Heavy quarkonium production

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<u>Outline</u>

- Introduction
- Exclusive production of heavy quarkonium
 - $e^+e^- \rightarrow J/\psi + \eta_c$
 - $e^+e^- \rightarrow J/\psi + J/\psi$
 - $e^+e^- \rightarrow \eta_c + \gamma$
- Inclusive production of heavy quarkonium
 - $p\bar{p} \to J/\psi + X$, $ep \to J/\psi + X$, $e^-e^+ \to J/\psi + X$ - $gg \to J/\psi \Upsilon$
- Summary



Heavy quarkonium decay

- Early stage calculation of P-wave heavy quarkonium decay was plagued with IR divergence.
- It has been resolved by the color-octet mechanism of NRQCD factorization approach.
- Inclusive charm production in χ_b decays:



(a) $b\bar{b}_{1}(^{3}P_{J}) \rightarrow c\bar{c}g \propto C_{J}^{(c)}, C_{8}^{(c)}$ (b) $b\bar{b}_{8}(^{3}S_{1}) \rightarrow g^{*} \rightarrow c\bar{c} \propto C_{8}^{(c)} \Rightarrow d\Gamma^{(c)} = dC_{J}^{(c)}(\Lambda) \frac{\langle \mathcal{O}_{1}(^{3}P_{J})\rangle}{m_{b}^{4}} + dC_{8}^{(c)} \frac{\langle \mathcal{O}_{8}(^{3}S_{1})\rangle^{(\Lambda)}}{m_{b}^{2}}$ $\mathcal{O}_{8}(^{3}S_{1}) = \mathcal{O}_{8}(^{3}S_{1})^{(\Lambda)} + \frac{(4\pi e^{-\gamma})^{\epsilon}}{\epsilon_{\rm UV}} \frac{2C_{F}\alpha_{s}}{3\pi N_{c}m_{b}^{2}} \sum_{J=0}^{2} \mathcal{O}_{1}(^{3}P_{J})$

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Exclusive vs. Inclusive

	Exclusive	Inclusive
Incident beam	lepton	lepton, hadron
Color-singlet	dominant	may be (sub)dominant
Color-octet	suppressed	may be (sub)dominant
QCD corrections	substantial	substantial
Relativistic corrections	large*	not so large**
Feed-down	-	moderate
	$^*e^+e^- \to J/\psi\eta_c$	$^{**}e^+e^- \rightarrow J/\psi c\bar{c}$

• Exclusive production of heavy quarkonium provides a unique chance to investigate effects of relativistic corrections.

• Color-octet mechanism should be proved in the inclusive mode, but compete with effects from the color-singlet contributions as well as from the feed-down.

Feed down



	S-wave		P-wave
2S+1LJ	¹ S ₀	³ S ₁	$^{1}P_{1} ^{3}P_{J,(J=0,1,2)}$
charmonium	n η _c	J/ψ	$h_c \chi_{cJ}$
bottomoniu	m η _b	Y	$h_b \chi_{bJ}$
c + anti-b	B _c	B_{c}^{*}	

Color singlet vs. Color octet



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Exclusive production

Relativistic corrections to S-wave quarkonium

 parameterized by ratios of matrix elements of higher orders in v to the leading-order one.

$$\langle q^{2n} \rangle_{J/\psi} = rac{\langle J/\psi(\lambda) | \psi^{\dagger}(-rac{i}{2}\overleftrightarrow{D})^{2n} \sigma \cdot \epsilon(\lambda)\chi | 0 \rangle}{\langle J/\psi(\lambda) | \psi^{\dagger} \sigma \cdot \epsilon(\lambda)\chi | 0
angle}$$

• Gremm-Kapustin relation

Gremm,Kapustin,PLB407,323(1997)

$$\langle \boldsymbol{q}^2 \rangle \approx \epsilon_{\mathsf{B}} m_c = (M_H - 2m_c) m_c$$

 ϵ_B :binding energy

large uncertainty in the charm quark mass

$$m_c = 1.4 \pm 0.2 \; \mathrm{GeV}$$

$$-0.35 < \langle q^2 \rangle < 0.84.$$

- even the sign is unknown.

