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Study of Higgs Self Couplings of a Supersymmetric E_6 Model at the International Linear Collider

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We study the Higgs self couplings of a supersymmetric E_6 model that has two Higgs doublets and two Higgs singlets. The lightest scalar Higgs boson in the model may be heavier than 112 GeV, at the one-loop level, where the negative results for the Higgs search at the LEP2 experiments are taken into account. The contributions from the top and scalar top quark loops are included in the radiative corrections to the one-loop mass of the lightest scalar Higgs boson in the effective potential approximation. The effect of the Higgs self couplings may be observed in the production of the lightest scalar Higgs bosons in e^+e^- collisions at the International Linear Collider (ILC) via the double Higgs-strahlung process. For the center of mass energy of 500 GeV with an integrated luminosity of 500 fb^{-1} and an efficiency of 20%, we expect that at least 5 events of the lightest scalar Higgs boson may be produced at the ILC via the double Higgs-strahlung process.

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I. INTRODUCTION

It is well accepted that one of the primary reasons to introduce supersymmetry is to elucidate the hierarchy problem of the standard model (SM) [1-3]. In the SM, the existence of the neutral Higgs boson is generally explained within the context of the origin of electroweak symmetry breaking. The breaking of the electroweak gauge symmetry is induced by the self interactions of the SM Higgs field via the nontrivial vacuum expectation value. Without supersymmetry, the mass of the SM Higgs boson would acquire quadratically divergent loop corrections, requiring a cut-off scale at very high energy to confine the Higgs mass at the electroweak scale.

If supersymmetry is a symmetry of nature, the loop contributions of the particles may be canceled by those of superpartners. However, since experiments have yet to observe any superparticles that have the same mass as the quark or the lepton, supersymmetry cannot be exact. In the presence of inexact supersymmetry, the one-loop corrections to the mass of the Higgs boson are not so small, but are as large as 30% of the tree-level values due to incomplete cancellation.

There are a number of phenomenologically interesting supersymmetric models that embrace the SM. A characteristic of these supersymmetric standard models is the requirement of at least two Higgs doublets in order to give masses separately to the up-quark sector and the down-quark sector. The key role of these Higgs fields is essentially the same as the SM Higgs field: breaking of the electroweak gauge symmetry via self interactions.

The simplest one is the minimal supersymmetric standard model (MSSM) [4], which has just two Higgs doublets and has been extensively studied. The Higgs sector of the MSSM can be extended by introducing Higgs singlet fields [5,6]. In the next-to-minimal supersymmetric standard model (NMSSM), there is one additional Higgs singlet. The motivation for introducing additional Higgs singlets is mainly to solve the μ problem of the MSSM [7]. In general, in terms of the vacuum expectation value of a neutral Higgs singlet, the μ parameter of the MSSM

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might be generated dynamically.

Besides the MSSM and beyond, there are string-motivated supersymmetric models, such as the supersymmetric E_6 model, in which the gauge symmetry is decomposed from E_6 to $SU(2) \times U(1) \times U_1(1) \times U_2(1)$. The two extra $U(1)$ symmetries would be mixed to yield new linearly orthogonal combinations, $U(1)'$ and $U(1)''$. Thus, it is a rank-6 supersymmetric model, with two extra neutral gauge bosons. Its Higgs sector has two Higgs doublets and two Higgs singlets. The phenomenology of the Higgs sector of this model has been investigated by several authors [8-11]. Some of us have elsewhere studied the possibility of detecting its neutral Higgs bosons at the Large Hadron Collider, assuming explicit CP violation in its Higgs sector [11].

The self interactions of the Higgs fields are induced by cubic and quartic couplings in the Higgs potential. Thus, if the Higgs phenomenology of a model is to be examined, it is important to know the nature of the Higgs self interactions and to study the possibility of measuring them. This kind of work was performed some years ago in the SM and in the MSSM by Djouadi *et al.* [12,13].

In this article, we study the Higgs self couplings of the supersymmetric E_6 model investigated in Refs. 8 and 9, which we will call the SUSY E_6 model hereafter. We study the possibility of discovering neutral scalar Higgs bosons of the SUSY E_6 model at the International Linear Collider (ILC). We calculate the cross section of the neutral Higgs bosons produced from the double Higgs-strahlung process in e^+e^- collisions. We find that, for reasonable parameter values, the cubic Higgs coupling of the lightest scalar Higgs boson is smaller than the corresponding coupling of the SM Higgs boson. Thus, the production cross section of the lightest scalar Higgs boson via the double Higgs-strahlung process is smaller than that of the corresponding SM Higgs boson. The radiative corrections to the mass of the lightest scalar Higgs boson due to the top and the stop quark loops, combined with the negative experimental result for the neutral scalar boson search at LEP2 [14], suggest that the lightest scalar Higgs boson of the present model is heavier than 112 GeV. At the International Linear Collider (ILC) with a center-of-mass energy of 500 GeV, an integrated luminosity of 500 fb^{-1}, and an efficiency of 20%, we expect at least 5 events of the double Higgs-strahlung process to be observed.

II. HIGGS POTENTIAL

The SUSY E_6 model has two Higgs-doublet superfields, $\mathcal{H}_1$ and $\mathcal{H}_2$, and two Higgs singlet superfields, $\mathcal{N}_1$ and $\mathcal{N}_2$. The superpotential of the model is given by [8,9]

$$\mathcal{W} \approx h_t Q^T \mathcal{H}_2 t_R^c + \lambda \mathcal{H}_1 \mathcal{H}_2 \mathcal{N}_1 \ , \tag{1}$$

where t_R^c is the right-handed top-quark superfield, Q is the left-handed quark doublet superfield of the third generation, h_t is the dimensionless Yukawa coupling coefficient for the top quark, and λ is a dimensionless coupling constant. The bottom-quark superfield is absent as we take only the top-quark superfield. Note that $\mathcal{N}_2$ is absent in the superpotential because the gauge symmetry of the model prohibits its coupling to the Higgs doublet superfields.

The Higgs fields of the model are two Higgs doublets, $H_1 = (H_1^0, H^-)$ and $H_2 = (H^+, H_2^0)$, and two Higgs singlets, N_1 and N_2. The μ parameter of the MSSM is dynamically generated in this model in terms of the vacuum expectation value of N_1. In terms of these Higgs fields, the Higgs potential at the tree level can be written as [9]

$$\begin{aligned} V_0 &= m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_3^2|N_1|^2 + m_4^2|N_2|^2 \\ &- (\lambda A H_1 H_2 N_1 + \text{H.c.}) + |\lambda|^2[|H_1|^2|H_2|^2 \\ &+ |H_1|^2|N_1|^2 + |H_2|^2|N_1|^2] \\ &+ \left(\frac{g_2^2}{2} - |\lambda|^2\right)|H_1^\dagger H_2|^2 + \frac{g_1^2+g_2^2}{8}(|H_1|^2 - |H_2|^2)^2 \\ &+ \frac{g_1^{'2}}{72}[C_\theta(|H_1|^2 + 4|H_2|^2 - 5|N_1|^2 - 5|N_2|^2) \\ &- \sqrt{15}S_\theta(|H_1|^2 - |N_1|^2 + |N_2|^2)]^2 \\ &+ \frac{g_1^{''2}}{72}[S_\theta(|H_1|^2 + 4|H_2|^2 - 5|N_1|^2 - 5|N_2|^2) \\ &+ \sqrt{15}C_\theta(|H_1|^2 - |N_1|^2 + |N_2|^2)]^2 \ , \end{aligned} \tag{2}$$

where m_i (i = 1, 2, 3, 4) are the mass parameters, $g_1^{'}$ and $g_1^{''}$ are, respectively, the gauge coupling coefficients of $U(1)'$ and $U(1)''$, A is the trilinear soft SUSY-breaking parameter with mass dimension, $C_\theta = \cos\theta$, and $S_\theta = \sin\theta$, with θ being the mixing angle between $U(1)'$ and $U(1)''$.

