Measurement of Branching Fractions for $\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau$ and $\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$ Decays in the Belle Experiment

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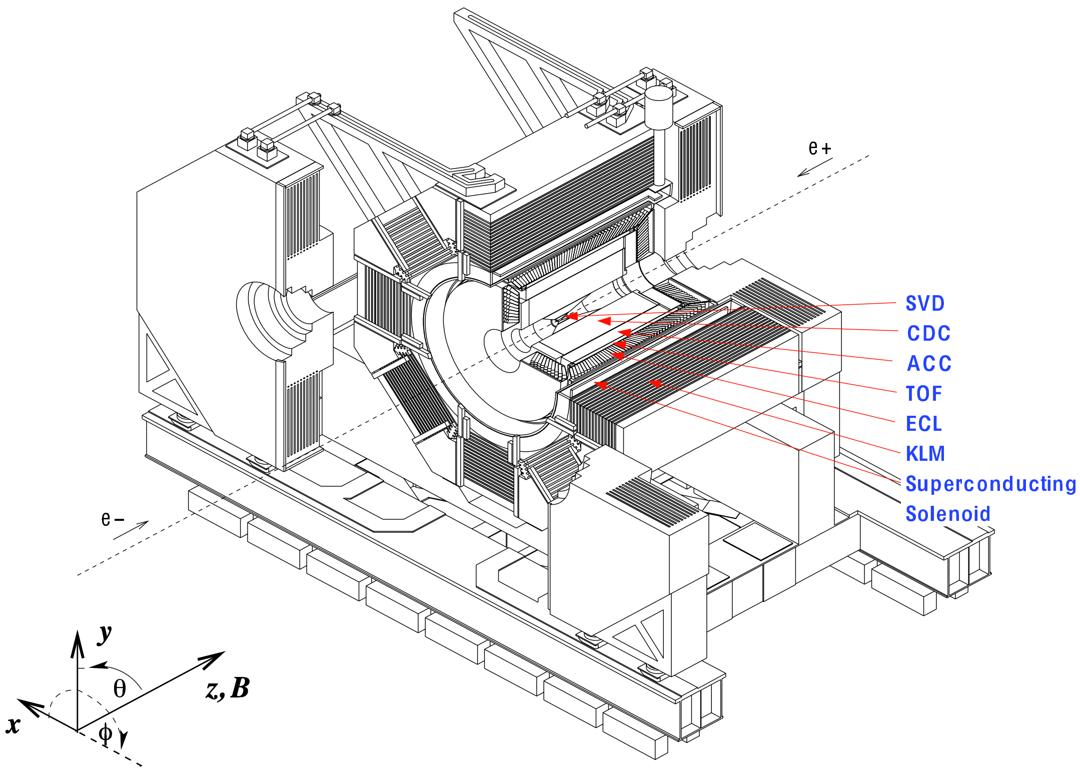
2010.04.14. KISTI

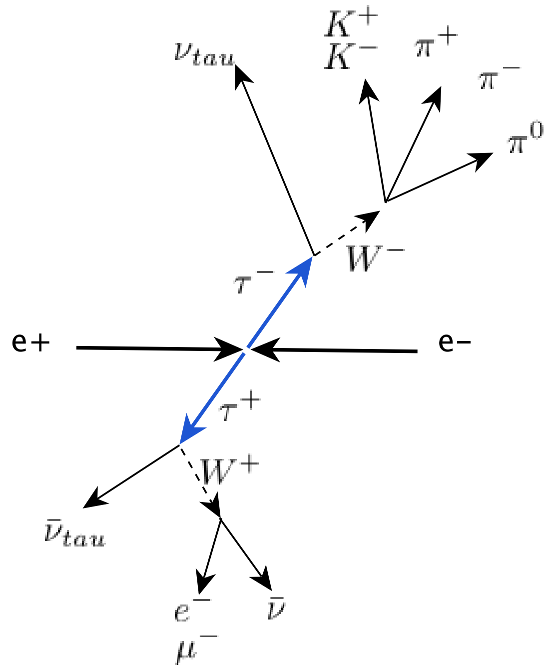
Introduction

1. tau is only lepton massive enough to decay into hadrons

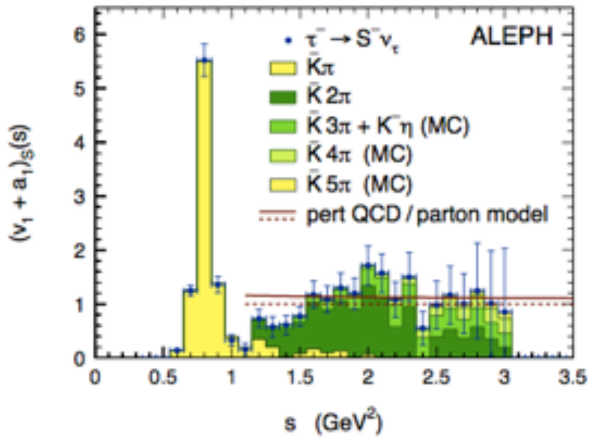
2. tau decay into 3 particles (included $\pi^{0)}$ has been studied since the discovery of tau lepton.

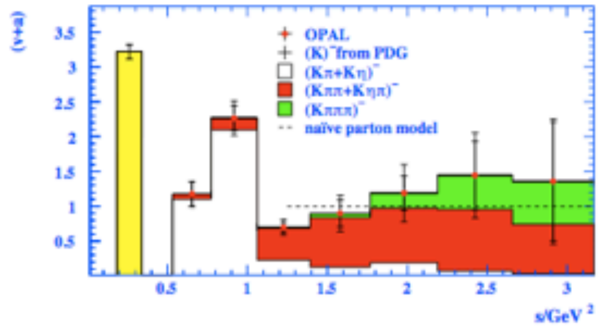
3. final state with a strangeness particle, has recently been interest, since its potential to use in $|V_{us}|$ and strange quark mass m_s measurement.





Strangeness spectral function





$$\text{ALEPH: } |V_{us}| = 0.2204 \pm 0.0028_{\text{exp}} \pm 0.0003_{\text{th}} \pm 0.0001_{\text{m}_s}$$

$$\text{J.Prades from OPAL data: } |V_{us}| = 0.2219 \pm 0.0034, \;\; m_s = (81 \pm 20) \text{ MeV}$$

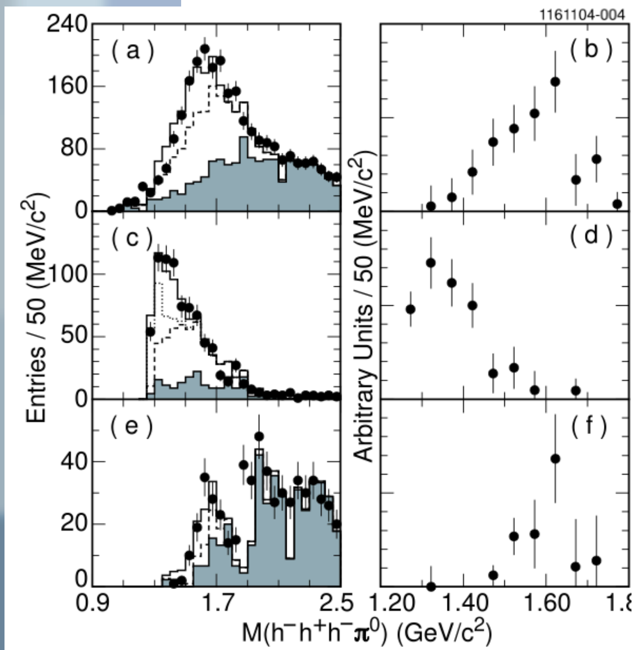
$$m_s - \text{J.G.Körner,A.Pivovarov,2001-2005}, \;\; m_s = (130 \pm 27) \text{ MeV}$$

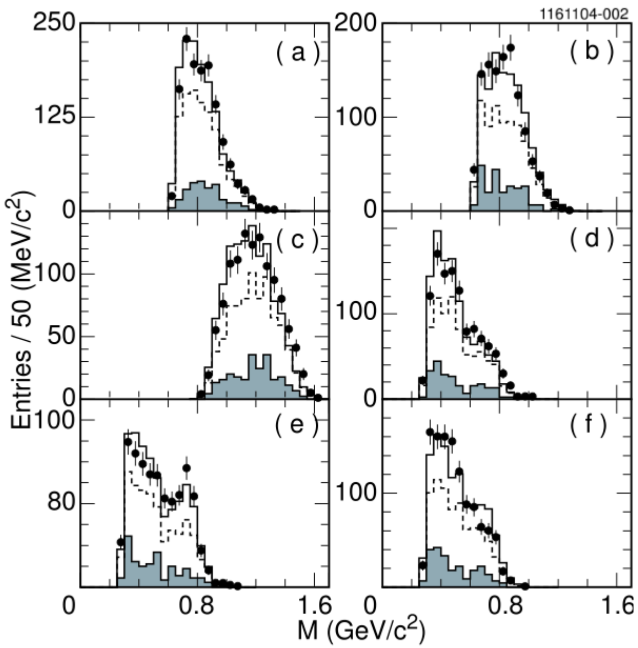
$$v_1^{\mathrm{S}}(s)/a_1^{\mathrm{S}}(s) = \frac{m_\tau^2}{6|V_{\mathrm{us}}|^2 S_{\mathrm{ew}}} \left(1 - \frac{s}{m_\tau^2}\right)^{-2} \left(1 + \frac{2s}{m_\tau^2}\right)^{-1} \frac{B(\tau \to (V/A)^{(S=-1,J=1)}\nu_\tau)}{B(\tau \to e^-\nu_e\nu_\tau)} \frac{1}{N_{\mathrm{V/A}}} \frac{\mathrm{d}N_{\mathrm{V/A}}}{\mathrm{d}s}$$

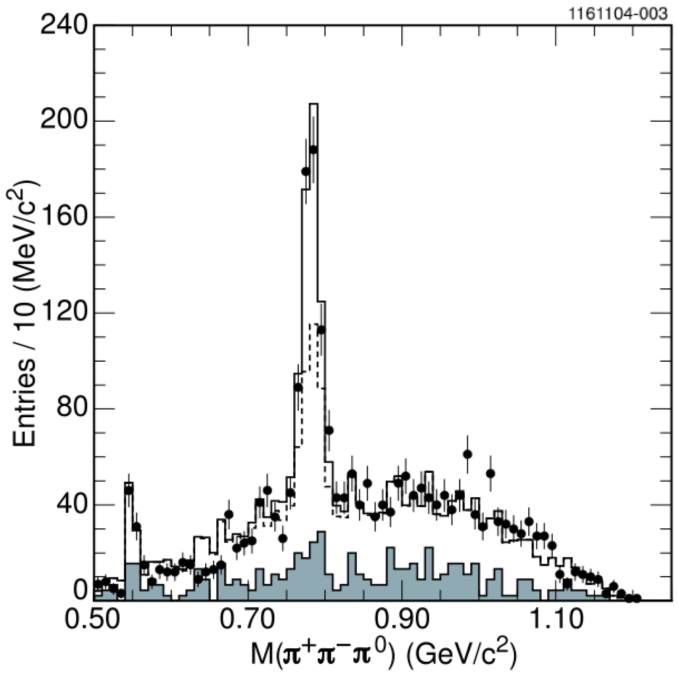
$|V_{us}|$ can be determined from the branching fractions of net strangeness decay modes as well as the spectral functions of those decays.

