Resummation of Relativistic Corrections to $J/\psi \to e^+e^-$

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Outline

- Quarkonium electromagnetic current
- A new method to compute
  1-loop NRQCD corrections to all orders in v
- Resummation of relativistic corrections
- Summary

Quarkonium Electromagnetic Current

- Full QCD definition:

$$(-iee_Q)i\mathcal{A}_H^\mu = \langle 0|J_{\rm EM}^\mu|H\rangle, \;\; J_{\rm EM}^\mu = (-iee_Q)\bar{\psi}\gamma^\mu\psi$$

- NRQCD factorization formula

$$i\mathcal{A}_H^i = \sqrt{2m_H}\sum_n c_n \langle 0|\mathcal{O}_n^i|H\rangle$$

$\mathcal{O}_n^i$: NRQCD operators involving $Q\bar{Q}$ decay in terms of two-component Pauli spinor fields

c_n : Short-distance coefficients that are free of IR divergences and calculable perturbatively

- Short-distance coefficients can be computed by perturbative matching:

$$i\mathcal{A}^i_{Q\bar{Q}_1} = \sum_n c_n \langle 0|\mathcal{O}^i_n|Q\bar{Q}_1\rangle$$

$\langle 0|\mathcal{O}^i_n|Q\bar{Q}_1\rangle$: perturbative NRQCD matrix element

- We only consider $Q\bar{Q}$ operators.
- Matching coefficients at order α_s are determined as

$$\text{order } \alpha_s^0 : \; i\mathcal{A}^{i(0)}_{Q\bar{Q}_1} = \sum_n c_n^{(0)} \langle 0|\mathcal{O}^i_n|Q\bar{Q}_1\rangle^{(0)}$$

$$\text{order } \alpha_s^1 : \; i\mathcal{A}^{i(1)}_{Q\bar{Q}_1} = \sum_n c_n^{(1)} \langle 0|\mathcal{O}^i_n|Q\bar{Q}_1\rangle^{(0)} + \sum_n c_n^{(0)} \langle 0|\mathcal{O}^i_n|Q\bar{Q}_1\rangle^{(1)}$$

finite

$\left[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}\right]_{\mathrm{NRQCD}}$

IR sensitive

- General structure of full-QCD amplitude for a color-singlet $Q\bar{Q}$ pair:

$$
\begin{aligned}
i\mathcal{A}^i_{Q\bar{Q}_1} &= \bar{v}(p_2)(G\gamma^i + Hq^i)u(p_1) \\
&= G\eta^\dagger\sigma^i\xi - \left[\frac{G}{E(E+m)} + \frac{H}{E}\right] q^i\eta^\dagger \boldsymbol{q}\cdot\boldsymbol{\sigma}\xi.
\end{aligned}
$$

$\mathcal{O}^i_A = \chi^\dagger\sigma^i\psi$ (S wave)

$\mathcal{O}^i_B = \chi^\dagger(-\frac{i}{2}\nabla^i)(-\frac{i}{2}\boldsymbol{\nabla}\cdot\boldsymbol{\sigma})\psi$ (S & D wave)

- IR divergence emerges from order α_s

IR divergence

$$
\begin{aligned}
G &= 1 + \frac{\alpha_s C_F}{4\pi}\left\{2\left[(1+\delta^2)L(\delta) - 1\right]\frac{1}{\epsilon_{\rm IR}} + (1+\delta^2)\left[\frac{\pi^2}{\delta} - \frac{i\pi}{\delta}\frac{1}{\epsilon_{\rm IR}}\right] + \cdots\right\}, \\
H &= \frac{\alpha_s C_F}{4\pi}\frac{1-\delta^2}{m}\left[-\frac{i\pi}{\delta} + \cdots\right].
\end{aligned}
$$

Coulomb divergence

$$L(\delta) = \frac{1}{2\delta}\log\left(\frac{1+\delta}{1-\delta}\right), \qquad \delta = \frac{v}{\sqrt{1+v^2}}$$

Long-distance contribution

$\left[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}\right]_{\rm NRQCD}$ **must be subtracted.**