Relativistic corrections

- ϵ_B can be determined in the potential model.
- generalized Gremm-Kapustin relation

 $\langle q^{2n}
angle pprox \left(m_c \epsilon_{
m B}
ight)^n pprox \langle q^2
angle^n.$ Bodwin,Kang,Lee,PRD74,014014(2006)

• the short-distance coefficients can be resummed.

$$A = \sum_{n} \underbrace{\frac{1}{n!}}_{n} \frac{\partial}{\partial q^2} \overset{n}{\longrightarrow} H(q^2) \underbrace{_{q^2=0}}_{q^2=0} \langle q^{2n} \rangle \langle \mathcal{O}_1 \rangle^{1/2}$$
short-distance coeff.

$$A = H(q^2)|_{q^2 = \langle q^2
angle} \langle \mathcal{O}_1
angle^{1/2}$$

• S-wave color-singlet NRQCD matrix elements are determined by the electromagnetic decay width and the potential model method. Bodwin,Chung,Kang,Lee,Yu,PRD77,094017(2008)

$$\langle \mathcal{O}_1 \rangle_{J/\psi} = 0.440^{+0.067}_{-0.055} \,\text{GeV}^3, \quad \langle q^2 \rangle_{J/\psi} = 0.441^{+0.140}_{-0.140} \,\text{GeV}^2$$

$$\implies \langle v^2 \rangle_{J/\psi} = 0.225^{+0.106}_{-0.088} \qquad 10$$

 $e^+e^- \rightarrow J/\psi + \eta_c$

$J/\psi car c$ (fb)	η_c	χ_{c0}	$\eta_c(2S)$
Belle [PRD70, 071102]	$25.6 \pm 2.8 \pm 3.4$	$6.4\pm1.7\pm1.0$	$16.5 \pm 3.0 \pm 2.4$
BABAR [PRD72,031101]	$17.6 \pm 2.8^{+1.5}_{-2.1}$	$10.3 \pm 2.5^{+1.4}_{-1.8}$	$16.4 \pm 3.7^{+2.4}_{-3.0}$
Braaten, Lee[PRD67, 054007]	3.78 ± 1.26	2.40 ± 1.02	1.57 ± 0.52
Liu, He, Chao [PLB557, 45]	5.5	6.7	3.7

• NRQCD at LO in α_s and v predicts a smaller value by an order of magnitude.

• The long-standing puzzle was resolved by the large QCD corrections and relativistic corrections.

 $e^+e^- \to J/\psi + \eta_c$

• QCD diagram

• QED fragmentation diagram





• QED contributions increase the cross section by about 19%.

 $e^+e^- \rightarrow J/\psi + \eta_c$

• K factor from QCD corrections is 1.96.

Zhang,Gao,Chao,PRL96,092001(2006); Gong,Wang,PRD77,054028(2008)

- Relativistic corrections can come from
 - direct corrections to the short-distance process.
 - indirect corrections through the NRQCD ME.



Bodwin,Lee,Yu,PRD77,094018(2008)

- A class of relativistic corrections are resummed.
 - resummation effects are about 12%.
- A few refinements:
 - VMD is used to calculate the photon fragmentation diagrams.
 - Running of EM coupling

- interference between the relativistic corrections and NLO corrections in $\alpha_{s}.$



Bodwin,Lee,Yu,PRD77,094018(2008)

• It seems fair to say that the discrepancy between theory and experiments has been resolved.

 $e^+e^- \rightarrow J/\psi + J/\psi$

• Two-photon exchange due to C parity conservation.



• may be comparable to $\sigma_{e^+e^- \rightarrow J/\psi + \eta_c}$.