Recent CLEO III results

Reference : Phys.Rev.Lett.94:241802,2005







invariant mass spectra

$\tau^- \rightarrow K^-\pi^+\pi^-\pi^0\nu_\tau$ (ex.ω)

non-ω resonant

$\tau^- \rightarrow K^-\pi^+\pi^-\pi^0\nu_\tau$

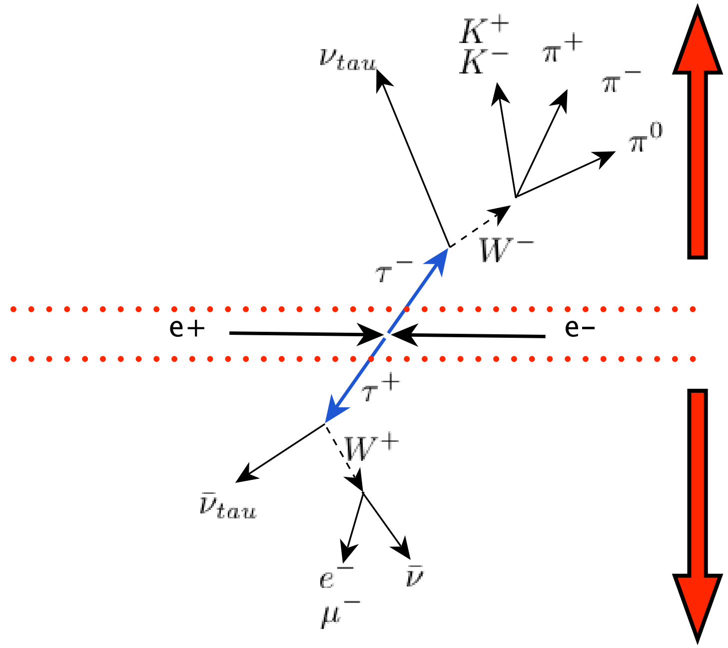
The branching fraction fractions for $\mathcal{B}(\tau^- \rightarrow K^-\pi^+\pi^-\pi^0\nu_\tau,\ \mathrm{ex}.K^0,\omega)$ and $\mathcal{B}(\tau^- \rightarrow K^-\omega\nu_\tau) \times \mathcal{B}(\omega \rightarrow \pi^+\pi^-\pi^0)$ can be combined to yield, $\mathcal{B}(\tau^- \rightarrow K^-\pi^+\pi^-\pi^0\nu_\tau,\ \mathrm{ex}.K^0) = (7.4 \pm 0.8 \pm 1.1) \times 10^-4$. (based on 7.6/fb).
This result is significantly more precise than OPAL's result.
They measured only branching fraction, not measured spectral function.

table of event selection

Title	Cut
Number of track Total charge	Number of good charged track $= 4$ sum of charged track $= 0$
Missing mass cut	$1.0GeV < M_{miss,CM} < 7.0GeV$ $30 < \theta_{miss,CM} < 150$
Particle ID	Electron : $prob(e) > 0.9$ Muon : $prob(\mu) > 0.8$ and no electron Kaon : $prob(K/pi) > 0.9$ Pion : $prob(pi/K) > 0.1$
3-1 prong	$\|thrust\| > 0.9$ Lepton and hadrons should locate opposite side based on the thrust
Inv. mass cut	$M_{inv,sig} < 1.8GeV$ $M_{inv,tag} < 1.8GeV$
Final Event selection	$N(K_s) = 0$ $N(\pi^0) \neq 0$ $N(E_\gamma)_{sig} \neq 0$

Event Selection

Two-hemisphere in e+e- CMS system



signal side
- 3 charged particle
  one charged kaon
  two charged pion
- 1 pi0
  No additional gamma with E_γ > 0.2GeV

tag side
- one electron or one muon

Used data : 71.1 /fb (exp 9 to 19)

Event selections :
- Start from the generic tau-pair selection
- Events are separated into two hemisphere with respect to the Thrust axis.
- Both $\tau^- \rightarrow K^-\pi^+\pi^-\pi^0\nu_\tau$ and $\tau^- \rightarrow \pi^-\pi^+\pi^-\pi^0\nu_\tau$ events are analyzed simultaneously.
- Selection of $\tau^- \rightarrow K^-\pi^+\pi^-\pi^0\nu_\tau$ candidates.
  ($\tau^- \rightarrow \pi^-\pi^+\pi^-\pi^0\nu_\tau$ selection is similar except pion ID instead of Kaon ID).

Select charged particles

$$\tau^{\pm} \rightarrow X_{sf} X_{ss} X_{op}$$

final state of in this study,
- one kaon and two charged pion
- two same charge and one opposite charge
- two pions are not same charge

To determine final particles,
- put the X_{sf}, X_{ss} and X_{op}
- X_{sf} is same charge and slower particle
- X_{ss} is same charge and faster particle
- X_{op} is opposite charge particle

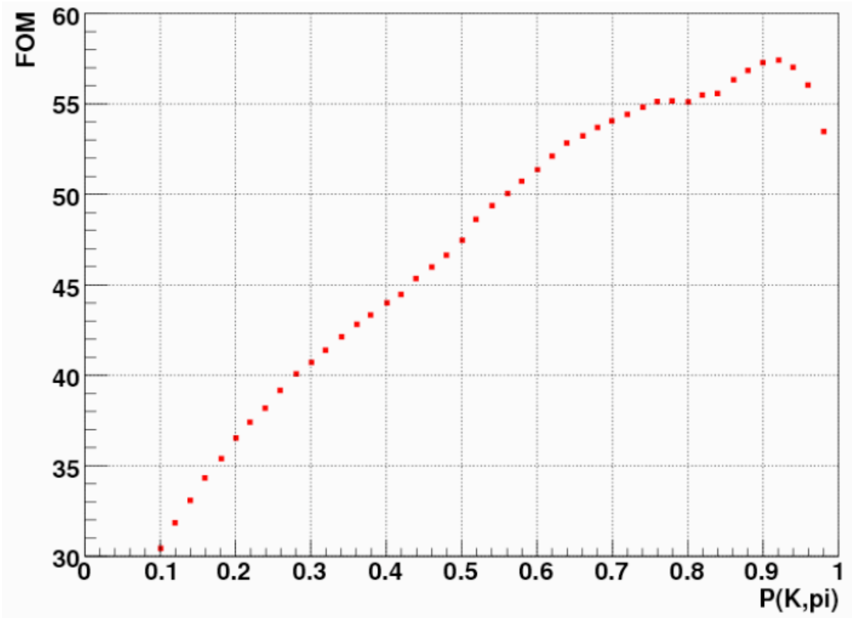
correct signal	wrong signal
Total charge = -1 $K^- \pi^- \pi^+$ $\pi^- K^- \pi^+$	**Total charge = -1** $\pi^- \pi^- K^+$
Total charge = +1 $K^+ \pi^+ \pi^-$ $\pi^+ K^+ \pi^-$	**Total charge = +1** $\pi^+ \pi^+ K^-$

Kaon/pion ID

To verify to usage of prob(K/π)>0.9 cut for Kaon selection.
Kaon ID is important in this analysis, because of large branching fraction for $\pi\pi\pi\pi^0$ decay mode than $K\pi\pi\pi^0$ decay mode.

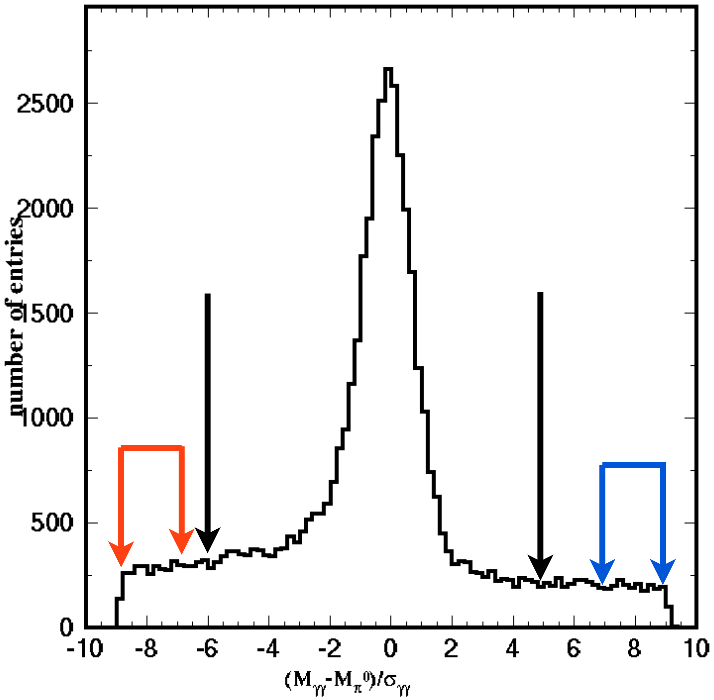
Cut:
- For Kaon selection : prob(K/π) > 0.9
- For Pion selection : prob(K/π) < 0.9 i.e. prob(π/K) > 0.1
- These condition is tight cut for Kaon and loose cut for Pion
- with changing prob(K/π) and prob(π/K) separately.
- I checked F.O.M (figure of merit).

$$F.O.M = \frac{S}{\sqrt{S+N}}$$



Selection of π^0 signal

The γγ invariant mass distribution for the different momentum region.
That has an asymmetric shape having the tail in the low mass side.