- $c_n^{(0)}$ is known to all orders in v^{2n}
  Bodwin, Chung, Kang, Lee, and Yu (PRD77, 094017)
- Determination of $c_n^{(1)}$

$$i\mathcal{A}^{i(1)}_{Q\bar{Q}_1} - \left[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}\right]_{\mathrm{NRQCD}} = \sum_n c_n^{(1)} \langle 0|\mathcal{O}_n^i|Q\bar{Q}_1\rangle^{(0)}$$

$$i\mathcal{A}^{i(1)}_{Q\bar{Q}_1} - \left[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}\right]_{\mathrm{NRQCD}} = \Delta G^{(1)}\eta^\dagger\sigma^i\xi - \left[\frac{\Delta G^{(1)}}{E(E+m)} + \frac{\Delta H^{(1)}}{E}\right] q^i\eta^\dagger \boldsymbol{q}\cdot\boldsymbol{\sigma}\xi$$

- In this work, we compute $c_n^{(1)}$ to all orders in v

References for $c_n^{(i)}$

	v^0	v^2	v^4	v^{all}
α_s^0	[1]	[1]	[2]	[X]
α_s^1	[3], [4]	[5]	[Y]	[Y]
α_s^2	[6], [7]			

[1] Bodwin, Braaten, Lepage, PRD 51, 1125
[2] Bodwin, Petrelli, PRD 66, 094011
[3] Barbieri et al., PL 57B, 455
[4] Celmaster, PRD 19, 1517
[5] Luke, Savage, PRD 57, 413
[6] Czarnecki, Melnikov, PRL 80, 2531
[7] Beneke, Signer, Smirnov, PRL 80, 2535

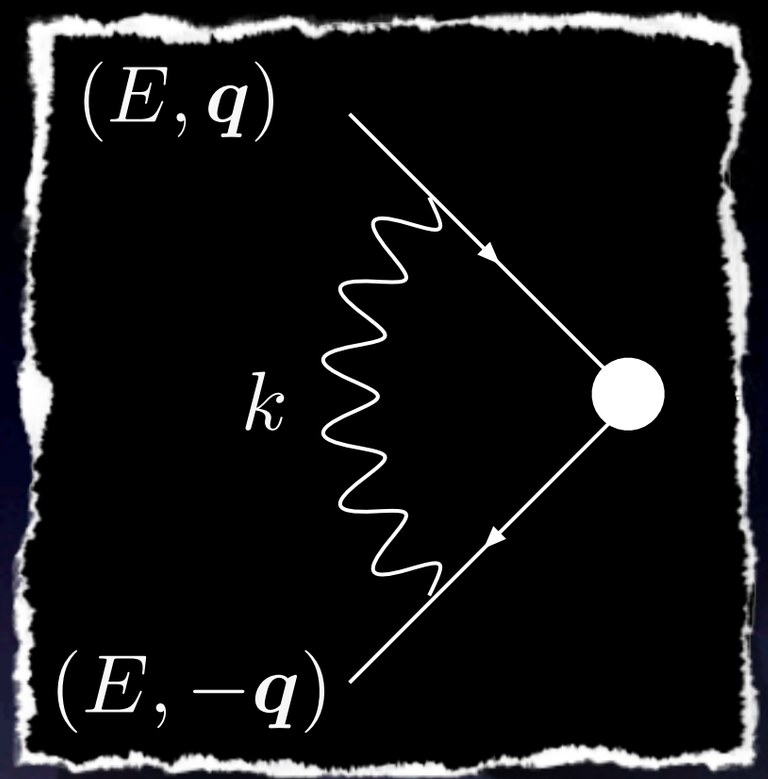
[X] Bodwin, Chung, Kang, Lee, Yu, PRD 77, 094017
[Y] This work

Computation of $\left[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}\right]_{\mathrm{NRQCD}}$

- Calculation of $\left[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}\right]_{\mathrm{NRQCD}}$ to higher orders in v requires knowledge of a huge number of
  - NRQCD operators,
  - the interactions,
  - and the Feynman rules to compute loop corrections.
- This becomes a daunting task at all orders in v.

1-loop NRQCD contribution

BBL Method to compute $\left[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}\right]_{\mathrm{NRQCD}}$



$\int_k$

i) Use NRQCD perturbation theory to construct amplitude.

ii) Integrate out k_0.

iii) Expand the integrand in powers of 3-momenta divided by m.

iv) Discard power-divergent scaleless integrals.

We propose a new method that replaces step i).

1-loop NRQCD contribution

A new method of calculation

- Interactions in NRQCD through infinite order in v is equivalent to QCD, but with the interactions re-arranged in an expansion in powers of v.
- Instead of using NRQCD perturbation theory, we expand the full-QCD integrand in powers of 3-momenta divided by m after evaluating k_0 integral.