Among the parameters in the Higgs potential, m_i may be expressed in terms of the other parameters by using the minimum equations obtained as the first derivatives of the Higgs potential with respect to the four neutral Higgs fields. After spontaneous breakdown of the electroweak gauge symmetry, we have two additional gauge bosons, $Z^{'}$ and $Z^{''}$, for the extra $U(1)'$ and $U(1)''$, respectively, and seven physical Higgs particles: four scalar Higgs bosons, one pseudoscalar Higgs bosons, and a pair of charged Higgs bosons. The vacuum expectation values of the neutral Higgs fields are given as $v_1 = \langle H_1 \rangle$, $v_2 = \langle H_2 \rangle$, $x_1 = \langle N_1 \rangle$, and $x_2 = \langle N_2 \rangle$. We introduce $\tan\beta = v_2/v_1$ and $v = \sqrt{v_1^2 + v_2^2} = 175$ GeV.

III. MASS SPECTRA

Now, let us calculate the masses of the relevant particles.

1. Masses of Neutral Gauge Bosons and Scalar Top Quarks

In the SUSY E_6 model, there are three neutral gauge bosons, Z, Z', and Z'', arising from $U(1)$, $U(1)'$, and $U(1)''$, respectively. Their square masses are given as the eigenvalues of a symmetric 3×3 matrix, $\mathrm{M^G}$. The elements of M^G are given explicitly as

$$\begin{aligned}
M^G_{11} &= m_Z^2 \ , \\
M^G_{22} &= \frac{20}{9} g_1^{'2} x_1^2 S_\theta^2 + \frac{5}{36} g_1^{'2} x_2^2 \left[8 + 7C_{2\theta} - \sqrt{15} S_{2\theta} \right] \\
&\quad + \frac{1}{9} g_1^{'2} v^2 \left[4 - C_{2\theta} + \sqrt{15}\cos(2\beta) S_{2\theta} \right] \ , \\
M^G_{33} &= \frac{20}{9} g_1^{''2} x_1^2 C_\theta^2 + \frac{5}{36} g_1^{''2} x_2^2 \left[C_\theta + \sqrt{15} S_\theta \right]^2 \\
&\quad + \frac{1}{9} g_1^{''2} v^2 \left[4 + C_{2\theta} - \sqrt{15}\cos(2\beta) S_{2\theta} \right] \ , \\
M^G_{12} &= \frac{1}{3} g_1^{'} m_Z v \left[\sqrt{3} C_\theta + \sqrt{5}\cos(2\beta) S_\theta \right] \ , \\
M^G_{13} &= \frac{1}{3} g_1^{''} m_Z v \left[\sqrt{5}\cos(2\beta) C_\theta - \sqrt{3} S_\theta \right] \ , \\
M^G_{23} &= \frac{10}{9} g_1^{'} g_1^{''} x_1^2 S_{2\theta} + \frac{1}{9} g_1^{'} g_1^{''} v^2 \left[\sqrt{15}\cos(2\beta) C_{2\theta} \right. \\
&\quad \left. + S_{2\theta} \right] - \frac{5}{36} g_1^{'} g_1^{''} x_2^2 \left[\sqrt{15} C_{2\theta} + 7 S_{2\theta} \right] \ , \qquad (3)
\end{aligned}$$

where $m_Z^2 = v^2(g_1^2 + g_2^2)/2$, g_1 and g_2 are the gauge coupling coefficients of $U(1)$ and $SU(2)$, respectively, $S_{2\theta} = \sin(2\theta)$ and $C_{2\theta} = \cos(2\theta)$.

A 3×3 orthogonal matrix U^G can be employed to diagonalize M^G in order to obtain three eigenvalues: namely, m_Z^2, $m_{Z'}^2$, and $m_{Z''}^2$. We note that the smallest eigenvalue, m_Z^2, is different from the squared mass of the neutral gauge boson in the SM. However, in the decoupling limit of the $U(1)$ symmetry from the extra $U(1)'$ and $U(1)''$, m_Z becomes the gauge boson mass of the SM. We sort them such that $m_Z < m_{Z'} < m_{Z''}$, where the mass of the lightest neutral gauge boson should be equal to the mass of the existing neutral gauge boson, 91.2 GeV. The mixings among them can be parametrized by three mixing angles: namely, α_1 between Z and Z', α_2 between Z' and Z'', and α_3 between Z and Z''. These mixing angles may be expressed in terms of the elements of U^G as

$$\alpha_1 = \arctan\left(\frac{U^G_{12}}{U^G_{22}} \right) \ , \qquad (4)$$

$$\alpha_2 = \arcsin\left(-U^G_{32} \right) \ , \qquad (5)$$

$$\alpha_3 = \arctan\left(-\frac{U^G_{31}}{U^G_{33}} \right) \ . \qquad (6)$$

We note that there are strong experimental constraints on the masses of the extra neutral gauge bosons and on the mixing angles between the extra gauge bosons and the SM Z boson. The results of a direct search in $p\bar{p}$ collisions at the Fermilab Tevatron provide the constraint that $m_{Z'}$, $m_{Z''} > 800$ GeV, and the electroweak precision measurements at LEP2 provide the constraint that $|\alpha_1|$, $|\alpha_3| < 3 \times 10^{-3}$ [15]. Since the mixings between $U(1)$ and $U(1)'$ or between $U(1)$ and $U(1)''$ are very small, the extra gauge symmetries are more or less decoupled from $SU(2) \times U(1)$.

Next, we calculate the scalar top quark masses at the tree level. They are obtained as

$$m^2_{\tilde{t}_1,\ \tilde{t}_2} = \frac{1}{2}(m_Q^2 + m_T^2) + m_t^2 + \frac{1}{4} m_Z^2 \cos 2\beta + G_t^{'} \mp \sqrt{X_t} \ , \qquad (7)$$

with

$$\begin{aligned}
G_t^{'} &= -\frac{g_1^{'2}}{4} \left(\frac{1}{3}\sqrt{\frac{5}{2}} S_\theta - \frac{1}{\sqrt{6}} C_\theta \right) \\
&\quad \times \left[\left(\frac{\sqrt{10}}{3} S_\theta + \sqrt{\frac{2}{3}} C_\theta \right) v^2 \cos^2\beta - \frac{2}{3}\sqrt{10} S_\theta x_1^2 \right. \\
&\quad + \left(\frac{\sqrt{10}}{3} S_\theta - \sqrt{\frac{2}{3}} C_\theta \right) v^2 \sin^2\beta \\
&\quad \left. - \left(\frac{1}{3}\sqrt{\frac{5}{2}} S_\theta - \frac{5}{\sqrt{6}} C_\theta \right) x_2^2 \right] \\
&\quad - \frac{g_1^{''2}}{4} \left(\frac{1}{3}\sqrt{\frac{5}{2}} C_\theta + \frac{1}{\sqrt{6}} S_\theta \right) \\
&\quad \times \left[\left(\frac{\sqrt{10}}{3} C_\theta - \sqrt{\frac{2}{3}} S_\theta \right) v^2 \cos^2\beta - \frac{2}{3}\sqrt{10} C_\theta x_1^2 \right. \\
&\quad + \left(\frac{\sqrt{10}}{3} C_\theta + \sqrt{\frac{2}{3}} S_\theta \right) v^2 \sin^2\beta \\
&\quad \left. - \left(\frac{1}{3}\sqrt{\frac{5}{2}} C_\theta + \frac{5}{\sqrt{6}} S_\theta \right) x_2^2 \right] \ , \qquad (8)
\end{aligned}$$

