \Delta_{Cor}\right],$$

$$\eta\left[\sigma Q_R^{\rho t} + \frac{1}{m^2} Q_K^{\rho t}\right]_{\xi_R} = \frac{\zeta^2}{2}\,\delta(J^0 - J^1)\,,$$

where $\mathbf{J} = (J^0, J^1, J^2)$ is given by

$$\mathbf{J} = \eta\left(\sigma\,\mathbf{L} + \frac{\zeta^2}{m^2}\left[2\mathbf{L}\times\mathbf{L}' + \mathbf{X}^2\,\mathbf{L}'' + \frac{1}{8}\Big(\mathbf{X}'^2 - 4\mathbf{X}\cdot\mathbf{X}''\Big)\mathbf{L}\right]\right).$$

Finally, mass and angular momentum in NMG can be obtained by

$$M = \frac{1}{4G}\sqrt{-\det g}\,\eta\left[\sigma Q_R^{rt} + \frac{1}{m^2}Q_K^{rt}\right]_{\xi_T,\, r\to\infty},$$

$$J = \frac{1}{4G}\sqrt{-\det g}\,\eta\left[\sigma Q_R^{rt} + \frac{1}{m^2}Q_K^{rt}\right]_{\xi_R,\, r\to\infty}.$$

Mass of New Type black hole can be calculated

$$M = \frac{1}{4\zeta GL}\left[-\frac{\zeta^2}{2}\Delta J^2\right] = \frac{b^2-4c}{16G}$$

which is consistent with the result by AdS/CFT correspondence and satisfies the simple form of the first law of black hole.

New Type Black Holes in NMG

$$ds^2 = L^2\left[-(r^2+br+c)dt^2 + \frac{dr^2}{(r^2+br+c)} + r^2 d\phi^2\right]$$

where

$$L^2 = \frac{1}{2m^2} = \frac{l^2}{2},$$

with outer and inner horizons at $r_\pm = \frac{1}{2}(-b \pm \sqrt{b^2-4c})$.

The scalar curvature of this metric $R = -\frac{6}{L^2} - \frac{2b}{L^2 r}$,

There is a curvature singularity at $r = 0$ when $b \neq 0$.

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New type black holes may be classified according to signs of parameters b and c. This classification may be tabulated by introducing a new parameter $q \equiv r_-/r_+$, as follows:

$q \equiv r_-/r_+$	b and c	r_+ and r_-
$q = 1$	$b < 0,\ c > 0\ (b^2 = 4c\ )$	$r_+ > 0\ , r_- > 0\ (r_+ = r_-)$
$0 < q < 1$	$b < 0,\ c > 0$	$r_+ > 0\ , r_- > 0\ (r_+ > r_-)$
$q = 0$	$b < 0,\ c = 0$	$r_+ > 0,\ r_- = 0$
$-1 < q < 0$	$b < 0,\ c < 0$	$r_+ > 0,\ r_- < 0\ (r_+ > \|r_-\|)$
$q = -1$	$b = 0,\ c < 0$	$r_+ > 0,\ r_- < 0\ (r_+ = -r_-)$
$q < -1$	$b > 0,\ c < 0$	$r_+ > 0,\ r_- < 0\ (r_+ < \|r_-\|)$

Note that $q = -1$ cases are nothing but non-rotating BTZ BHs.

Exact QNMs of New Type Black Hole

The EOM of scalar fields (mass m on new type black hole backgrounds $\nabla^2\Psi - m^2\Psi = 0$,

By the separation of variables $\Psi = R(r)e^{i\omega t + i\mu\phi}$ with the change of variable, $z \equiv \frac{r-r_+}{r-r_-}$, the radial equation of the EOM

$$R''(z) + \left[\frac{1}{z} - \frac{1}{z-1} + \frac{1}{z-z_0}\right]R'(z) + \frac{1}{z(z-1)(z-z_0)} \times \left[\frac{(z-1)(z-z_0)}{z(z_0-1)^2}\frac{\omega^2}{r_-^2} - \frac{(z-z_0)}{(z-1)}m^2 - \frac{(z-1)}{(z-z_0)}\frac{\mu^2}{r_-^2}\right]R(z) = 0,$$

with $z_0 \equiv r_+/r_- = 1/q$ and $' = d/dz$.

Wednesday, March 30,

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with $z_0 \equiv r_+/r_- = 1/q$ and $' = d/dz$.