pi0 signal region

$$-6 < \frac{(M_{\gamma\gamma} - M_{\pi^0})}{\sigma_{\gamma\gamma}} < 5$$

pi0 side-band region

$$-9 < \frac{(M_{\gamma\gamma} - M_{\pi^0})}{\sigma_{\gamma\gamma}} < -7$$

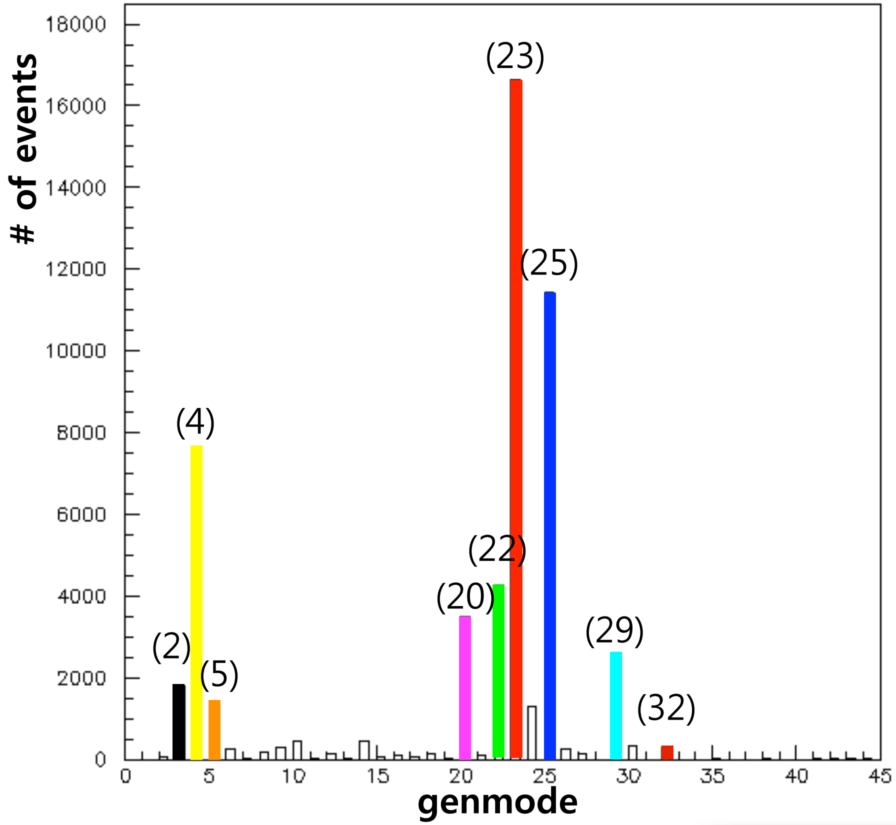
$$7 < \frac{(M_{\gamma\gamma} - M_{\pi^0})}{\sigma_{\gamma\gamma}} < 9$$

In order to select the pi0 signal, we used normalized γγ distribution.
$\sigma_{\gamma\gamma}$ is the averaged value of the r.m.s resolution.
The TRUE number of $\tau^- \to K^-\pi^+\pi^-\pi^0\nu_{tau}$ decay is thus detected as

$$N_{K^-\pi^+\pi^-\pi^0} = N^{signal}_{K^-\pi^+\pi^-\pi^0} - \frac{11}{4} N^{background}_{K^-\pi^+\pi^-\pi^0}$$

Event generate TAUOLA

genmode	mode	B.R	genmode	mode	B.R
0	$\tau^- \to e^-\bar{\nu}\nu_\tau$	0.19204	1	$\tau^- \to \mu\bar{\nu}\nu_\tau$	0.17370
2	$\tau^- \to \pi^-\nu_\tau$	0.10689	3	$\tau^- \to \pi^-\pi^0\nu_\tau(\rho)$	0.24908
4	$\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$	0.09203	5	$\tau^- \to \pi^-\pi^0\pi^0\pi^0\nu_\tau$	0.01019
14	$\tau^- \to \pi^-K_s\pi^0\nu_\tau$	0.00199	20	$\tau^- \to K^-K_s\pi^0\nu_\tau$	0.00075
22	$\tau^- \to \pi^-\pi^+\pi^-\nu_\tau$	0.08773	23	$\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$	0.04390
25	$\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau$	0.00120	29	$\tau^- \to \pi^-\pi^+\pi^-\pi^0\pi^0\nu_\tau$	0.00494
32	$\tau^- \to \pi^-\pi^+\pi^-\pi^0\pi^0\pi^0\nu_\tau$	0.00051			



Main decay modes of tau-lepton relevant in this analysis and their branching fraction used in the TAUOLA program.

"genmode" is a decay mode number assigned for our internal use.
The fraction of the event is proportional to the branching fraction implemented in the TAUOLA program.

The plot is the tau decay mode after apply the event selection explained so far.

Main background is from (4), (20), (22), (29) and etc.

Rejection of events : $\gamma \to e^+e^-$

$$\tau^- \to \pi^-\pi^0\pi^0\nu_\tau \to \pi^-\gamma\gamma\gamma\gamma,\ \gamma \to e^+e^-$$

pi0 decays to two gamma.

$$\pi^0 \to \gamma\gamma$$

electrons from $\pi^0 \to$ gamma gamma
If charged pion is mis-identified as Kaon and electrons are mis-identified as pion.

$\pi^-\pi^0\pi^0$ looks like $K^-\pi^+\pi^-\pi^0$ mode.

In order to reject which include $\gamma \to e^+e^-$ conversion, we using this condition.

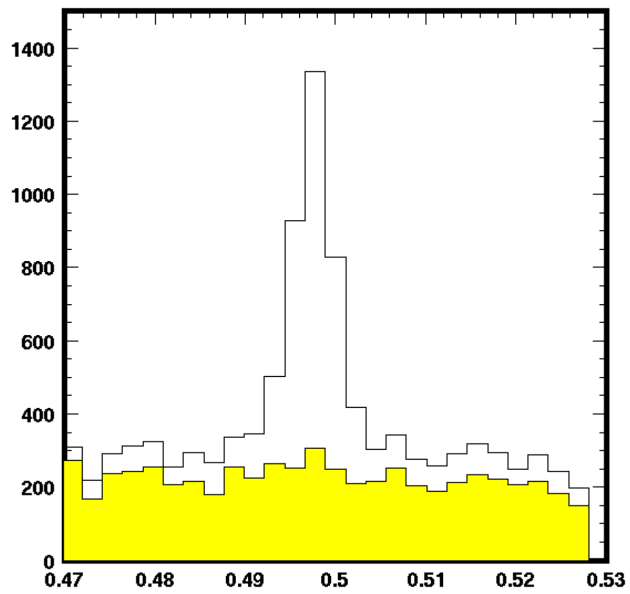
Cut : **M(e+e-)<0.1 GeV**

Rejection of events : K_s

K_s veto is more important since there is a significant background from $\tau \rightarrow K K_s \pi^0 \nu$ as well as $\tau \rightarrow \pi K_s \pi^0 \nu$

"good Ks" is selected from the following condition and the events are rejected if there is more than one "good Ks".

- "good Ks" condition :
    - (a) number of Ks candidates, (Ks >=1)
    - (b) Ks decay vertex position in the x-y plane (xyKs) unit is cm, (0.1<xyKs<20)
    - (c) Ks decay vertex position alone the beam axis. unit is cm, (abs(zKs)<20)
    - (d) Ks decay vertex position on 3 dimension from beam IP. unit is cm. (sqrt(disKs) > 0.4)



M(pi+pi-) invariant mass distribution for Ks candidates.
Yellow is background estimation from the good Ks condition

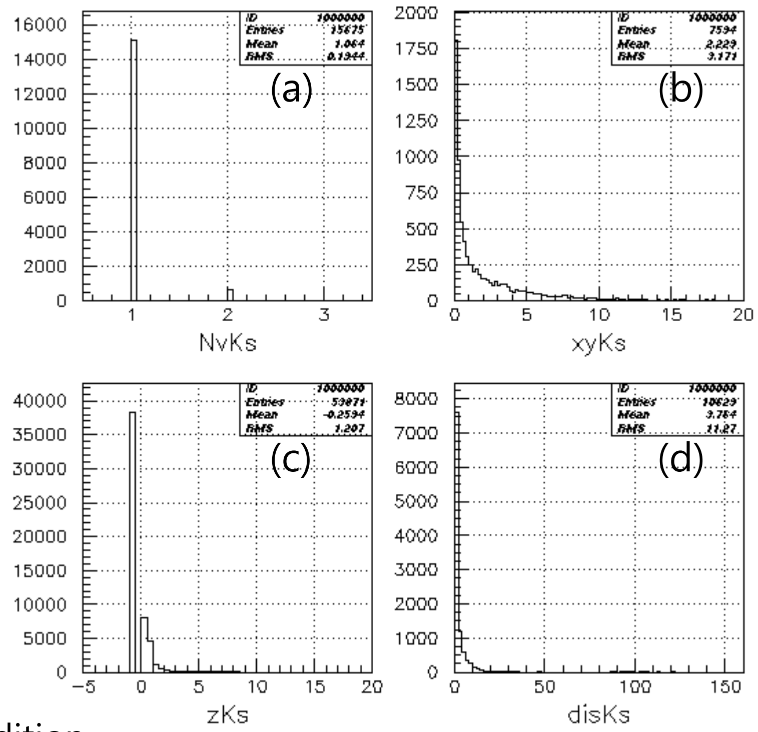


Table of event selection

$K^-\pi^+\pi^-\pi^0$

CLEO3 case,

$K^-\pi^+\pi^-\pi^0\nu_\tau$ (ex.K^0, ω)

= 833 +- 36

$K^-\omega$

= 500 +- 35

Selection Criteria	MC	DATA	$Q\bar{Q}$	$C\bar{C}$
Luminosity (fb^{-1})	67.42	72.13	144.75	174.49
After tau-pair event selection	4.03×10^7	4.54×10^7	2.03×10^6	1.25×10^6
Number of Track=3	5.81×10^6	6.99×10^6	6.77×10^5	4.93×10^5
Sum of Charge == ± 1	5.81×10^6	6.99×10^6	6.77×10^5	4.93×10^5
Kaon ID	3.24×10^5	7.14×10^5	1.63×10^5	8.07×10^4
$\pi^0 \neq 0$	5.40×10^4	1.64×10^5	5.04×10^4	2.82×10^4
No additional γs (Eg>0.2GeV)	4.44×10^4	1.25×10^5	3.34×10^4	1.70×10^4
one-track in tag-side	4.44×10^4	1.25×10^5	3.34×10^4	1.70×10^4
ID electron or muon in tag-side	1.88×10^4	3.57×10^4	8.78×10^2	3.46×10^3
N(high energy $\gamma > 0.2$GeV) ≤ 1	1.86×10^4	3.44×10^4	6.28×10^2	2.69×10^3
veto to $\gamma \to ee$ candidates	1.68×10^4	3.15×10^4	5.98×10^2	2.53×10^3
veto to K_s candidates	1.48×10^4	2.82×10^4	5.53×10^2	2.17×10^3
Final candidate (in π^0 region)	1.22×10^4	2.21×10^4	4.58×10^2	1.60×10^3
π^0 side band region	1.44×10^3	3.32×10^3	4.50×10^1	3.08×10^2