$$\begin{aligned}
X_t &= \left[\frac{1}{2}(m_Q^2 - m_T^2) + \left(\frac{2}{3} m_W^2 - \frac{5}{12} m_Z^2 \right) \cos 2\beta \right]^2 \\
&\quad + m_t^2 (A_t - \lambda x_1 \cot\beta)^2 \ , \qquad (9)
\end{aligned}$$

where m_Q and m_T are the soft SUSY breaking masses, $m_W^2 = v^2 g_2^2/2$, $m_t = h_t v_2$ is the top quark mass, A_t is the trilinear SUSY breaking parameter with mass dimension, $G_t^{'}$ is the D-term contribution from the extra $U(1)'$ and $U(1)''$, and X_t is the mixing term between $\tilde{t}_1$ and $\tilde{t}_2$.

2. Masses of Pseudoscalar and Scalar Higgs Bosons

Next, we evaluate the one-loop corrections to the mass spectra. The corrections are calculated by inserting the relevant tree-level masses into the one-loop effective potential of Coleman and Weinberg [16]. The top quark and the scalar top contribution to the one-loop effective potential is given by

$$V_1 = \sum_l \frac{n_l \mathcal{M}_l^4}{64\pi^2} \left[\log \frac{\mathcal{M}_l^2}{\Lambda^2} - \frac{3}{2} \right] \ , \qquad (10)$$

where Λ is the renormalization scale of the modified minimal subtraction scheme. The summation is over $l = \tilde{t}_1$, $\tilde{t}_2$, t and $n_l = 6$ for the scalar top quarks and $n_l = -12$ for the top quark.

There are four neutral Higgs fields in the SUSY E_6 model, arising from two Higgs doublets and two Higgs singlets. Among the four complex components of these neutral Higgs fields, three of them are gauged to generate masses of Z, Z', and Z''. The remaining one complex component becomes the physical pseudoscalar Higgs boson of the model. The mass of the pseudoscalar Higgs boson at the one-loop level is given as

$$m_A^2 = m_{A^0}^2 + m_{A^1}^2 , \qquad (11)$$

with

$$\begin{aligned} m_{A^0}^2 &= \frac{2\lambda A v}{\sin 2\alpha} , \\ m_{A^1}^2 &= -\frac{3\lambda m_t^2 A_t}{8\pi^2 v \sin 2\alpha \sin^2\beta} f(m_{\tilde{t}_1}^2,\, m_{\tilde{t}_2}^2) , \end{aligned} \qquad (12)$$

where $m_{A^0}^2$ is obtained from the tree-level Higgs potential V_0, $m_{A^1}^2$ is obtained from the one-loop Higgs potential V_1, $\tan\alpha = v \sin 2\beta/(2x_1)$, and

$$f(m_x^2,\, m_y^2) = \frac{1}{(m_y^2 - m_x^2)} \left[m_x^2 \log\frac{m_x^2}{\Lambda^2} - m_y^2 \log\frac{m_y^2}{\Lambda^2} \right] + 1 . \qquad (13)$$

The second derivatives of the Higgs potential with respect to the real components of these four neutral Higgs fields give us the mass matrix for the four scalar Higgs bosons. The 4×4 symmetric mass matrix M for the scalar Higgs bosons can be decomposed as

$$M = M^0 + M^1 , \qquad (14)$$

where M^0 is obtained from the tree-level Higgs potential V_0, and M^1 is obtained from the one-loop Higgs potential V_1. The explicit formulas for M_{ij}^0 $(i,\, j = 1,\, 2,\, 3,\, 4)$ are

$$\begin{aligned} M_{11}^0 &= m_Z^2 \cos^2\beta + m_{A^0}^2 \sin^2\beta \cos^2\alpha \\ &\quad + \frac{1}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v^2 \cos^2\beta \\ &\quad + \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) v^2 \cos^2\beta \\ &\quad - \frac{\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v^2 \cos^2\beta , \\ M_{22}^0 &= m_Z^2 \sin^2\beta + m_{A^0}^2 \cos^2\beta \cos^2\alpha \\ &\quad + \frac{8}{9}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v^2 \sin^2\beta , \\ M_{33}^0 &= m_{A^0}^2 \sin^2\alpha + \frac{25}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) x_1^2 \\ &\quad + \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) x_1^2 \\ &\quad - \frac{5\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta x_1^2 , \\ M_{44}^0 &= \frac{25}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) x_2^2 + \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) x_2^2 \\ &\quad + \frac{5\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta x_2^2 , \\ M_{12}^0 &= (\lambda^2 v^2 - m_Z^2/2) \sin 2\beta - m_{A^0}^2 \cos\beta \sin\beta \cos^2\alpha \\ &\quad + \frac{1}{9}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v^2 \sin 2\beta \\ &\quad - \frac{\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v^2 \sin 2\beta , \\ M_{13}^0 &= 2\lambda^2 v x_1 \cos\beta - m_{A^0}^2 \sin\beta \cos\alpha \sin\alpha \\ &\quad - \frac{5}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v x_1 \cos\beta \\ &\quad - \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) v x_1 \cos\beta \\ &\quad + \frac{\sqrt{15}}{3}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v x_1 \cos\beta , \\ M_{14}^0 &= -\frac{5}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v x_2 \cos\beta \\ &\quad + \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) v x_2 \cos\beta \\ &\quad + \frac{2\sqrt{15}}{3}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v x_2 \cos\beta , \\ M_{23}^0 &= 2\lambda^2 v x_1 \sin\beta - m_{A^0}^2 \cos\beta \cos\alpha \sin\alpha \\ &\quad - \frac{10}{9}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v x_1 \sin\beta \\ &\quad + \frac{2\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v x_1 \sin\beta , \\ M_{24}^0 &= -\frac{10}{9}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) v x_2 \sin\beta \\ &\quad - \frac{2\sqrt{15}}{9}(g_1^{'2} - g_1^{''2}) C_\theta S_\theta v x_2 \sin\beta , \\ M_{34}^0 &= \frac{25}{18}(g_1^{'2} C_\theta^2 + g_1^{''2} S_\theta^2) x_1 x_2 \\ &\quad - \frac{5}{6}(g_1^{'2} S_\theta^2 + g_1^{''2} C_\theta^2) x_1 x_2 . \end{aligned} \qquad (15)$$

and the explicit formulas for M_{ij}^1 $(i, j = 1,\, 2,\, 3,\, 4)$ are obtained by inserting the Higgs field dependent masses of the top quark and scalar top quarks at the tree level into V^1 as

$$\begin{aligned} M_{ij}^1 &= \frac{3}{32\pi^2 v^2} W_i W_j \frac{g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2} \\ &\quad + \frac{3}{32\pi^2 v^2} A_i A_j \log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{\Lambda^4}\right) \\ &\quad + \frac{3}{32\pi^2 v^2} (W_i A_j + A_i W_j) \frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)} + D_{ij} , \end{aligned} \qquad (16)$$

with

$$g(m_x^2, m_y^2) = \frac{m_y^2 + m_x^2}{m_x^2 - m_y^2} \log\frac{m_y^2}{m_x^2} + 2 , \qquad (17)$$