 $e^+e^- \rightarrow J/\psi + J/\psi$

• originally suggested to resolve the $e^+e^- \rightarrow J/\psi + \eta_c$ puzzle. Bodwin,Lee,Braaten,PRL90,162001(2003)

$H_1 + H_2$	σ (fb)
$J/\psi + J/\psi$	6.65 ± 3.02
$J/\psi + \psi(2S)$	5.52 ± 2.50
$\psi(2S) + \psi(2S)$	1.15 ± 0.52

• No evidence at Belle.



Belle, PRD70, 071102(2004)

$$\sigma[e^+e^- \to J/\psi + J/\psi] \times \mathcal{B}_{>2}[J/\psi] < 9.1 \text{ fb},$$

$$\sigma[e^+e^- \to J/\psi + \psi(2S)] \times \mathcal{B}_{>2}[\psi(2S)] < 5.2 \text{ fb}.$$

• disfavored by the angular distribution analysis of $J/\psi + \eta_c$ events at Belle.

$\underline{e^+e^- \to J/\psi + J/\psi}$

• use a VMD (vector-meson-dominance) model for the photon fragmentation diagram.

 $\langle J/\psi(\lambda)|J^{\mu}(x=0)|0\rangle = g_{J/\psi\gamma}\epsilon^{\mu}(\lambda)^{*},$ $\Gamma[J/\psi \to e^{+}e^{-}] = \frac{4\pi\alpha^{2}g_{J/\psi\gamma}^{2}}{3m_{J/\psi}^{3}}.$

• The predictions are below the upper bounds at Belle. Bodwin,Braaten,Lee,Yu,PRD74,074014(2006)

cross section	$J/\psi + J/\psi$	$J/\psi + \psi(2S)$	$\psi(2S)+\psi(2S)$
fragmentation	2.52 ± 0.13	1.81 ± 0.06	0.32 ± 0.02
interference	-0.98 ± 0.48	-1.09 ± 0.60	-0.30 ± 0.19
non fragmentation	0.15 ± 0.16	0.23 ± 0.29	0.09 ± 0.14
total	1.69 ± 0.35	0.95 ± 0.36	0.11 ± 0.09

 $e^+e^- \rightarrow J/\psi + J/\psi$

• Recently, the NLO corrections to $e^+e^- \rightarrow J/\psi + J/\psi$ were calculated. Gong,WangPRL100,181803(2008)

$m_c(\text{GeV})$	μ	$\alpha_s(\mu)$	$\sigma_{LO}(\mathrm{fb})$	σ_{NLO} (fb)	σ_{NLO}/σ_{LO}
1.5	m_c	0.369	7.409	-2.327	-0.314
1.5	$2m_c$	0.259	7.409	0.570	0.077
1.5	$\sqrt{s}/2$	0.211	7.409	1.836	0.248
1.4	m_c	0.386	9.137	-3.350	-0.367
1.4	$2m_c$	0.267	9.137	0.517	0.057
1.4	$\sqrt{s}/2$	0.211	9.137	2.312	0.253

- inconsistent with the previous calculation?
- need to include relativistic corrections. (in progress)

$$e^+e^- \to \eta_c + \gamma$$

• C=+1 quarkonium : $\eta_c, \eta_c(2S), \chi_{cJ}, \eta_b, \chi_{bJ}$



- test ground for production of the color-singlet heavy quarkonium.
- test for the convergence of velocity expansion for the $\eta_c(2S)$ decay into two photons.
- QCD corrections and relativistic corrections are not available yet.