$\pi^-\pi^+\pi^-\pi^0$

Selection Criteria	MC	DATA	QQ	CC
Luminosity (fb^{-1})	67.42	72.13	144.75	174.49
After tau-pair event selection	4.03×10^7	4.54×10^7	2.03×10^6	1.01×10^6
Number of Track=3	5.81×10^6	6.99×10^6	6.77×10^5	3.99×10^5
Sum of Charge == ± 1	5.81×10^6	6.99×10^6	6.77×10^5	3.99×10^5
Kaon ID	5.31×10^6	5.77×10^6	3.99×10^5	2.55×10^5
$\pi^0 \neq 0$	9.47×10^5	1.16×10^6	1.46×10^5	9.81×10^4
No additional γs (Eg>0.2GeV)	8.13×10^5	9.55×10^5	8.84×10^4	5.62×10^4
one-track in tag-side	8.13×10^5	9.55×10^5	8.84×10^4	5.62×10^4
ID electron or muon in tag-side	3.45×10^5	3.50×10^5	2.16×10^3	1.39×10^4
N(high energy $\gamma > 0.2$GeV) ≤ 1	3.41×10^5	3.42×10^5	1.51×10^3	1.06×10^4
veto to $\gamma \to ee$ candidates	3.09×10^5	3.12×10^5	1.41×10^3	9.71×10^3
veto to K_s candidates	2.89×10^5	2.91×10^5	1.27×10^3	8.16×10^3
Final candidate (in π^0 region)	2.54×10^5	2.45×10^5	1.01×10^3	6.18×10^3
π^0 side band region	1.96×10^4	2.55×10^4	1.36×10^2	1.08×10^3

PID efficiency correction

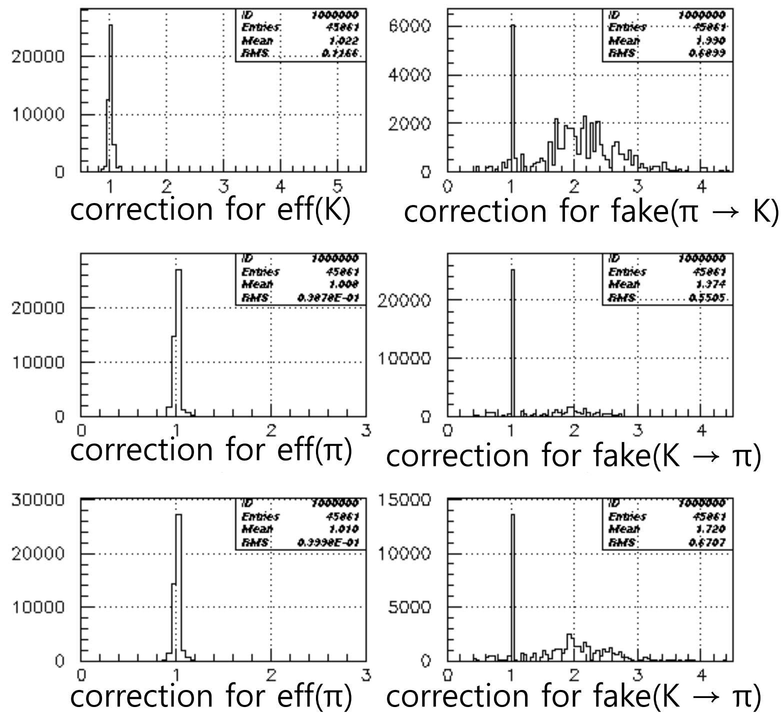
Since imperfect detector simulation may mean that the reconstruction (or selection) efficiencies differ between real data and Monte Carlo sample, some efficiency corrections are needed to apply to the Monte Carlo sample.

$$\frac{\eta^K_{data}}{\eta^K_{MC}} \quad \text{and} \quad \frac{\eta^\pi_{\mathrm{data}}}{\eta^\pi_{\mathrm{MC}}}$$

efficiency correction factor for Kaon and Pion

$$\frac{fake(\pi^- \to K^-)_{data}}{fake(\pi^- \to K^-)_{MC}} \quad \text{and} \quad \frac{\mathrm{fake}(\mathrm{K}^- \to \pi^-)_{\mathrm{data}}}{\mathrm{fake}(\mathrm{K}^- \to \pi^-)_{\mathrm{MC}}}$$

fake rate correction factor for Kaon and Pion



1. Read the momentum for 3 charged hadron tracks.

2. Multiply three efficiency or fake ratio to evaluate the efficiency or fake rate of current event.

cont.

efficiency correction factor for Kaon = 1.0205 +- 0.0100
efficiency correction factor for Pion = 0.9823 +- 0.0045
fake rate correction factor for Kaon = 1.0540 +- 0.1550
fake rate correction factor for Pion = 2.1218 +- 0.0858

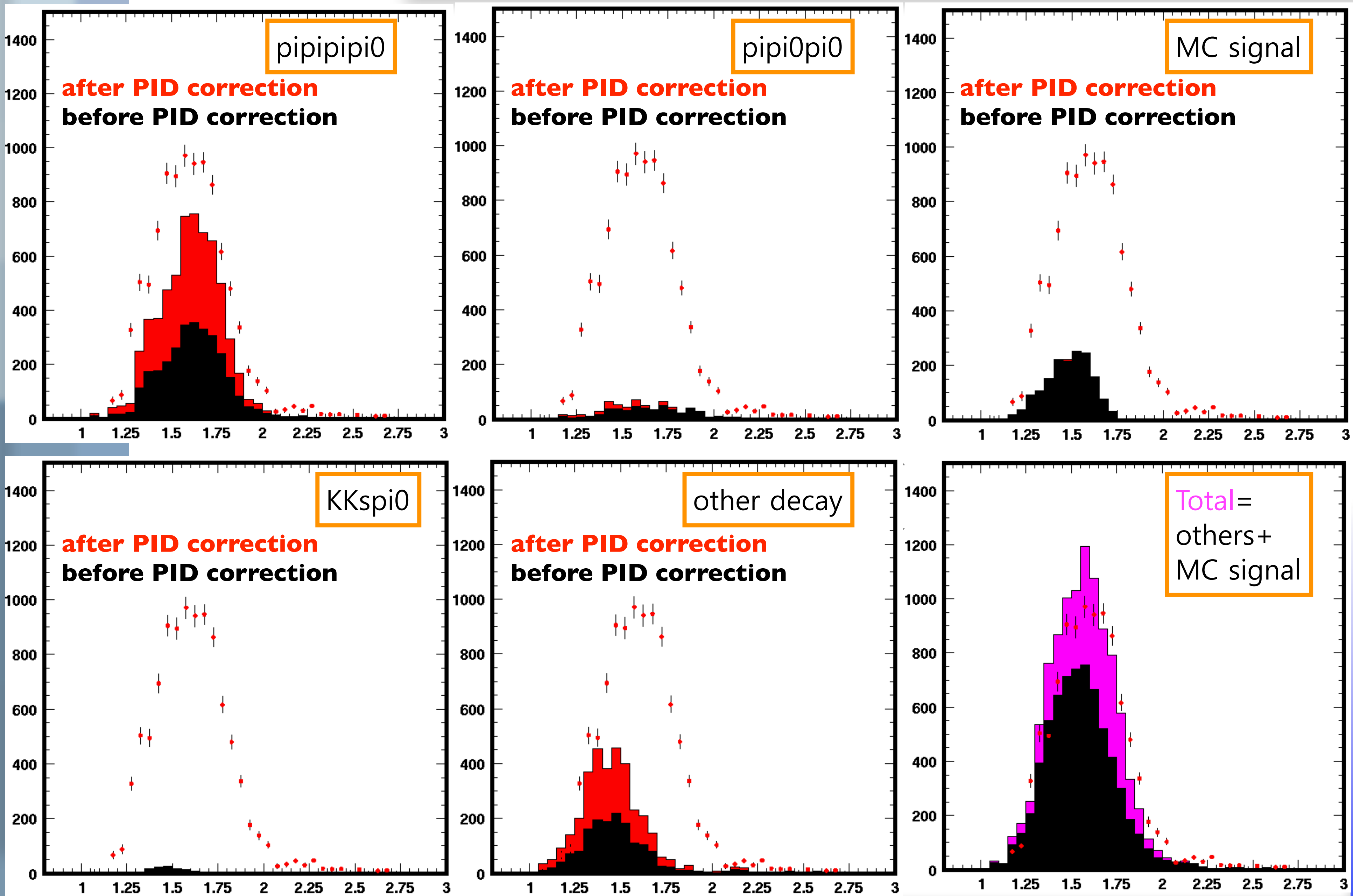
Kpipipi0 mode case,

1. $K\pi\pi\pi^0$, $KK_s\pi^0$
   : applied all efficiency correction factor for each particle
2. $\pi\pi\pi\pi^0$, $\pi\pi^0$, $\pi\pi^0\pi^0$, $\pi\pi\pi$, $\pi\pi\pi\pi^0\pi^0$
   : pion is miss-identified as kaon, applied fake rate factor for Kaon and efficiency correction factor for pion.
3. other(with Kaon) : applied with case 1.
4. other(without Kaon) : applied with case 2.