$$\begin{aligned} W_1 &= \frac{2m_t^2 x_1 \lambda \Delta_1}{\sin\beta} + \cos\beta \Delta_2 , \\ W_2 &= -\frac{2m_t^2 A_t \Delta_1}{\sin\beta} - \sin\beta \Delta_2 , \\ W_3 &= \frac{2m_t^2 \lambda v \Delta_1}{\tan\beta} , \end{aligned}$$

$$
\begin{aligned}
W_4 &= 0 , \\
A_1 &= \frac{1}{2}\cos\beta(4G_1v^2 + m_Z^2) , \\
A_2 &= \frac{2m_t^2}{\sin\beta} + 2G_2v^2\sin\beta - \frac{m_Z^2}{2}\sin\beta , \\
A_3 &= 2G_3x_1v , \\
A_4 &= 2G_4x_2v , \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_1 &= \lambda x_1\cot\beta - A_t , \\
\Delta_2 &= \left(\frac{4}{3}m_W^2 - \frac{5}{6}m_Z^2\right) \\
&\times\left[m_Q^2 - m_T^2 + \left(\frac{4}{3}m_W^2 - \frac{5}{6}m_Z^2\right)\cos 2\beta\right], \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
G_1 &= \frac{g_1^{'2}}{36}(4C_{2\theta} - 1) - \frac{g_1^{''2}}{36}(4C_{2\theta} + 1) , \\
G_2 &= \frac{g_1^{'2}}{36}(\sqrt{15}S_{2\theta} + C_{2\theta} - 4) \\
&- \frac{g_1^{''2}}{36}(\sqrt{15}S_{2\theta} + C_{2\theta} + 4) , \\
G_3 &= -\frac{g_1^{'2}}{18}(\sqrt{15}C_\theta - 5S_\theta)S_\theta \\
&+ \frac{g_1^{''2}}{18}(\sqrt{15}S_\theta + 5C_\theta)C_\theta , \\
G_4 &= \frac{g_1^{'2}}{72}(10 - 3\sqrt{15}S_{2\theta} + 5C_{2\theta}) \\
&+ \frac{g_1^{''2}}{72}(10 + 3\sqrt{15}S_{2\theta} - 5C_{2\theta}) , \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
D_{11} &= m_{A^1}^2\sin^2\beta\cos^2\alpha \\
&- \frac{3\cos^2\beta}{16\pi^2v^2}\left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6}\right)^2 f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) , \\
D_{22} &= m_{A^1}^2\cos^2\beta\cos^2\alpha \\
&- \frac{3\sin^2\beta}{16\pi^2v^2}\left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6}\right)^2 f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \\
&- \frac{3m_t^4}{4\pi^2v^2\sin^2\beta}\log\left(\frac{m_t^2}{\Lambda^2}\right) , \\
D_{33} &= m_{A^1}^2\sin^2\alpha , \\
D_{12} &= \frac{3\sin 2\beta}{32\pi^2v^2}\left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6}\right)^2 f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) , \\
D_{13} &= -m_{A^1}^2\sin\beta\cos\alpha\sin\alpha \\
&- \frac{3m_t^2\lambda^2x_1\cos\beta}{8\pi^2v\sin^2\beta}f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) , \\
D_{23} &= -m_{A^1}^2\cos\beta\cos\alpha\sin\alpha , \\
D_{i4} &= 0 , \quad (i = 1, 2, 3, 4) . \qquad (21)
\end{aligned}
$$

The eigenstates of M are four physical scalar Higgs bosons of the SUSY E_6 model, denoted as S_i (i = 1, 2, 3, 4), and the corresponding eigenvalues are their squared masses, denoted as $m_{S_i}^2$. The masses of these four scalar Higgs bosons are ordered as $m_{S_1} < m_{S_2} < m_{S_3} < m_{S_4}$.

For the numerical analysis, we establish the parameter space as follows: $1 < \tan\beta \le 30$, $0 < \lambda \le 0.83$, $10 < A <$ 400 GeV, $0 < \theta < \pi/2$, $100 \le x_1, x_2 \le 1500$ GeV, $100 \le$ m_Q, m_T, and $A_t \le 1000$ GeV. Furthermore, we put m_t = 175 GeV, and we assume that the scalar top quarks are heavier than the top quark. We set the lower bound on the effective $\mu^e = \lambda x_1$ parameter to be 150 GeV in order to take into account the experimental lower bound on the lighter chargino mass at the LEP2 experiments.

We also take into account the experimental results for the Higgs search at the LEP2 experiments. Recently, the collaborations of the LEP2 experiments reported a model-independent upper bound on the Higgs coupling coefficient to a pair of Z bosons at the 95% confidence level [14]. The result may be interpreted as the coefficient of the SUSY E_6 model. The coupling coefficient of S_i to a pair of Z bosons of the model, normalized to the corresponding SM coupling coefficient, can be written as

$$
G_{ZZS_i} \approx \cos\beta O_{1i} + \sin\beta O_{2i} , \qquad (22)
$$

where O_{ij} are the ij-th element of the 4×4 orthogonal matrix that diagonalizes the mass matrix for the neutral scalar Higgs bosons.

For a given set of parameter values, we evaluate m_{S_i} and G_{ZZS_i}. In this way, by using a Monte Carlo method, we explore 10^5 points of the established parameter space. In Fig. 1, we show the result of evaluations of m_{S_1} and G_{ZZS_1}. The SM coupling coefficient is 1.0 because it is normalized to itself. We find that G_{ZZS_1} is larger than 0.9 for most of the parameter space, which tells us that S_1 is more or less equivalent to the SM Higgs boson. Since G_{ZZS_i} (i = 1, 2, 3, 4) satisfy a sum rule of $\sum_{i=1}^4 G_{ZZS_i}^2 \approx 1$, the pair of Z bosons couples nearly exclusively to S_1 in most of the parameter space, and the couplings to the other heavier scalar Higgs bosons are negligibly small.

The result in Fig. 1 also shows that m_{S_1} is in the range of 112 to 142 GeV at the one-loop level. The upper bound on m_{S_1} is determined primarily by the maximum value of the SUSY breaking scale and the LEP2 constraints on the SM Higgs coupling coefficient to a pair of S bosons. The masses of heavier scalar Higgs bosons are 135 GeV $< m_{S_2} <$ 897 GeV, 800 GeV $< m_{S_3} <$ 1155 GeV, and 1033 GeV $< m_{S_4} <$ 2828 GeV. Also, the masses of the pseudoscalar Higgs boson of the SUSY E_6 model at the one-loop level are estimated to be $120 < m_A < 2828$ GeV for the parameter values we consider.