Define $R(z) = z^{\alpha}(1-z)^{\beta}(z_0 - z)^{\gamma}\mathcal{H}(z)$ and choosing parameters α, β and γ without loss of generality as

$$\alpha = \frac{i\omega}{(1-1/z_0)r_+}\,, \quad \beta = 1 - \sqrt{1+m^2} \;\text{ and }\; \gamma = \frac{\mu}{r_+}\sqrt{z_0}\,,$$

We get a differential equation for $\mathcal{H}(z)$ in the form of

$$\mathcal{H}''(z) + \left[\frac{2\alpha+1}{z} + \frac{2\beta-1}{z-1} + \frac{2\gamma+1}{z-z_0}\right]\mathcal{H}'(z) + \Big[z_0(\alpha+\beta-2\alpha\beta-\beta^2)$$
$$-\alpha-\gamma-2\alpha\gamma-\gamma^2 + \left\{(\alpha+\beta+\gamma)^2-\alpha^2\right\}z\Big]\frac{\mathcal{H}(z)}{z(z-1)(z-z_0)} = 0\,,$$

Heun's differential equation.

$$\mathcal{H}''(z) + \left[\frac{\nu}{z} + \frac{\delta}{z-1} + \frac{\epsilon}{z-z_0}\right]\mathcal{H}'(z) + \frac{(\lambda\xi z - \eta)}{z(z-1)(z-z_0)}\mathcal{H}(z) = 0\,,$$

with the condition $\epsilon = \lambda + \xi - \nu - \delta + 1$.

Wednesday, March 30,

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- The asymptotic QNM frequencies of scalar fields on new type black holes.
- Use Stokes line method : matching the approximate solutions of the radial equation by the analytic continuation of the radial coordinate to the complex plane.
- It is sufficient to consider the asymptotic solutions near the infinity, the origin and the event horizon.
- The approximate solutions are matched with appropriate boundary conditions.
- Through these matchings, one can read off the QNM frequencies of scalar fields on new type black holes.

- Solve the scalar perturbation equation in each region.
- The tortoise coordinates x as follows $\frac{dx}{dr} = \frac{1}{(r-r_+)(r-r_-)}$, With $x(r=0) = x_0$, one obtains

$$x = \frac{1}{r_+ - r_-} \ln \frac{r - r_+}{r - r_-} + x_0 \,,$$

where

$$x_0 \equiv \frac{1}{r_+ - r_-} \ln \left(\frac{r_-}{r_+} \right) = \frac{1}{2\kappa_+} \ln \left(\frac{r_-}{r_+} \right) .$$

In each region, the tortoise coordinate x behaves like

$$x \simeq \begin{cases} \frac{r}{r_+ r_-} & r \to 0 \\ -\infty & r \to r_+ \\ x_0 - \frac{1}{r} & r \to \infty \end{cases} .$$

By denoting $\Phi(x(r)) \equiv \sqrt{r}R(r)$, the radial equation of scalar field equation can be written as

$$\left[-\frac{d^2}{dx^2} - \omega^2 + U(x) \right] \Phi(x) = 0\,,$$

where the potential term is given by

$$U(x) = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)\left[\mu^2 + m^2 r^2 + \frac{1}{4}\Big(3r^2 - (r_+ + r_-)r - r_+ r_-\Big)\right].$$

The potential term in each region becomes

$$U(x) \simeq \begin{cases} \frac{r_+ r_-}{r^2}\left(\mu^2 - \frac{r_+ r_-}{4}\right) = \frac{j^2-1}{4x^2}\,, \quad j^2 \equiv \frac{4\mu^2}{r_+ r_-} & r \to 0 \ \ (x \to 0) \\ 0 & r \to r_+ \ \ (x \to -\infty) \\ \left(\frac{3}{4} + m^2\right) r^2 = \frac{j_\infty^2 - 1}{4(x-x_0)^2}\,, \quad j_\infty \equiv 2\sqrt{1+m^2} & r \to \infty \ \ (x \to x_0) \end{cases} .$$

The radial function, $\Phi(x)$:

$$\Phi(x) = \begin{cases} A_+\sqrt{2\pi\omega x}J_{\frac{j}{2}}(\omega x) + A_-\sqrt{2\pi\omega x}J_{-\frac{j}{2}}(\omega x)\,, & r \to 0 \ \ (x \to 0) \\ C_+\sqrt{2\pi\omega(x_0-x)}J_{\frac{j_\infty}{2}}(\omega(x_0-x)) & \\ \qquad +C_-\sqrt{2\pi\omega(x_0-x)}J_{\frac{-j_\infty}{2}}(\omega(x_0-x))\,, & r \to \infty \ \ (x \to x_0) \\ D_+e^{i\omega x} + D_-e^{-i\omega x}\,, & r \to r_+ \ \ (x \to -\infty) \end{cases}$$

where $A_\pm$,$C_\pm$ and $D_\pm$ are complex constants and $J_{\pm\frac{j}{2}}(\omega x)$ and $J_{\pm\frac{j_\infty}{2}}(\omega(x_0-x))$ represent first kind Bessel functions.