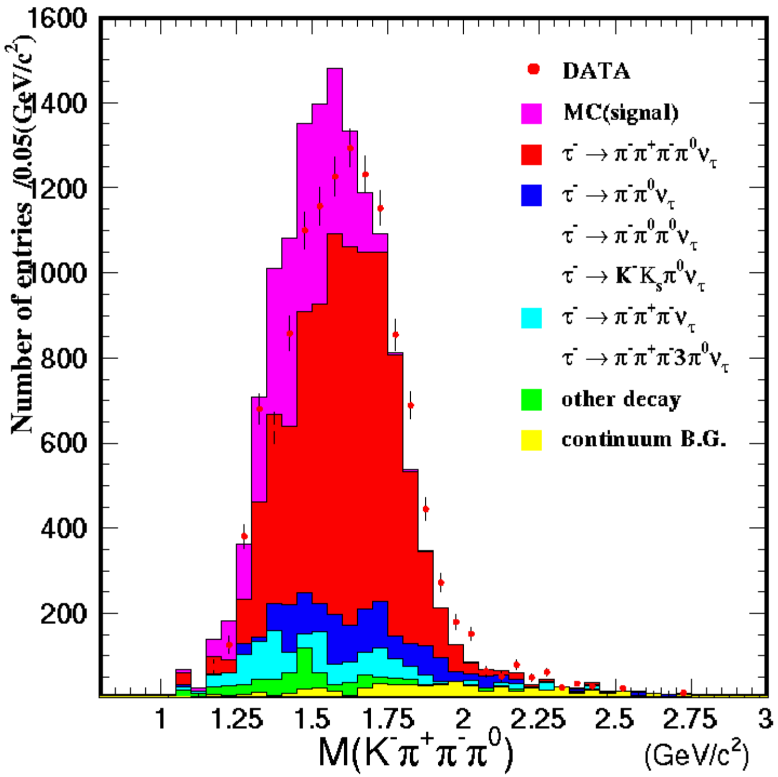
pipipipi0 mode case,

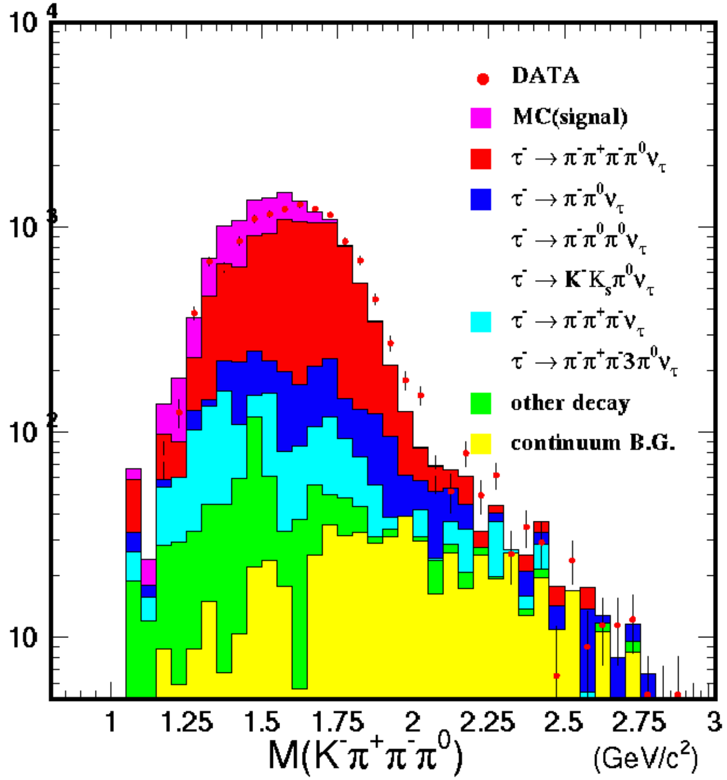
1. $K\pi\pi\pi^0$, $KK_s\pi^0$
   : kaon is miss-identified as pion, applied fake rate factor for pion and efficiency correction factor for pion.
2. $\pi\pi\pi\pi^0$, $\pi\pi^0$, $\pi\pi^0\pi^0$, $\pi\pi\pi$, $\pi\pi\pi\pi^0\pi^0$, $\pi\pi\pi\pi^0\pi^0\pi^0$, $\pi K_s\pi^0$
   : applied all efficiency correction factor for each particle
3. other(with Kaon) : applied with case 1.
4. other(without Kaon) : applied with case 2.

cont.


pipipipi0
after PID correction
before PID correction
pipi0pi0
after PID correction
before PID correction
MC signal
after PID correction
before PID correction
KKspi0
after PID correction
before PID correction
other decay
after PID correction
before PID correction
Total=
others+
MC signal

$\mathcal{M}(K^-\pi^+\pi^-\pi^0)$ for $\tau^- \to K^-\pi^+\pi^-\pi^0\nu$





Decay mode	f_i
$\pi^-\pi^0$	0.001
$\pi^-\pi^0\pi^0$	0.070
$K^-K_s\pi^0$	0.012
$\pi^-\pi^+\pi^-$	0.013
$\pi^-\pi^+\pi^-\pi^0\pi^0$	0.051
other decay w/ Kaon	0.020
other decay w/o Kaon	0.014
uds	0.023
charm	0.022
SUM	0.226
df_i	0.010
df_i/f_i	0.046

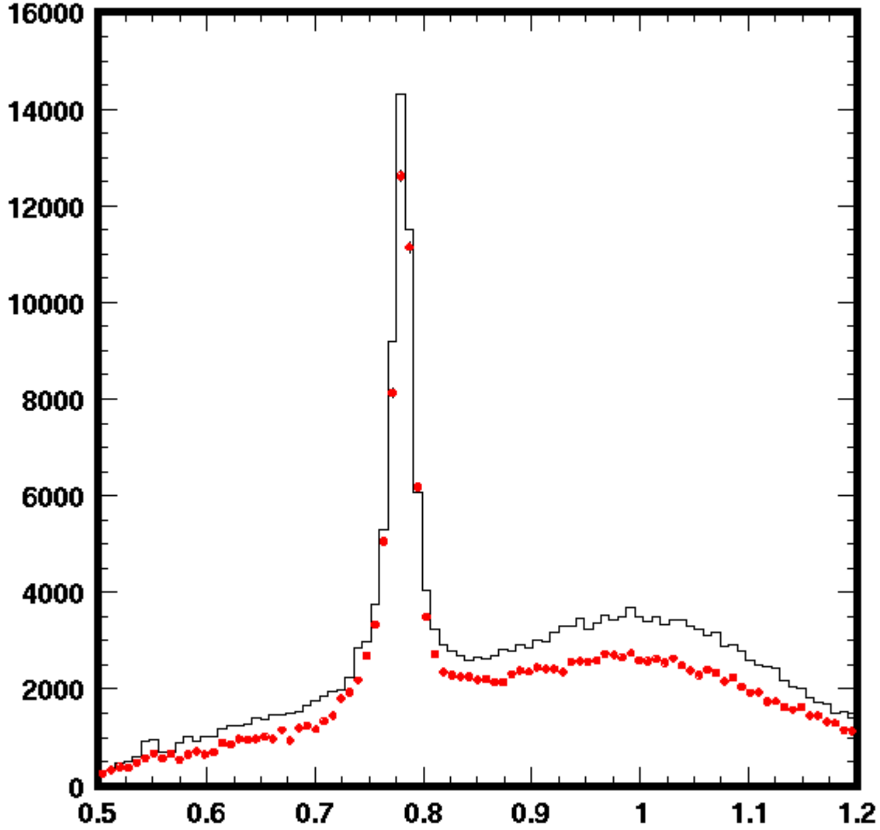
Background fraction for
$\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau$

yield = 1.22 X 10^4 events

$f = 0.226 \pm 0.01\ (4.5\%)$

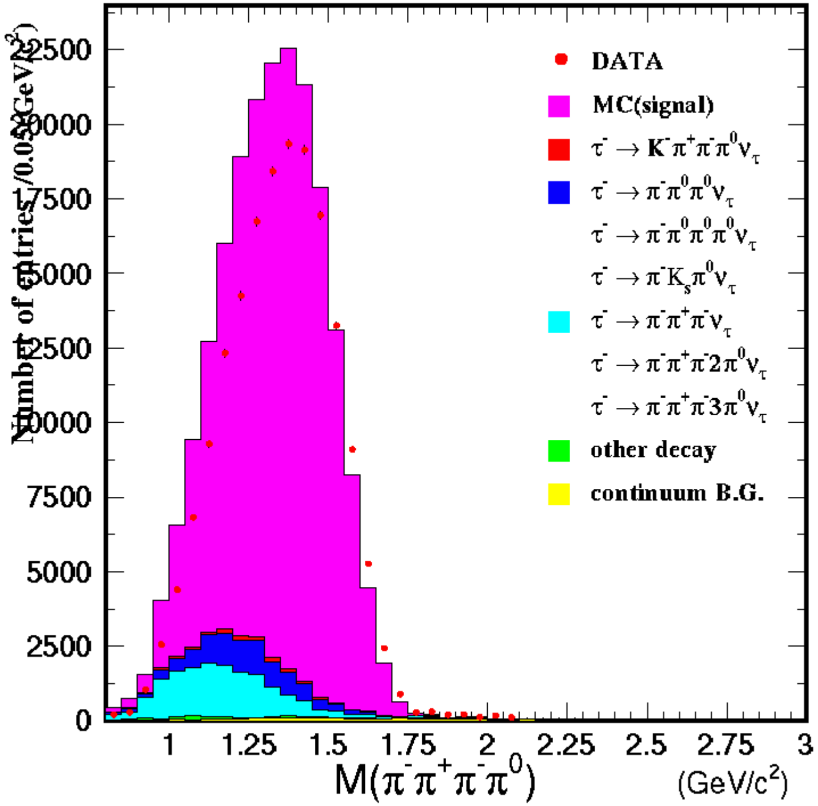
MC is normalized to the data luminosity

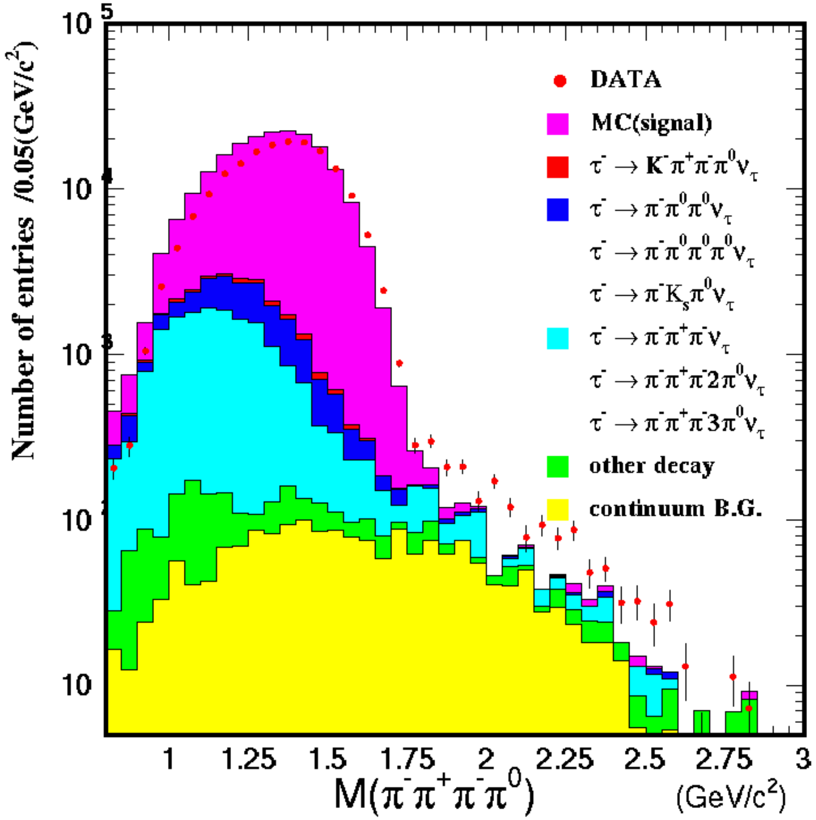
$\mathcal{M}(\pi^+\pi^-\pi^0)$ distribution



$\pi^+\pi^-\pi^0$ invariant mass spectrum in $\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$ events.
This figure clearly shows an ω signal.