The coupling coefficient G_{ZZS_1} is crucial to calculate the production cross section for the Higgs-strahlung process, $\sigma(e^+e^- \to Z \to ZS_1)$. In Fig. 2, we show the production cross section of S_1 via the Higgs-strahlung process in e^+e^- collisions with $\sqrt{s}$ = 500 GeV, where the corresponding SM cross section is also shown as a solid curve, as a function of the SM Higgs boson mass. The values of the relevant parameters are the same as in Fig. 1 and were chosen randomly in the established

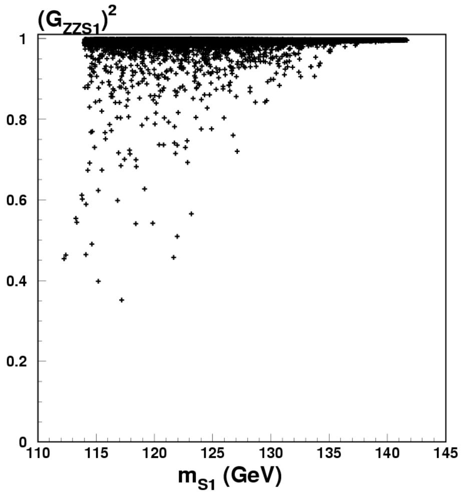

Fig. 1. The squared coupling coefficient $G^2_{ZZS_1}$ against m_{S_1}, normalized to the SM coefficient, for 10^5 sets of parameter values that vary within the parameter space established as $1 < \tan\beta \leq 30$, $0 < \lambda \leq 0.83$, $10 < A < 400$ GeV, $0 < \theta < \pi/2$, $100 \leq x_1,\ x_2 \leq 1500$ GeV, $100 \leq m_Q,\ m_T$, and $A_t \leq 1000$ GeV. The squared SM coefficient, normalized to itself, is a solid line at 1.0. Notice that most of the points are distributed close to the solid line and that m_{S_1} is between 112 and 142 GeV.

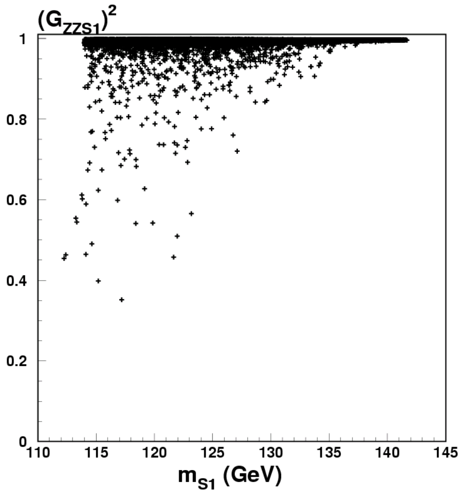

Fig. 2. The squared coupling coefficient $G^2_{ZZS_1}$ against m_{S_1}, normalized to the SM coefficient, for 10^5 sets of parameter values that vary within the parameter space established as $1 < \tan\beta \leq 30$, $0 < \lambda \leq 0.83$, $10 < A < 400$ GeV, $0 < \theta < \pi/2$, $100 \leq x_1,\ x_2 \leq 1500$ GeV, $100 \leq m_Q,\ m_T$, and $A_t \leq 1000$ GeV. The squared SM coefficient, normalized to itself, is a solid line at 1.0. Notice that most of the points are distributed close to the solid line and that m_{S_1} is between 112 and 142 GeV.

parameter space.

One may notice in Fig. 2 that most of the 10^5 points are distributed quite close to the solid curve. In other words, S_1 of the SUSY E_6 model behaves almost the same as the SM Higgs boson in the Higgs-strahlung process in e^+e^- collisions. One may also notice in Fig. 2 that the absolute minimum of the production cross section is about 19 fb. That is, the production cross section is larger than 19 fb for any parameter values. It is worthwhile to compare this absolute lower bound on the production cross section with the corresponding results of other models. We studied the same subject in the NMSSM [17]. We calculated the absolute lower bound on the production cross section of the lightest scalar Higgs boson via the Higgs-strahlung process in the NMSSM and obtained a value of about 15 fb (see the dashed curve of Fig. 3(a) in Ref. 17). Thus, the absolute lower bound on the production cross section of the SUSY E_6 model is, in fact, larger than the corresponding value in the NMSSM.

Usually, for a supersymmetric model with several Higgs singlets, the production cross section for the lightest scalar Higgs boson tends to decrease as the number of Higgs singlets increases because the probability of production should be shared with other heavier scalar Higgs bosons. In this respect, the lower bound on the production cross section of the SUSY E_6 model should be smaller than that of the NMSSM because the SUSY E_6 model has one more Higgs singlet than the NMSSM. However, the result is opposite to the usual anticipation. We think that the main reason for this result is that S_1 of the SUSY E_6 model is more similar to the SM Higgs boson than the lightest scalar Higgs boson of the NMSSM, at least in the Higgs-strahlung process in e^+e^- collisions.

IV. PRODUCTION OF SCALAR HIGGS BOSONS VIA THE DOUBLE HIGGS-STRAHLUNG PROCESS

The cubic and quartic couplings of Higgs bosons contribute when a process involves multiple Higgs bosons. Typically, the double Higgs production process in e^+e^- collisions depends on the cubic couplings of Higgs bosons and the quartic couplings of Higgs-Higgs-gauge-gauge bosons. For practical simplicity, we only consider double S_1 productions: $e^+e^- \to ZS_1S_1$. The double productions of heavier scalar Higgs bosons are kinematically suppressed, so the production cross sections are very small. The relevant Feynman diagrams for the double S_1 production are shown in Fig. 3. Notice that quartic coupling and cubic coupling are present in Fig. 3.

In principle, all four scalar Higgs bosons of the SUSY E_6 model may participate in Fig. 3 as intermediate particles. The contributions of the heavier scalar Higgs particles as intermediate particles depend on two coupling coefficients: G_{ZZS_i} at the one end and $G_{S_iS_1S_1}$ at the other end (i = 2, 3, 4). However, as we have observed before, G_{ZZS_i} (i = 2, 3, 4), the coupling coefficients of heavier scalar Higgs bosons to the Z boson pair, are negligibly smaller than G_{ZZS_1}. Thus, the contributions of the heavier scalar Higgs particles as intermediate parti-

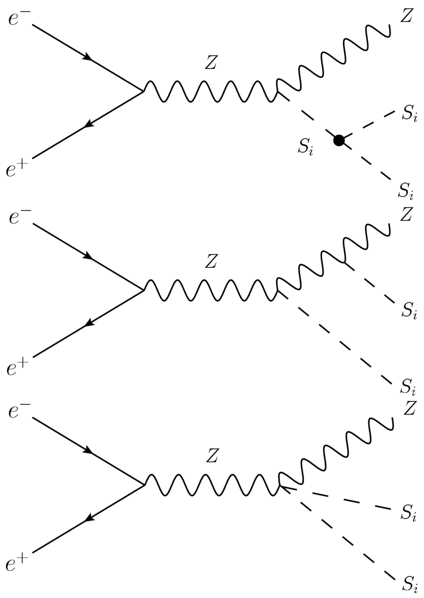

Fig. 3. Feynman diagrams for the double Higgs-strahlung process, $e^+e^- \to Z \to ZS_1S_1$. Notice that the Higgs cubic coupling and the Higgs-Higgs-gauge-gauge quartic coupling are present.

cles may be neglected, and we only consider S_1 as the intermediate particle in Fig. 3 in our numerical analysis. Consequently, only S_1 participates in Fig. 3.