- The Dirichlet boundary condition near the spatial infinity : $C_- = 0$.
- The condition of $D_- = 0$ comes from the boundary condition for QNMs near the event horizon r_+: $\Phi(x)$ should contain only ingoing mode $e^{i\omega x}$ at there.

We consider $r_- < 0$ case only.
Using the asymptotic expansion of the first kind Bessel function in the region of $|\omega x| \to \infty$, we get

$$\begin{aligned}
\Phi_o(x) &\sim e^{-i\frac{\pi}{4}}\left(A_+ e^{-i\frac{j\pi}{4}} + A_- e^{i\frac{j\pi}{4}}\right)e^{i\omega x} + e^{i\frac{\pi}{4}}\left(A_+ e^{i\frac{j\pi}{4}} + A_- e^{-i\frac{j\pi}{4}}\right)e^{-i\omega x}\,, \\
\Phi_\infty(x) &\sim e^{i\frac{\pi}{4}}\left(C_+ e^{-i\omega x_0} e^{i\frac{j_\infty \pi}{4}}\right)e^{i\omega x} + e^{-i\frac{\pi}{4}}\left(C_+ e^{i\omega x_0} e^{-i\frac{j_\infty \pi}{4}}\right)e^{-i\omega x}\,.
\end{aligned}$$

Matching these two functions gives us a relation

$$A_+ \cos\left[\omega x_0 - \frac{(j_\infty + j)\pi}{4}\right] + A_- \cos\left[\omega x_0 - \frac{(j_\infty - j)\pi}{4}\right] = 0\,.$$

By matching this solution near the origin with the solution near the event horizon one obtains

$$A_+ e^{-i\frac{3j\pi}{4}} + A_- e^{i\frac{3j\pi}{4}} = 0\,.$$

The existence of nontrivial solutions implies

$$\omega x_0 = \left(n + \frac{1}{2} + \frac{j_\infty}{4}\right)\pi - \frac{i}{2}\ln\left[2\cos\left(\frac{j\pi}{2}\right)\right]\,.$$

Since we know the exact form of x_0 we can obtain the QNM frequencies analytically.

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Substituting $\kappa_+ = (1-q)r_+/2 = 2\pi T_H$ we obtain

$$x_0 = \frac{1}{4\pi T_H}\ln(q) = \frac{1}{4T_H}\left(i + \frac{1}{\pi}\ln|q|\right).$$

Rewriting j in terms of q as

$$j = i\frac{1-q}{\sqrt{|q|}}\frac{\mu}{2\pi T_H}.$$

QNM frequencies of scalar perturbation on new type black holes can be written as

$$\omega_{QNM} = i\frac{4\pi^3 T_H}{\pi^2 + (\ln|q|)^2}\left(n + \frac{1}{2} + \frac{1}{2}\sqrt{1+m^2}\right) + i\frac{2\pi T_H \ln|q|}{\pi^2 + (\ln|q|)^2}\ln\left[2\cosh\left(\frac{1-q}{\sqrt{|q|}}\frac{\mu}{4T_H}\right)\right]$$
$$-\frac{4\pi^2 T_H \ln|q|}{\pi^2 + (\ln|q|)^2}\left(n + \frac{1}{2} + \frac{1}{2}\sqrt{1+m^2}\right) + \frac{2\pi^2 T_H}{\pi^2 + (\ln|q|)^2}\ln\left[2\cosh\left(\frac{1-q}{\sqrt{|q|}}\frac{\mu}{4T_H}\right)\right].$$

In the limit of the non rotating BTZ black hole case i.e. $q = -1$ QNM frequencies become the simple form such as

$$\omega_{QNM} = i4\pi T_H \left(n + \frac{1}{2} + \frac{1}{2}\sqrt{1+m^2} \right) + 2T_H \ln \left[2\cosh\left(\frac{\mu}{2T_H} \right) \right].$$

This is exactly the same as form as the BTZ QNM.

Summary

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- Black holes in 3D : BTZ, Warped BTZ, New Type BH
- New Type Black Holes have many properties to be calculated (Mass, Angular Momentum)
- Quasinormal modes of New Type Black Hole

- Outlook
  - Quantization of New Type Black Hole Area
  - Beyond AdS/CFT

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