$\mathcal{M}(\pi^-\pi^+\pi^-\pi^0)$ for $\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu$





Decay mode	f_i
$\pi^-\pi^0\pi^0$	0.037
$\pi^-\pi^0\pi^0\pi^0$	0.004
$\pi^-K_s\pi^0$	0.002
$\pi^-\pi^+\pi^-$	0.007
$\pi^-\pi^+\pi^-\pi^0\pi^0$	0.068
$\pi^-\pi^+\pi^-\pi^0\pi^0\pi^0$	0.005
other decay w/ Kaon	0.001
other decay w/o Kaon	0.004
uds	0.002
charm	0.006
SUM	0.137
df_i	0.001
df_i/f_i	0.037

Background fraction for $\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$

yield = 2.45 X 10^5 events

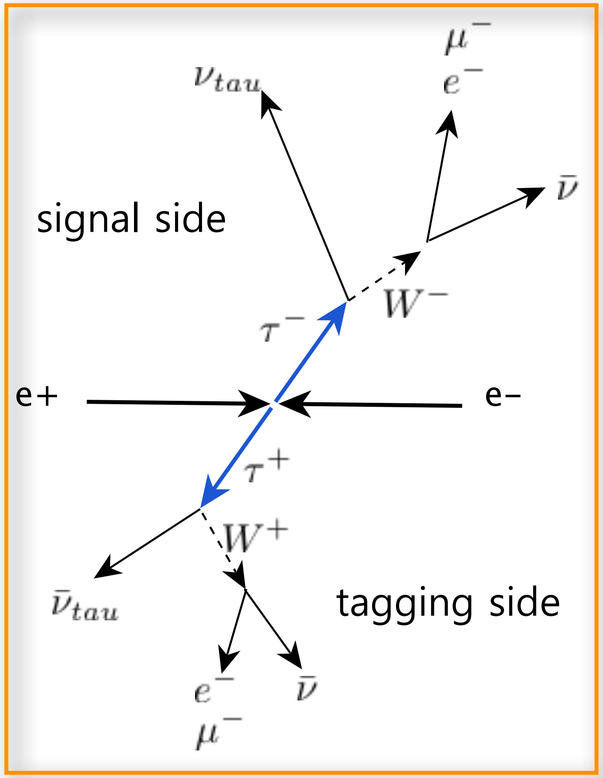
$f = 0.137 \pm 0.005$ (3.7%)

MC is normalized to the data luminosity
MC mass distribution is taken from $e^+e^- \to 4\pi$ data

{e,mu} event selection

In order to determine the Branching Fraction, we needs to know the number of tau-pair events.
To know precise tau-pair evens, we have to check number of leptonic decay events.

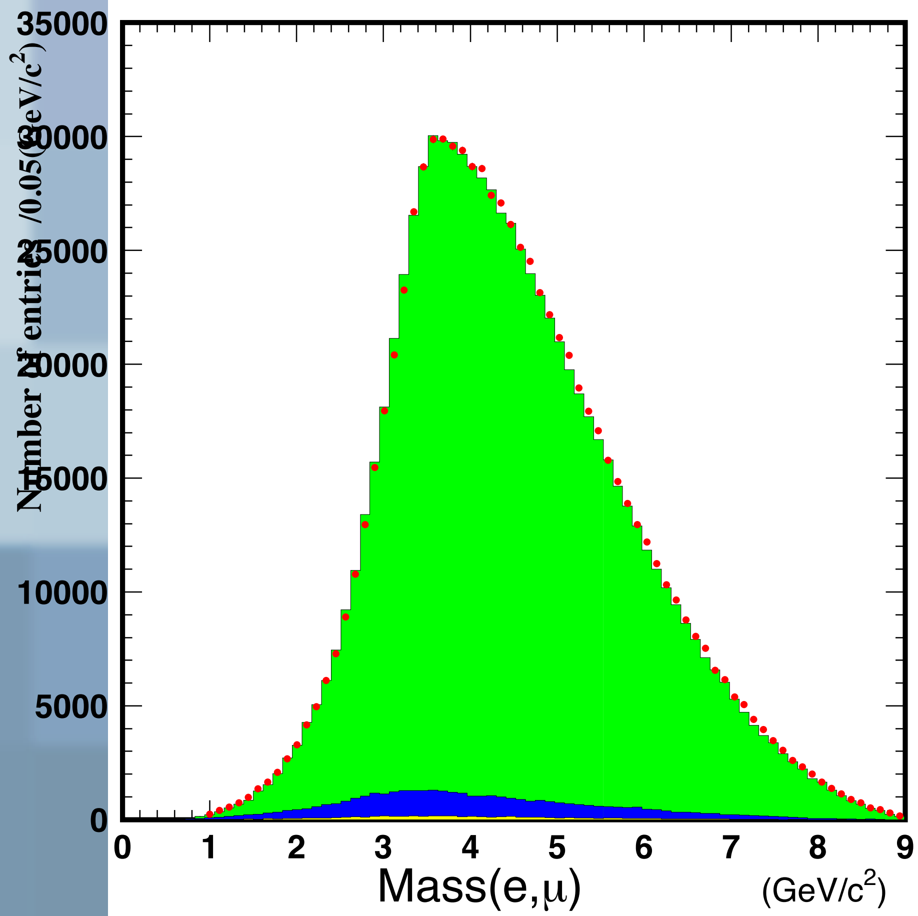
one τ decays $\tau \longrightarrow e\bar{\nu}\nu$
the other τ decays $\tau \longrightarrow \mu\bar{\nu}\nu$



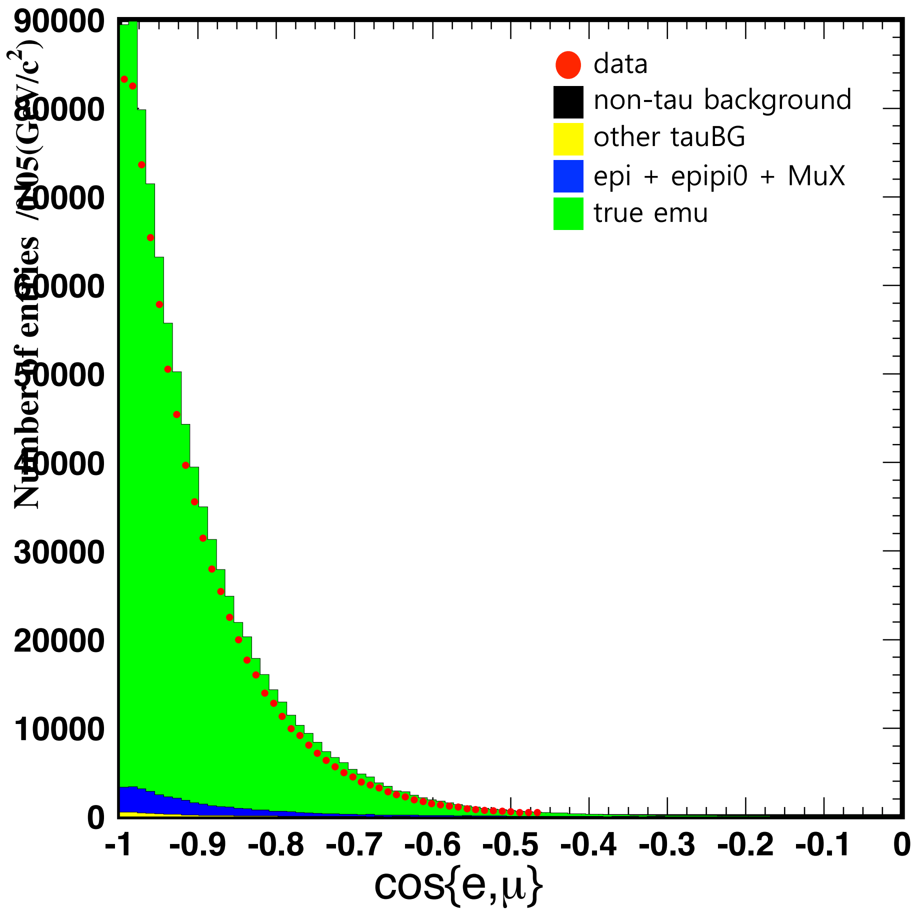
$$\mathcal{B}_{\tau\rightarrow e\bar{\nu}\nu} \cdot \mathcal{B}_{\tau\rightarrow\mu\bar{\nu}\nu} = \frac{N_{sig,e\mu}}{2 \cdot \epsilon_{e\mu} \cdot N_{\tau\tau}}$$

$\mathcal{B}_{\tau\rightarrow e\bar{\nu}\nu}$ is the branching fraction of $\tau \rightarrow e\nu_\tau\bar{\nu_\tau}$
$\mathcal{B}_{\tau\rightarrow\mu\bar{\nu}\nu}$ is the branching fraction of $\tau \rightarrow \mu\nu_\tau\bar{\nu_\tau}$
$N_{sig,e\mu}$ is the number of $\{e,\mu\}$ events,
$\epsilon_{e\mu}$ is the detection efficiency and
$N_{\tau\tau}$ is the number of tau-pair events.

Distribution of {e,mu}



Invariant mass of the electron and muon system.



Cosine of the angle between electron and muon

MC is normalized to the data luminosity

cont.

	MC	DATA	QQ	CC
Call All emusel	2.02×10^7	2.27×10^7	1.02×10^6	6.23×10^5
1-1 events	1.43×10^7	1.57×10^7	3.39×10^5	1.30×10^5
e-e events	1.09×10^5	3.38×10^5	4.00×10^0	6.76×10^2
$\mu - \mu$ events	3.22×10^5	4.62×10^5	1.26×10^2	1.18×10^3
e-μ events	9.12×10^5	8.77×10^5	6.40×10	2.31×10^3
no.of γ in e-side$\leq$1	8.98×10^5	8.60×10^5	1.60×10	6.87×10^2
no.of γ in μ-side$\leq$1	8.58×10^5	8.26×10^5	5	1.59×10^2
generated $[e, \mu]$ event	4.13×10^6			
true $[e, \mu]$ event	8.22×10^5			
$[e, X]$ event	3.33×10^4			
$[\mu, X]$ event	2.69×10^3			
$[X, X]$ event	1.02×10^2			
$[e, e]$ event	1.00×10			
$[\mu, \mu]$ event	4.50×10			
$[e, \pi]$ event	1.50×10^4			
$[\pi, \mu]$ event	8.40×10^2			
$[\pi, \pi]$ event	1.10×10			
$[e, \pi\pi^0]$ event	1.42×10^4			

$$\text{efficiency of } \{e, \mu\} = \frac{\text{Number of true } \{e,\mu\}}{\text{Number of generated } \{e,\mu\}}$$

$\{e, \mu\}$ efficiency	$\pm$ 19.9%
B.G fraction for tau B.G	$\pm$ 4.22%
B.G fraction for non-tau B.G	$\pm$ 0.007%

Branching Fraction

1. Correlations between Kpipipi0 and pipipipi0 taken into account.
   pipipipi0 is most dominant background mode.

2. {e,mu} events are used for Br measurement.

$$\mathcal{B}_i = \frac{N_i^{true}}{2 \cdot N_{\tau\tau} \cdot (\mathcal{B}_{\tau\to e\bar{\nu}\nu} + \mathcal{B}_{\tau\to\mu\bar{\nu}\nu})}$$

$$\mathcal{B}_i = N_i^{true} \times \frac{\epsilon_{e\mu}}{N_{sig,e\mu}} \times \frac{\mathcal{B}_{\tau\to e\bar{\nu}\nu} \cdot \mathcal{B}_{\tau\to\mu\bar{\nu}\nu}}{(\mathcal{B}_{\tau\to e\bar{\nu}\nu} + \mathcal{B}_{\tau\to\mu\bar{\nu}\nu})}$$

$$\begin{pmatrix} N^{obs}_{K^-\pi^+\pi^-\pi^0\nu_\tau}(1 - f^{other}_{background}) \\ N^{obs}_{\pi^-\pi^+\pi^-\pi^0\nu_\tau}(1 - f^{other}_{background}) \end{pmatrix} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{pmatrix} \begin{pmatrix} N^{true}_{K^-\pi^+\pi^-\pi^0\nu_\tau} \\ N^{true}_{\pi^-\pi^+\pi^-\pi^0\nu_\tau} \end{pmatrix}$$