$\frac{e^+e^- \to \eta_c + \gamma}{1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-$							
Н	σ (fb)	$\sigma_{\rm cut}~({\rm fb})$	E_{γ} (GeV)				
$egin{array}{l} \eta_c \ \eta_c(2S) \ \eta_b \end{array}$	$82.0^{+21.4}_{-19.8}$ $49.2^{+9.4}_{-7.4}$ $2.5^{+0.2}_{-0.2}$	$59.7^{+15.6}_{-14.4}$ $35.8^{+6.8}_{-5.4}$ $1.8^{+0.1}_{-0.1}$	4.87 4.66 1.12				
χ_{c0} χ_{c1} χ_{c2}	$1.3^{+0.2}_{-0.2} \\ 13.7^{+3.4}_{-3.1} \\ 5.3^{+1.6}_{-1.3}$	$1.0^{+0.1}_{-0.1} \\ 10.2^{+2.6}_{-2.3} \\ 4.0^{+1.3}_{-1.0}$	4.74 4.71 4.69				
χ _{b0} χ _{b1} χ _{b2}	$\begin{array}{c} 0.6^{+0.4}_{-0.2} \\ 2.8^{+1.1}_{-0.7} \\ 3.0^{+1.4}_{-0.9} \end{array}$	$\begin{array}{r} 0.4\substack{+0.3\\-0.2}\\ 2.3\substack{+0.9\\-0.6}\\ 2.4\substack{+1.1\\-0.7}\end{array}$	0.95 0.66 0.57				

Chung,Lee,Yu,PR78,074022(2008)

Inclusive production

Color-octet contribution to J/w production



(c) & (d) are fitted to the data, but the resulting NP MEs are of the order predicted by the scaling rules of NRQCD. F.Maltoni, QWG'07

Polarization of J/ ψ at Tevatron Run I



- agree in intermediate p_T .
- disagree in the hightest p_T bin.
- error bars are large.

$$\alpha = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$$

Polarization of J/ ψ at Tevatron Run II



- Prompt polarization does not show the trend to the transverse polarization.
- CDF Run II data disagree with both CDF Run I and NRQCD.

NLO corrections to color-singlet contribution



- NLO corrections : virtual (129 diags.) + real.
- Fragmentation approximation disregarded. (direct calculation)
- Incomplete NNLO corrections.
- Improved in shape and size, but smaller than data.
 → still needs

F.Maltoni, QWG'07

ψ' production



• a small gap at large p $_{T}$.

• Polarization is not explained by NNLO corrections.

• feed-down, color octet? Lansberg, QWG'08

J/ψ production in ep collisions

J/ψ production in ep collisions

Color singlet



Color octet



NLO corrections



NLO corrections in the CS model

• the NLO corrections in CS model explain the data?





choose a more natural choice of the scale:





- the cross sections differential in z and P_T underestimate the data
- the longitudinal polarization at large P_T is not seen in the data

J/ψ production in e^+e^- collisions



• NLO corrections in α_s in CSM were completed.

$m_c(\text{GeV})$	$\alpha_s(\mu)$	$\sigma^{(0)}(\mathrm{pb})$	$a(\hat{s})$	$\sigma^{(1)}(\mathrm{pb})$	$\sigma^{(1)}/\sigma^{(0)}$
1.4	0.267	0.341	2.35	0.409	1.20
1.5	0.259	0.308	2.57	0.373	1.21
1.6	0.252	0.279	2.89	0.344	1.23

	$\mu = 2.8 \text{GeV}$	$\mu = 2.8 \text{GeV}$	$\mu = 5.3 {\rm GeV}$	$\mu = 5.3 {\rm GeV}$
	LO	NLO	LO	NLO
$\sigma(gg)$	0.57	0.67	0.36	0.53
$\sigma(c\bar{c})$	0.38	0.71	0.24	0.53
$R_{c\bar{c}}$	0.40	0.51	0.40	0.50

Ma.,Zhang,Chao('08);Gong,Wang('09)

the color-octet contributions?

at NLO

		$par{p}$	e^-p	e^-e^+
σ not enough		not enough	may explain	
CS	α	disagree	inconsistent at high P_T	
<u> </u>	σ may be needed		may be needed	unnecessary?
CO	α	not completed	not yet	

- the CS contributions could not explain the data.
- room for the color-octet contributions?

Need to find the process in which the CO mechanism is indeed dominant.

double quarkonium production at the LHC

Motivations for double quarkonium production

- need further tests for the CS and CO mechanisms in NRQCD.
- predicted to test the color-octet mechanism at the Tevatron. Brager, Fleming, Phillips('96); Qiao('02)
- recently extended to the LHC.

Li,Zhang,Chao(0903.2250);Qiao,Sun,Sun(0903.0954)

• clean signals.