In order to calculate the production cross section of the lightest scalar Higgs boson, we have to know its cubic and quartic coupling coefficients in the SUSY E_6 model. The cubic coupling coefficient of S_i can be written as

$$G_{S_iS_iS_i} = G^0_{S_iS_iS_i} + G^1_{S_iS_iS_i} \ , \tag{23}$$

where $G^0_{S_iS_iS_i}$ is the tree-level coefficient and $G^1_{S_iS_iS_i}$ is the one-loop correction. Explicitly, the tree-level cubic coupling coefficient of S_1 is given as

$$\begin{aligned} G^0_{S_iS_iS_i} &= 12\lambda^2[(O^2_{1i}+O^2_{2i})O_{3i}x_1 + O_{1i}(O^2_{2i}+O^2_{3i}) \\ &\quad \times v\cos\beta + O_{2i}(O^2_{1i}+O^2_{3i})v\sin\beta] \\ &\quad +3v(g_1^2+g_2^2)(O^2_{1i}-O^2_{2i}) \times (O_{1i}\cos\beta \\ &\quad -O_{2i}\sin\beta) - \frac{6m^2_{A^0}\sin 2\alpha}{v} O_{1i}O_{2i}O_{3i} \\ &\quad +G^{'}_{S_iS_iS_i} + G^{''}_{S_iS_iS_i} \ , \end{aligned} \tag{24}$$

with

$$\begin{aligned} G^{'}_{S_iS_iS_i} &= \frac{1}{3}g_1^{'2}[C_\theta(O^2_{1i}+4O^2_{2i}-5O^2_{3i}-5O^2_{4i}) \\ &\quad -\sqrt{15}(O^2_{1i}-O^2_{3i}+O^2_{4i})S_\theta] \\ &\quad \times[\sqrt{15}S_\theta(O_{3i}x_1 - O_{4i}x_2 - O_{1i}v\cos\beta) \\ &\quad -C_\theta(5O_{3i}x_1 + 5O_{4i}x_2 - O_{1i}v\cos\beta \\ &\quad -4O_{2i}v\sin\beta)] \ , \\ G^{''}_{S_iS_iS_i} &= G^{'}_{S_iS_iS_i}\left[g_1^{'} \to g_1^{''}, C_\theta \to S_\theta, S_\theta \to -C_\theta\right], \end{aligned} \tag{25}$$

where $G^{'}_{S_iS_iS_i}$ is the D-term contribution of the $U(1)'$ and $G^{''}_{S_iS_iS_i}$ is the D-term contribution of the $U(1)''$ at the tree level.

The one-loop correction to the cubic coupling of the scalar Higgs boson is given as

$$\begin{aligned} G^1_{S_iS_iS_i} &= -\frac{9}{32\pi^2}Y_{ii}Y_i\log\left(\frac{m_t^2}{\Lambda^2}\right) - \frac{3}{32\pi^2 m_t^2}Y_i^3 \\ &\quad +\frac{9}{64\pi^2}A_{ii}A_i\log\left(\frac{m^2_{\tilde t_1}m^2_{\tilde t_2}}{\Lambda^4}\right) \\ &\quad -\frac{9}{32\pi^2}W_i^3\frac{\log(m^2_{\tilde t_1}m^2_{\tilde t_2}/\Lambda^4)}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})^4} \\ &\quad +\frac{9}{64\pi^2}W_{ii}W_i\frac{\log(m^2_{\tilde t_1}m^2_{\tilde t_2}/\Lambda^4)}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})^2} \\ &\quad +\frac{9}{64\pi^2}\left[A_{ii}W_i + W_{ii}A_i\right]\frac{\log(m^2_{\tilde t_2}/m^2_{\tilde t_1})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})} \\ &\quad -\frac{9}{32\pi^2}W_i^2A_i\frac{\log(m^2_{\tilde t_2}/m^2_{\tilde t_1})}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})^3} \\ &\quad +\frac{3}{64\pi^2 m^2_{\tilde t_1}}\left[A_i - \frac{W_i}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})}\right]^3 \\ &\quad +\frac{3}{64\pi^2 m^2_{\tilde t_2}}\left[A_i + \frac{W_i}{(m^2_{\tilde t_2}-m^2_{\tilde t_1})}\right]^3 , \end{aligned} \tag{26}$$

where

$$\begin{aligned} Y_i &= \frac{2m_t^2 O_{2i}}{v\sin\beta}, \\ Y_{ii} &= \frac{2m_t^2 O^2_{2i}}{v^2\sin^2\beta}, \\ A_i &= 2g_1^{'}O_{3i}x_1 + 2g_1^{''}O_{4i}x_2 + 2g_1O_{1i}v\cos\beta \\ &\quad +2g_2O_{2i}v\sin\beta + \frac{2m_t^2}{v\sin\beta}O_{2i} \\ &\quad +\frac{(O_{1i}\cos\beta - O_{2i}\sin\beta)m_Z^2}{2v}, \\ A_{ii} &= 2g_1O^2_{1i} + 2g_2O^2_{2i} + \frac{2m_t^2}{v^2\sin^2\beta}O^2_{2i} \\ &\quad +2g_1^{'}O^2_{3i} + 2g_1^{''}O^2_{4i} + \frac{(O^2_{1i}-O^2_{2i})\,m_Z^2}{2v^2} \ , \\ W_i &= \frac{O_{1i}\cos\beta - O_{2i}\sin\beta}{v}\left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6}\right) \\ &\quad \times\left[m_Q^2 - m_T^2 + \cos(2\beta)\left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6}\right)\right] \\ &\quad +\frac{2m_t^2}{v\sin\beta}(O_{1i}x_1\lambda + O_{3i}v\lambda\cos\beta - A_tO_{2i}) \\ &\quad (x_1\lambda\cot\beta - A_t), \\ W_{ii} &= \frac{2(O_{1i}\cos\beta - O_{2i}\sin\beta)^2}{v^2}\left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6}\right)^2 \\ &\quad +\frac{O^2_{1i}-O^2_{2i}}{v^2}\left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6}\right) \\ &\quad \times\left[m_Q^2 - m_T^2 + \cos(2\beta)\left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6}\right)\right] \\ &\quad +2\left(\frac{m_t}{v\sin\beta}\right)^2\Big[O^2_{3i}v^2\lambda^2\cos^2\beta \\ &\qquad -2O_{3i}v\lambda\{(A_tO_{2i} - 2O_{1i}x_1\lambda)\cos\beta \\ &\qquad +A_tO_{1i}\sin\beta\} + (A_tO_{2i} - O_{1i}x_1\lambda)^2\Big] \end{aligned} \tag{27}$$

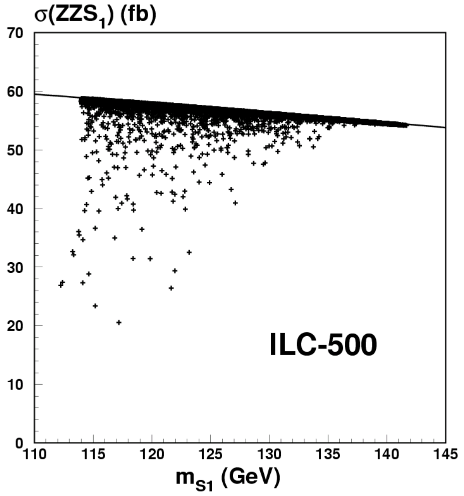

Fig. 4. The production cross section $\sigma(ZZS_1)$ against m_{S_1}, via the Higgs-strahlung process in e^+e^- collisions with $\sqrt{s} = 500$ GeV, for the same 10^5 sets of parameter values as in Fig. 1. The solid curve is the corresponding SM cross section as a function of the SM Higgs boson mass.

The corresponding SM cubic coupling coefficient, G_{hhh}, may be obtained from the SM Higgs potential $\mu^2\phi^\dagger\phi/2 + \lambda(\phi^\dagger\phi)^2/4$ as $6m_h^2/v$, where $v = \langle\phi\rangle$ and m_h is the mass of SM Higgs boson. Conventionally, the SM cubic coupling coefficient is normalized such that [13] $\bar{G}_{hhh} = G_{hhh}v/(2m_Z^2)$. In Fig. 3, we also plot $\bar{G}_{hhh}^2$ as a function of m_h (the solid curve).

In order to compare the cubic coupling coefficient of the SUSY E_6 model with the SM cubic coupling coefficient, we may normalize $G_{S_iS_iS_i}$ in units of $2m_Z^2/v$ as $[v/(2m_Z^2)]G_{S_iS_iS_i}$.