ϵ_{11} : detection efficiency of $\tau \to K\pi\pi\pi^0\nu$
ϵ_{12} : efficiency (fake rate) $\tau \to \pi\pi\pi\pi^0\nu$ is mis-reconstructed as $\tau \to K\pi\pi\pi^0\nu$
ϵ_{21} : efficiency (fake rate) $\tau \to K\pi\pi\pi^0\nu$ is mis-reconstructed as $\tau \to \pi\pi\pi\pi^0\nu$
ϵ_{22} : detection efficiency of $\tau \to \pi\pi\pi\pi^0\nu$ efficiency

cont.

$$
\begin{aligned}
\mathcal{B}_1 &= A\left(\frac{\epsilon_{22}}{\Delta}n_1(1-f_1) - \frac{\epsilon_{12}}{\Delta}n_2(1-f_2)\right)\,, \\
\mathcal{B}_2 &= A\left(\frac{\epsilon_{11}}{\Delta}n_2(1-f_2) - \frac{\epsilon_{21}}{\Delta}n_1(1-f_1)\right)\,,
\end{aligned}
$$

with

$$
\begin{aligned}
\Delta &= \epsilon_{11}\epsilon_{22} - \epsilon_{12}\epsilon_{21} \\
A &= \frac{\epsilon_{e\mu}}{N_{\mathrm{sig},e\mu}} \times \frac{\mathcal{B}_e \cdot \mathcal{B}_\mu}{\mathcal{B}_e + \mathcal{B}_\mu}.
\end{aligned}
$$

where,
index i and j are $\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau$ and $\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$ decays

Variable	Value
$N^{\mathrm{obs}}_{mydecay}$	$(1.30 \pm 0.01) \times 10^4$
$N^{\mathrm{obs}}_{mytpimode}$	$(1.75 \pm 0.004) \times 10^5$
$N_{e,\mu}$	$(8.3 \pm 0.01) \times 10^5$
ϵ_{11}	$(5.3 \pm 0.005) \times 10^-2$
ϵ_{22}	$(8.3 \pm 0.058) \times 10^-2$
ϵ_{21}	$(3.6 \pm 0.16) \times 10^-3$
ϵ_{12}	$(21.2 \pm 0.07) \times 10^-3$
$\epsilon_{e\mu}$	0.199
$\mathcal{B}_{\tau\to e\bar{\nu}\nu}$	0.1785
$\mathcal{B}_{\tau\to e\bar{\nu}\nu}$	0.1736
$f_{\tau\to K\pi\pi\pi^0}$	0.227 ± 0.010
$f_{\tau\to\pi\pi\pi\pi^0}$	0.137 ± 0.005

Systematic Error Estimation

Source	$\tau \to K^-\pi^+\pi^-\pi^0\nu_\tau$	$\tau \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$
Tracking efficiency	$\pm$ 1.83	$\pm$ 2.03
PID efficiency	$\pm$ 3.54	$\pm$ 1.53
Background estimation	$\pm$ 4.06	$\pm$ 0.61
Fake rate	$\pm$ 11.59	$\pm$ 0.18
π^0 efficiency	$\pm$ 1.83	$\pm$ 2.02
Trigger efficiency	$\pm$ 0.91	$\pm$ 1.01
Gamma veto	$\pm$ 0.91	$\pm$ 1.01
e,μ normalization	$\pm$ 0.59	$\pm$ 0.59
MC statistic	$\pm$ 1.10	$\pm$ 0.22
Total (%)	$\pm$ 13.15	$\pm$ 3.64

- Basic formula for error estimation.

$$\underbrace{\begin{pmatrix} \Delta^2_{\mathcal{B}_1}, & cov(\mathcal{B}_1,\mathcal{B}_2) \\ cov(\mathcal{B}_2,\mathcal{B}_1), & \Delta^2_{\mathcal{B}_2} \end{pmatrix}}_{M} = \underbrace{\begin{pmatrix} \frac{\partial \mathcal{B}_1}{\partial \alpha_1}, & \frac{\partial \mathcal{B}_1}{\partial \alpha_2} \\ \frac{\partial \mathcal{B}_2}{\partial \alpha_1}, & \frac{\partial \mathcal{B}_2}{\partial \alpha_2} \end{pmatrix}}_{^tD} \cdot \underbrace{\begin{pmatrix} \Delta^2_{\alpha 1}, & cov(\alpha_1,\alpha_2) \\ cov(\alpha_2,\alpha_1), & \Delta^2_{\alpha 2} \end{pmatrix}}_{X} \cdot \underbrace{\begin{pmatrix} \frac{\partial \mathcal{B}_1}{\partial \alpha_1}, & \frac{\partial \mathcal{B}_2}{\partial \alpha_1} \\ \frac{\partial \mathcal{B}_1}{\partial \alpha_2}, & \frac{\partial \mathcal{B}_2}{\partial \alpha_2} \end{pmatrix}}_{D},$$

where, X is the matrixes M and X of the parameters
D is the matrix of the derivative.
^tM = M and ^tX = X, but $^tD \neq$ D.

cont.

1. Tracking efficiency : used tracking group's code (~1% per track for hadron track)
   - there are 2 tracks in {e,mu} events, i.e, 1% X 2 tracks = 2% (+-1.83% , +-2.03%)

2. PID efficiency : efficiency correction to data/MC relative efficiency and corresponding uncertainty is calculated for each track.
   - efficiency and its error for Kaon = 1.022 +- 0.01
   - efficiency and its error for Pion = 0.991 +- 0.005
   - error for Kpipipi0 = 0.01+2X0.005 = 0.020
   - error for pipipipi0 = 3X0.005 = 0.015

   (+-3.54%, +-1.53%)

3. Background estimation : $f = 0.137 \pm 0.005\ (3.7\%)$, $f = 0.226 \pm 0.01\ (4.5\%)$

   (+-4.06%, +-0.61%)

4. Fake rate : uncertainties of the pion and kaon fake rate.
   - fake($\pi \rightarrow K$) = 2.12 +- 0.0807 (4%) <- we assumed 50% bigger uncertainty (6%)
   - fake($K \rightarrow \pi$) = 1.056 +- 0.16 (15%)

   (+-11.59%, +-0.18%)

5. π^0 efficiency :

   different momentum ranges and different π^0 and γ selection,
   absolute efficiency calibration. (+-1.83%, +-2.02%)

cont.

6. Trigger efficiency : Maximum deviation from average trigger efficiency from signal MC.
   (+-0.91%, +-1.01%)

7. Gamma veto : used 0.2GeV cut to calculate BR, deviation of BR is used for systematics
   (+-0.91%, +-1.01%)

8. {e,mu} normalization : uncertainty of branching ratio of the leptonic decay of τ is taken into account. The leptonic decay branching ratio is (28.8 +- 0.589)%
   (+-0.59%, +-0.59%)

9. MC statistic : statistical error of efficiency
   (+-1.10%, +-0.22%)

Branching Fraction

$\tau \to K^-\pi^+\pi^-\pi^0\nu_\tau$	
List of systematic error	dBr/Br (%)
Tracking efficiency	$\pm$ 1.83
PID efficiency	$\pm$ 3.54
Background estimation	$\pm$ 4.06
Fake rate	$\pm$ 11.59
π^0 efficiency	$\pm$ 1.83
Trigger efficiency	$\pm$ 0.91
Gamma veto	$\pm$ 0.91
e,μ normalization	$\pm$ 0.59
MC statistic	$\pm$ 1.10
Total systematic	$\pm$ 13.15
Statistical error	$\pm$ 2.63

$\tau \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$	
List of systematic error	dBr/Br (%)
Tracking efficiency	$\pm$ 2.03
PID efficiency	$\pm$ 1.53
Background estimation	$\pm$ 0.61
Fake rate	$\pm$ 0.18
π^0 efficiency	$\pm$ 2.02
Trigger efficiency	$\pm$ 1.01
Gamma veto	$\pm$ 1.01
e,μ normalization	$\pm$ 0.59
MC statistic	$\pm$ 0.22
Total systematic	$\pm$ 3.64
Statistical error	$\pm$ 0.25

$$\begin{aligned}
\mathcal{B}(\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau) &= (1.53 \pm 0.04(\mathrm{stat}) \pm 0.19(\mathrm{sys})) \times 10^{-3}, \\
\mathcal{B}(\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau) &= (4.20 \pm 0.01(\mathrm{stat}) \pm 0.14(\mathrm{sys})) \times 10^{-2}.
\end{aligned}$$

Correlation coefficient

List of covariant errors	cov_{12}
statistical errors	-9.3947×10^{-10}
tracking efficiency	-1.9568×10^{-8}
PID efficiency	-2.371×10^{-8}
background estimation	-4.4444×10^{-9}
fake rate	-6.8394×10^{-9}
pi^0 efficiency	-1.9568×10^{-8}
trigger efficiency	-4.8921×10^{-9}
γ veto	-4.8921×10^{-9}
$\{e, \mu\}$ normalization	1.8343×10^{-9}
MC statistic	0
Total covariant errors(sys + stat)	-8.208×10^{-8}
Correration coefficient, ρ_{12}^{tot}	-0.31485

$$cov(\mathcal{B}_i, \mathcal{B}_j) = \left(\frac{\partial \mathcal{B}_i}{\partial \alpha_1}\frac{\partial \mathcal{B}_j}{\partial \alpha_1}\right)\Delta_{\alpha 1}^2 + \left(\frac{\partial \mathcal{B}_i}{\partial \alpha_2}\frac{\partial \mathcal{B}_j}{\partial \alpha_2}\right)\Delta_{\alpha 2}^2 + \left(\frac{\partial \mathcal{B}_i}{\partial \alpha_1}\frac{\partial \mathcal{B}_j}{\partial \alpha_2} + \frac{\partial B_i}{\partial \alpha_2}\frac{\partial B_j}{\partial \alpha_1}\right) cov(\alpha_1, \alpha_2).$$

$$\rho_{12}^{tot} = \frac{cov_{12}}{\sigma_1^{tot} \times \sigma_2^{tot}}$$

In order to calculate the correlation coefficients, the sum of correlations from these different sources calculated to evaluate the correlation coefficients,

Correlation coefficient	$\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau$
$\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$	-0.31485

the value is not percentage, the value is greater than -1 and less than 1.

Therefore, uncertainty of the fake rate (π → K) is dominated for the branching fraction of $\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau$.