4 μ 's events for $J/\psi J/\psi$, $J/\psi \Upsilon$, $\Upsilon \Upsilon$ production.

• $gg \rightarrow J/\psi \chi_c$ and $gg \rightarrow J/\psi \eta_c$ are forbidden in the CS model, but allowed in the CO model.

Color singlet



- $gg \to Q_1 Q_2$ dominates over $q\bar{q} \to Q_1 Q_2$.
- 31 Feynman diagrams for $J/\psi J/\psi$ and $B_c(B_c^*)\bar{B}_c(\bar{B}_c^*)$ production.
- 39 Feynman diagrams for $\eta_c \eta_c$ production.

Color octet



• gluon fragmentation to an color-octet $c\bar{c}$ pair dominates.

$$d\hat{\sigma}_{\mathcal{Q}_1+\mathcal{Q}_2} = \int_0^1 dz_1 \int_0^1 dz_2 D_{g\to\mathcal{Q}_1}(z_1, m_{Q_1}) D_{g\to\mathcal{Q}_2}(z_2, m_{Q_2}) d\hat{\sigma}_{gg}(E_1/z_1, E_2/z_2),$$

• gluon fragmentation function is

$$D_{g \to \mathcal{Q}}(z, \mu^2) = \sum d_{g \to n}(z, \mu^2) \langle \mathcal{O}_n^H \rangle.$$
$$d_{g \to \underline{8}^3 S_1} = \frac{\pi \alpha_s(2m_Q)}{24m_Q^3} \delta(1-z).$$

Cross section



Qiao,Sun,Sun(0903.0954)

a : color-octet b : unpolarized (CS) c,d,e : polarizated (CS)

$\sigma(\text{events})$	$p_{Tcut}=3 \text{ GeV}$	$p_{Tcut}=4~{ m GeV}$	$p_{Tcut}{=}5~{\rm GeV}$	$p_{Tcut}{=}6~{\rm GeV}$	$p_{Tcut}{=}7~{\rm GeV}$
$\perp \perp$	5.83 pb(58324)	1.74 pb(17425)	0.56 pb(5607)	$0.20 \mathrm{pb}(1981)$	$0.077 \mathrm{pb}(767)$
	2.55 pb(25543)	$0.83 \mathrm{pb}(8262)$	$0.28 \mathrm{pb}(2786)$	$0.10 \mathrm{pb}(1014)$	$0.040 \mathrm{pb}(401)$
⊥	3.95 pb(39425)	$0.94 \mathrm{pb}(9445)$	$0.24 \mathrm{pb}(2380)$	0.066 pb(660)	$0.020 \mathrm{pb}(204)$
tot	12.33pb(123319)	3.51 pb(35131)	1.08 pb(10773)	$0.37 \mathrm{pb}(3656)$	$0.14 \mathrm{pb}(1372)$
$\perp_8\perp_8$	2.90pb(29022)	1.82 pb(18205)	1.15 pb(11461)	0.74pb(7399)	$0.49 \mathrm{pb}(4925)$

$J/\psi + \Upsilon$ production at the LHC

gluon fragmentation



• gluon fragmentation into a heavy quarkonium may be dominant.

$$\frac{d\hat{\sigma}}{dP_T^2} \sim \frac{\alpha_s^4 v_{J/\psi}^4 v_{\Upsilon}^4}{P_T^4}$$

color singlet

• the color-singlet yield appears at α_s^6 order.



• QED contributions are suppressed.



- further suppression factor from PDF.

cross section

• in the gluon fragmentation approximation

- this mode could be observed soon at the LHC.
- if we could not observe, then the color-octet mechanism should be suppressed.

color octet

- the gluon fragmentation approximation is a bad approximation.
- the full calculation is being carried out. (36 diagrams)



• are these diagrams really suppressed?

Summary

• Heavy quarkonium production provides a test for the color-singlet and color-octet mechanism.

- Exclusive production of heavy quarkonium : relativistic corrections
- Inclusive production of heavy quarkonium : color-octet mechanism