In Fig. 4 we plot the square of the cubic coupling coefficient, normalized to $2m_Z^2/v$, against m_{S_1}, by varying the values of the relevant parameters within the established space. We find that the cubic coupling coefficient of the SUSY E_6 model is smaller than the corresponding SM coefficient. This behavior of the SUSY E_6 model is similar to that of the MSSM, where the cubic coupling coefficient of the lightest scalar Higgs boson is smaller than the corresponding coefficient of the SM Higgs boson [18]. On the other hand, it is different from the two Higgs doublet model, where the SM cubic coupling coefficient may be smaller than the cubic coupling coefficient of the lightest scalar Higgs boson [19]. We also find that the cubic coupling coefficients of other scalar Higgs bosons, $G_{S_iS_iS_i}$ (i = 2, 3, 4), are smaller than the SM cubic coupling coefficient.

Next, let us consider the quartic coupling coefficient of Higgs-Higgs-gauge-gauge bosons of the SUSY E_6 model. When normalized to the corresponding SM quartic coupling coefficient, $G_{ZZS_iS_i}$ of a S_i pair to a Z boson pair is given as

$$G_{ZZS_iS_i} \approx O_{1i}^2 + O_{2i}^2 \qquad (28)$$

where mixing between the Z boson and the extra neutral gauge bosons is neglected. Since the mixing angles $|\alpha_2|$ and $|\alpha_3|$ are very small, this is justified in practice.

Now, we are ready to calculate the differential cross section for the double Higgs-strahlung process in e^+e^- collisions of the lightest scalar Higgs boson of the SUSY E_6 model. We obtain

$$\frac{d\sigma}{dx_1dx_2}(e^+e^- \to Z \to ZS_1S_1) = \frac{\sqrt{2}G_F^3m_Z^6}{384\pi^3 s}\frac{v_e^2 + a_e^2}{(1-\mu_Z)^2}\mathcal{Z}\,, \qquad (29)$$

where $x_i = 2E_i/\sqrt{s}$ is the scaled energy of the i-th S_1, with E_i being the energy of the i-th S_1 and $\sqrt{s}$ being the center-of-mass (c.m.) energy of the e^+e^- system (i = 1, 2), $v_e = -1 + 4\sin^2\theta_W$ and $a_e = -1$ are, respectively, the vector and the axial-vector Z charges of the incoming electron, $\mu_Z = m_Z^2/s$ is the square of the reduced Z boson mass, and $\mathcal{Z}$ is given by

$$\begin{aligned}\mathcal{Z} = {} & \mathcal{Z}_1 G_{ZZS_1}^2 G_{S_1S_1S_1}^2 + \mathcal{Z}_2 G_{ZZS_1}^4 + \mathcal{Z}_3 G_{ZZS_1S_1}^2 \\ & + \mathcal{Z}_{12} G_{ZZS_1}^3 G_{S_1S_1S_1} \\ & + \mathcal{Z}_{13} G_{ZZS_1} G_{S_1S_1S_1} G_{ZZS_1S_1} \\ & + \mathcal{Z}_{23} G_{ZZS_1}^2 G_{ZZS_1S_1}\,, \end{aligned} \qquad (30)$$

with

$$\begin{aligned}\mathcal{Z}_1 = {} & \mu_Z\frac{(y_1+y_2)^2 + 8\mu_Z}{4(y_3-\mu_{HZ})^2}\,, \\ \mathcal{Z}_2 = {} & \mu_Z\frac{(y_1+y_2)^2+8\mu_Z}{(y_1+\mu_{HZ})^2} + \mu_Z\frac{(y_1+y_2)^2+8\mu_Z}{(y_2+\mu_{HZ})^2} \\ & + \frac{2\mu_Z[(y_1+y_2)^2+8\mu_Z]}{(y_1+\mu_{HZ})(y_2+\mu_{HZ})} \\ & - \frac{y_1\mu_H(y_1-4\mu_Z+y_1\mu_Z)}{(y_1+\mu_{HZ})^2\mu_Z} \\ & - \frac{y_2\mu_H(y_2-4\mu_Z+y_2\mu_Z)}{\mu_Z(y_2+\mu_{HZ})^2} \\ & + \frac{1}{(y_1+\mu_{HZ})^2}\Big[y_1(y_1-1)(\mu_Z-y_1) \\ & - y_2(1+y_1)(y_1+\mu_Z) + 2\mu_Z(1-4\mu_H+\mu_Z)\Big] \\ & + \frac{1}{4(y_1+\mu_{HZ})^2\mu_Z}\Big[(y_1-1)^2(\mu_Z-y_1)^2 \\ & - \mu_Z^2 + \mu_Z(1-4\mu_H)(\mu_Z-4\mu_H)\Big] \\ & + \frac{1}{(y_2+\mu_{HZ})^2}\Big[y_2(y_2-1)(\mu_Z-y_2) \\ & - y_1(1+y_2)(y_2+\mu_Z) + 2\mu_Z(1-4\mu_H+\mu_Z)\Big] \\ & + \frac{1}{4\mu_Z(y_2+\mu_{HZ})^2}\Big[(y_2-1)^2(\mu_Z-y_2)^2 \\ & - \mu_Z^2 + \mu_Z(1-4\mu_H)(\mu_Z-4\mu_H)\Big] \\ & + \frac{1}{(y_1+\mu_{HZ})(y_2+\mu_{HZ})}\Big[y_1(y_1-1)(\mu_Z-y_1) \\ & - y_2(1+y_1)(y_1+\mu_Z) + 2\mu_Z(1-4\mu_H+\mu_Z)\Big] \end{aligned}$$

$$
\begin{aligned}
& +\frac{1}{(y_1+\mu_{HZ})(y_2+\mu_{HZ})}\Big[y_2(y_2-1)(\mu_Z-y_2) \\
& -y_1(1+y_2)(y_2+\mu_Z)+2\mu_Z(1-4\mu_H+\mu_Z)\Big] \\
& +\frac{1}{2\mu_Z(y_1+\mu_{HZ})(y_2+\mu_{HZ})} \\
& \times\Big\{\mu_Z^2+4\mu_H\mu_Z(1+4\mu_H+\mu_Z) \\
& +y_1y_2\left[1+y_1y_2+\mu_Z^2+4\mu_H(1+\mu_Z)\right] \\
& +(1+y_3+2\mu_Z)[\mu_Z(y_3-8\mu_H+\mu_Z) \\
& -y_1y_2(1+\mu_Z)]\Big\} ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{Z}_3 &= \frac{(y_1+y_2)^2}{4\mu_Z}+2 , \\
\mathcal{Z}_{12} &= \frac{1}{(y_3-\mu_{HZ})(y_1+\mu_{HZ})}\Big\{\mu_Z\left[(y_1+y_2)^2+8\mu_Z\right] \\
& +\frac{1}{2}[y_1(y_1-1)(\mu_Z-y_1)-y_2(1+y_1)(y_1+\mu_Z) \\
& +2\mu_Z(1-4\mu_H+\mu_Z)]\Big\} \\
& +\frac{1}{(y_2+\mu_{HZ})(y_3-\mu_{HZ})}\Big\{\mu_Z\left[(y_1+y_2)^2+8\mu_Z\right] \\
& +\frac{1}{2}[y_2(y_2-1)(\mu_Z-y_2)-y_1(1+y_2)(y_2+\mu_Z) \\
& +2\mu_Z(1-4\mu_H+\mu_Z)]\Big\} , \\
\mathcal{Z}_{13} &= \frac{\left[(y_1+y_2)^2+8\mu_Z\right]}{2(y_3-\mu_{HZ})} , \\
\mathcal{Z}_{23} &= \frac{(y_1+y_2)^2+8\mu_Z}{y_1+\mu_{HZ}}+\frac{(y_1+y_2)^2+8\mu_Z}{y_2+\mu_{HZ}} \\
& +\frac{1}{2(y_1+\mu_{HZ})\mu_Z}\Big[y_1(y_1-1)(\mu_Z-y_1) \\
& -y_2(1+y_1)(y_1+\mu_Z)+2\mu_Z(1-4\mu_H+\mu_Z)\Big] \\
& +\frac{1}{2(y_2+\mu_{HZ})\mu_Z}\Big[y_2(y_2-1)(\mu_Z-y_2) \\
& -y_1(1+y_2)(y_2+\mu_Z) \\
& +2\mu_Z(1-4\mu_H+\mu_Z)\Big], \qquad (31)
\end{aligned}
$$