Summary

Decay mode	Experiments	Branching Ratio
$\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau(\mathrm{ex}.K_s)$	CLEO3	$(7.4 \pm 0.8 \pm 1.1) \times 10^{-4}$
	ALEP	$(6.1 \pm 3.9 \pm 1.8) \times 10^{-4}$
	Belle	$(1.53 \pm 0.04(\mathrm{stat.}) \pm 0.19(\mathrm{sys.})) \times 10^{-3}$
$\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau(\mathrm{ex}.K_s)$	ALEP	$(4.598 \pm 0.057 \pm 0.064) \times 10^{-2}$
	CLEO	$(4.19 \pm 0.10 \pm 0.21) \times 10^{-2}$
	Belle	$(4.20 \pm 0.01(\mathrm{stat.}) \pm 0.14(\mathrm{sys.})) \times 10^{-2}$

Compare with the previous results,

1. We measure $\mathcal{B}(\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau)$ in the world best accuracy (12%).

2. The error is dominated by the uncertainty of fake(π→K) rate.

3. $\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau$ decay is around factor of 2 bigger than previous results although their errors are large.

4. Our result for $\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$ decay is consistent with the CLEO results but smaller than the LEP results.

BackUp

continue

$\Gamma(K^-\pi^+\pi^-\pi^0\nu_\tau(\text{ex.}K^0))/\Gamma_{\text{total}}$ $\Gamma_{88}/\Gamma = (\Gamma_{89}+0.226\Gamma_{128})/\Gamma$

Data marked "avg" are highly correlated with data appearing elsewhere in the Listings, and are therefore used for the average given below but not in the overall fits. "f&a" marks results used for the fit and the average.

VALUE (units 10^{-4})	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
8.1±1.2 OUR FIT					
7.3±1.2 OUR AVERAGE					
$7.4\pm0.8\pm1.1$	f&a	[1] ARMS	05	CLE3	7.6 fb^{-1}, $E^{ee}_{\text{cm}} = 10.6$ GeV
$7.5\pm2.6\pm1.8$	avg	[2] RICHICHI	99	CLEO	$E^{ee}_{\text{cm}} = 10.6$ GeV
$6.1\pm3.9\pm1.8$	f&a	BARATE	98	ALEP	1991–1995 LEP runs
• • • We do not use the following data for averages, fits, limits, etc. • • •					
<17	95	ABBIENDI	00D	OPAL	1990–1995 LEP runs

[1] Not independent of ARMS 05 $\Gamma(\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau(\text{ex.}K^0,\omega))$ / Γ_{total} and $\Gamma(\tau^- \to K^-\omega\nu_\tau)$ / Γ_{total} values.

[2] Not independent of RICHICHI 99 $\Gamma(\tau^- \to K^-h^+\pi^-\nu_\tau(\text{ex.}K^0))/\Gamma(\tau^- \to \pi^-\pi^+\pi^-\nu_\tau(\text{ex.}K^0))$, $\Gamma(\tau^- \to K^-K^+\pi^-\nu_\tau)/\Gamma(\tau^- \to \pi^-\pi^+\pi^-\nu_\tau(\text{ex.}K^0))$ and BALEST 95C $\Gamma(\tau^- \to h^-h^-h^+\nu_\tau(\text{ex.}K^0))/\Gamma_{\text{total}}$ values.

$\Gamma(\pi^-\pi^+\pi^-\pi^0\nu_\tau(\text{ex.}K^0))/\Gamma_{\text{total}}$ $\Gamma_{69}/\Gamma = (\Gamma_{70}+0.888\Gamma_{150}+0.017\Gamma_{152})/\Gamma$

VALUE (%)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
4.48 ±0.06 OUR FIT Error includes scale factor of 1.2.					
4.55 ±0.13 OUR AVERAGE Error includes scale factor of 1.6.					
$4.598\pm0.057\pm0.064$	16k	[1] SCHAEL	05C	ALEP	1991-1995 LEP runs
$4.19\pm0.10\pm0.21$		[2] EDWARDS	00A	CLEO	4.7 fb^{-1} $E^{ee}_{\text{cm}} = 10.6$ GeV

[1] SCHAEL 05C quote $(4.590\pm0.057\pm0.064)$%. We add 0.008% to remove their correction for $\tau^- \to \pi^-\pi^0\omega\nu_\tau \to \pi^-\pi^0\pi^+\pi^-\nu_\tau$ decays. See footnote to SCHAEL 05C $\Gamma(\tau^- \to e^-\overline{\nu}_e\nu_\tau)/\Gamma_{\text{total}}$ measurement for correlations with other measurements.

[2] EDWARDS 00A quote $(4.19\pm0.10)\times10^{-2}$ with a 5% systematic error.

```
------------------------------------------------
 Branching fraction evaulation:      cal_br()
------------------------------------------------
 K2pip0:  No. of observed events.    =  12987.8 events
          background fraction.       =  0.226854
 3pip0:  No. of observed events.     =  174920 events
          background fraction.       =  0.137374
 efficiencies:  K2pipi0              =  0.0528287
                3pipi0               =  0.0828678
 feed-down  3pipi0->K2pipi0  e_12    =  0.0036443
            K2pipi0->3pipi0  e_21    =  0.0211962
         Delta=e_11*e_22- e_12*e_21  =  0.00430055
 {e-mu} events  No. of observed      =  826497 events
            efficiency               =  0.199
            Be                       =  0.1785
            Bmu                      =  0.1736
 Anorm=(e_emu/N_emu)*(BeBmu)/(Be+Bmu)=  2.11901e-08
..........................................
 Br(K2pipi0)                         =  0.00139059
 Br(3pipi0)                          =  0.0382285
---------------------------------------------
 Statistical errors:      cal_stat_error()
---------------------------------------------
 Statistical error for K2pipi0       =  3.65555e-05
           (relative errors dB/B =  2.62877 %)
 Statistical error for 3pipi0        =  9.43618e-05
           (relative errors dB/B =  0.246837 %)
 Covariant sta. error cov(Br_1,Br_2) =  -9.39466e-10
---------------------------------------------
 Systematic:tracking efficiency unc. cal_eff_trk()
---------------------------------------------
 Error of tracking  efficiency    (depsilon_ii):
                K2pipi0      =  0.00105657 (relative error de/epsilon =  2 %)
                3pipi0       =  0.00165736 (relative error de/epsilon =  2 %)
 Br uncertainty  for K2pipi0      =  2.53784e-05
           ( relative errors dB/B = 1.82501 %)
 Br uncertainty  for 3pipi0       =  0.00077106
           ( relative errors dB/B = 2.01698 %)
 Covariant  error   cov(Br_1,Br_2) =  -1.95683e-08

---------------------------------------------
 Systematic:PID efficiency unc. cal_eff_pid()
---------------------------------------------
 Error of pid efficiency    (depsilon_ii):
                K2pipi0      =  0.00105657 (relative error de/epsilon =  2 %)
                3pipi0       =  0.00124302 (relative error de/epsilon =  1.5 %)
 Br uncertainty  for K2pipi0      =  4.9224e-05
           ( relative errors dB/B = 3.53978 %)
 Br uncertainty  for 3pipi0       =  0.000583771
           ( relative errors dB/B = 1.52706 %)
 Covariant  error   cov(Br_1,Br_2) =  -2.37102e-08
---------------------------------------------
 Systematic:background uncertainty cal_bkg_error()
---------------------------------------------
 Error of background fraction (dfback):
                K2pipi0      =  0.0102084 (relative error df/fback =  4.5 %)
                3pipi0       =  0.00508283 (relative error df/fback =  3.7 %)
 Br uncertainty  for K2pipi0      =  5.64412e-05
           ( relative errors dB/B = 4.05879 %)
 Br uncertainty  for 3pipi0       =  0.000231846
           ( relative errors dB/B = 0.606474 %)
 Covariant  error   cov(Br_1,Br_2) =  -4.44442e-09
---------------------------------------------
 Systematic:PID fake rate  unc. cal_fake_rate()
---------------------------------------------
 Error of pid fake rate    (depsilon_ij):
          pi->      K          =  0.000218658 (relative error de/epsilon =  6 %)
          K  -> pi             =  0.00317943 (relative error de/epsilon =  15 %)
 Br uncertainty  for K2pipi0      =  0.000161113
           ( relative errors dB/B = 11.5859 %)
 Br uncertainty  for 3pipi0       =  6.817e-05
           ( relative errors dB/B = 0.178323 %)
 Covariant  error   cov(Br_1,Br_2) =  -6.83939e-09
```

Systematic:pi0 efficiency unc. cal_eff_pi0()

Error of pi0 efficiency (depsilon_ii):
K2pipi0 = 0.00105657 (relative error de/epsilon = 2 %)
3pipi0 = 0.00165736 (relative error de/epsilon = 2 %)
Br uncertainty for K2pipi0 = 2.53784e-05
(relative errors dB/B = 1.82501 %)
Br uncertainty for 3pipi0 = 0.00077106
(relative errors dB/B = 2.01698 %)
Covariant error cov(Br_1,Br_2) = -1.95683e-08

Systematic: trigger efficiency unc. cal_eff_trg()

Error of trigger efficiency (depsilon_ii):
K2pipi0 = 0.000528287 (relative error de/epsilon = 1 %)
3pipi0 = 0.000828678 (relative error de/epsilon = 1 %)
Br uncertainty for K2pipi0 = 1.26892e-05
(relative errors dB/B = 0.912504 %)
Br uncertainty for 3pipi0 = 0.00038553
(relative errors dB/B = 1.00849 %)
Covariant error cov(Br_1,Br_2) = -4.89208e-09

Systematic: gamma veto unc. cal_gam_veto()

Uncertainty of gamma veto/ev-sel (depsilon_ii):
K2pipi0 = 0.000528287 (relative error de/epsilon = 1 %)
3pipi0 = 0.000828678 (relative error de/epsilon = 1 %)
Br uncertainty for K2pipi0 = 1.26892e-05
(relative errors dB/B = 0.912504 %)
Br uncertainty for 3pipi0 = 0.00038553
(relative errors dB/B = 1.00849 %)
Covariant error cov(Br_1,Br_2) = -4.89208e-09

Systematic: emu_norm unc. cal_emu_norm()

Error of enorm
lepton branching dBl/Bl = 0.288018 %
lepton-ID uncertainty = 0.5 %
statistical error of Nsig.emu = 0.109997 %
total normalization error = 0.587413 %
Br uncertainty for K2pipi0 = 8.16852e-06
(relative errors dB/B = 0.587413 %)
Br uncertainty for 3pipi0 = 0.000224559
(relative errors dB/B = 0.587413 %)
Covariant error cov(Br_1,Br_2) = 1.83431e-09

Systematic: MC statistic cal_emu_norm()

MC stat. errors
k2pipi0 = 1.1017 %
3pipi0 = 0.223734 %
Br uncertainty for K2pipi0 = 1.53201e-05
(relative errors dB/B = 1.1017 %)
Br uncertainty for 3pipi0 = 8.553e-05
(relative errors dB/B = 0.223734 %)
Covariant error cov(Br_1,Br_2) = 0