where $x_3 = 2 - x_1 - x_2$ is the scaled energy of the Z boson, $y_1 = 1 - x_1$, $y_2 = 1 - x_2$, $\mu_H = m_{S_1}^2/s$ is the square of the reduced mass of S_1, and $\mu_{HZ} = \mu_H - \mu_Z$.

Then, we integrate the differential cross section to obtain the total production cross section for the double Higgs-strahlung process of S_1, $\sigma(ZS_1S_1)$, as a function of m_{S_1}. The values of the relevant parameters are the same as in Fig. 1, 10^5 points in the parameter space. The result is shown in Fig. 6, where we take $\sqrt{s} = 500$ GeV, the proposed c.m. energy of the first stage of the ILC (ILC-500).

For the sake of comparison, we also plot the corresponding SM cross section in Fig. 5. One may notice that most of the points are distributed close to the solid curve. This implies that S_1 behaves very much alike the SM Higgs boson with respect to the double Higgs-strahlung process. Also, the absolute minimum of the production cross section of the lightest scalar Higgs boson via the double Higgs-strahlung process is about 0.05 fb at the ILC-500. In other words, the SUSY E_6 model predicts that $\sigma(ZS_1S_1)$ is larger than 0.05 fb, whatever the parameter values. Therefore, we expect the ILC-500 to produce at least 5 S_1 events for the SUSY E_6 model via the double Higgs-strahlung process if the ILC-500 has an integrated luminosity of 500 fb^{-1} and an efficiency of 20%.

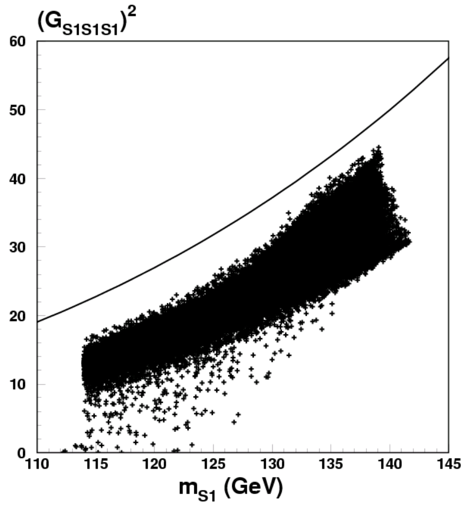

Fig. 5. Squared coupling coefficient $G^2_{S_1S_1S_1}$ against m_{S_1}, normalized to $2m_Z^2/v$, for same 10^5 sets of parameter values as in Fig. 1. The solid curve is the squared SM coefficient, normalized to $2m_Z^2/v$, as a function of the SM Higgs boson mass.

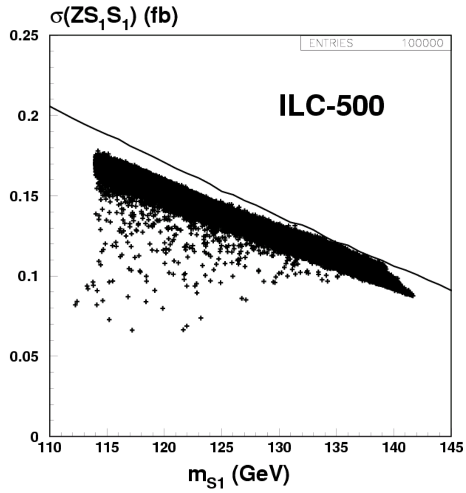

Fig. 6. Production cross section $\sigma(ZS_1S_1)$ against m_{S_1}, via the double Higgs-strahlung process in e^+e^- collisions with $\sqrt{s} = 500$ GeV, for the same 10^5 sets of parameter values as in Fig. 1. The solid curve is the corresponding SM cross section as a function of the SM Higgs boson mass.

V. CONCLUSIONS

We have studied a supersymmetric E_6 model, which has two Higgs doublets and two Higgs singlets. In particular, we have studied the production of the lightest scalar Higgs boson in e^+e^- collisions, via the Higgs-strahlung process and the double Higgs-strahlung process, for a reasonably established parameter space, at the one-loop level by considering the contributions from top and scalar top quark loops.

At the one-loop level, the mass of the lightest scalar Higgs boson of the SUSY E_6 model is estimated to be between 112 GeV and 142 GeV. Thus, its mass is comparable to that of the SM Higgs boson. Not only its mass but also its production cross sections in e^+e^- collisions via the Higgs-strahlung process and the double Higgs-strahlung process is found to be quite similar to those of the SM Higgs boson. For the most part, this similarity may be attributed to the experimental constraints on the large masses of the extra neutral gauge bosons and the tiny mixings between them and the Z boson, the electroweak precision measurement at the LEP2 experiment, and the direct search in $p\bar{p}$ collisions at the Tevatron. Since the extra neutral gauge bosons are heavier than 800 GeV and their mixing angles with the Z boson are less than 3×10^{-3}, they are practically decoupled from the Z boson and, hence, from S_1 in the Higgs-strahlung process and the double Higgs-strahlung process. In other words, we may assume without difficulty that only the Z boson is involved in these processes. We also find that the same experimental constraints set the lower bounds on x_1 and x_2, the vacuum expectation values of the two Higgs singlets, as about 1265 GeV.

Our study on the G_{ZZS_1} coupling coefficient, the production cross section $\sigma(e^+e^- \to Z \to ZZS_1)$ via Higgs-strahlung process, the $G_{S_1S_1S_1}$ cubic coupling coefficient, the $G_{ZZS_1S_1}$ quartic coupling coefficient, and the production cross section $\sigma(e^+e^- \to Z \to ZS_1S_1)$ via the double Higgs-strahlung process show clearly the similarity of S_1 of our model to the SM Higgs boson. Thus, the contributions of heavier scalar Higgs bosons are nearly negligible in these processes. Consequently, the experimental constraints on the extra neutral gauge bosons make the Higgs sector of the supersymmetric E_6 model with two Higgs doublets and two Higgs singlets very similar to the SM Higgs sector, as far as S_1 is concerned.

The absolute lower bound on the cross section for S_1 production in e^+e^- collisions via the Higgs-strahlung process at the ILC-500 is about 19 fb, and the absolute lower bound on the cross section for S_1S_1 production in e^+e^- collisions via the double Higgs-strahlung process at the ILC-500 is about 0.05 fb. If an integrated luminosity of 500 fb^{-1} and an efficiency of 20% are assumed for the ILC-500, at least 5 events of S_1 of our model might be explored at the ILC via the double Higgs-strahlung process.

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