Comparison with Method of Regions

- At lowest order in v, the method of regions [Beneke and Smirnov (NPB 1998)] has been used to compute order-α_s^2 correction to quarkonium EM current

Beneke, Signer, Smirnov (PRL 1998)

- In MoR, contribution from each region can be computed separately by expanding small parameters

= 1-loop NRQCD contribution

Comparison with Method of Regions

- With MoR, it is difficult to obtain a closed-form formula for the hard part valid to all orders in v.
- In our method, the NRQCD contribution becomes a simple series that can be resummed easily, yielding very compact expressions for the hard part.
- In dimensional regularization,
  our method is equivalent to calculating
  "soft + ultrasoft + potential" part by MoR.
- Our method also works for hard cut-off like lattice, but MoR does not.

Our final results

- IR pole cancels in $i\mathcal{A}^{i(1)}_{Q\bar{Q}_1} - \left[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}\right]_{\mathrm{NRQCD}}$
- UV pole is absorbed by redefining NRQCD operators in $\overline{\mathrm{MS}}$ scheme:

$$\Delta G^{(1)}_{\overline{\mathrm{MS}}} = \frac{\alpha_s C_F}{4\pi}\left\{2\left[(1+\delta^2)L(\delta)-1\right]\log\frac{\mu^2}{m^2} + 6\delta^2 L(\delta) - 4(1+\delta^2)K(\delta) - 4\right\}$$

$$\Delta H^{(1)} = \frac{\alpha_s C_F}{4\pi}\frac{2(1-\delta^2)}{m}L(\delta)$$

$$L(\delta) = \frac{1}{2\delta}\log\left(\frac{1+\delta}{1-\delta}\right) = 1 + \frac{\delta^2}{3} + O(\delta^4)$$

$$K(\delta) = \frac{1}{4\delta}\left[\mathrm{Sp}\left(\frac{2\delta}{1+\delta}\right) - \mathrm{Sp}\left(-\frac{2\delta}{1-\delta}\right)\right] = 1 + \frac{4\delta^2}{9} + O(\delta^4)$$

Exact cancellation of IR and non-analytic contributions to all orders in v

- QCD 1-loop corrections

IR divergence

$$G^{(1)} = \frac{\alpha_s C_F}{4\pi}\left\{2\left[(1+\delta^2)L(\delta)-1\right]\frac{1}{\epsilon_{\rm IR}} + (1+\delta^2)\left[\frac{\pi^2}{\delta} - \frac{i\pi}{\delta}\frac{1}{\epsilon_{\rm IR}}\right] + \cdots\right\},$$

$$H^{(1)} = \frac{\alpha_s C_F}{4\pi}\frac{1-\delta^2}{m}\left[-\frac{i\pi}{\delta} + \cdots\right].$$

Coulomb divergence

$$L(\delta) = \frac{1}{2\delta}\log\left(\frac{1+\delta}{1-\delta}\right)$$

- NRQCD 1-loop corrections

IR divergence

$$G^{(1)}_{\rm NRQCD} = \frac{\alpha_s C_F}{4\pi}\left\{2\left[(1+\delta^2)L(\delta)-1\right]\left(\frac{1}{\epsilon_{\rm IR}} - \frac{1}{\epsilon_{\rm UV}}\right) + (1+\delta^2)\left[\frac{\pi^2}{\delta} - \frac{i\pi}{\delta}\frac{1}{\epsilon_{\rm IR}}\right] + \cdots\right\},$$

$$H^{(1)}_{\rm NRQCD} = \frac{\alpha_s C_F}{4\pi}\frac{1-\delta^2}{m}\left(-\frac{i\pi}{\delta}\right).$$

Coulomb divergence

Resummation to all orders in v

- Generalized Gremm-Kapustin relation for spin-independent potential models

$$\left[\langle \boldsymbol{q}^{2n}\rangle_{H(^3S_1)}\right]_{\overline{\rm MS}} = \left[\langle \boldsymbol{q}^{2}\rangle_{H(^3S_1)}\right]^{n}_{\overline{\rm MS}}.$$

Bodwin, Kang, and Lee (PRD 2006)

$$\langle \boldsymbol{q}^{2n}\rangle_{H(^3S_1)} = \frac{\langle 0|\mathcal{O}^i_{An}|H(^3S_1)\rangle}{\langle 0|\mathcal{O}^i_{A0}|H(^3S_1)\rangle}$$

- This allows us to resum the relativistic corrections

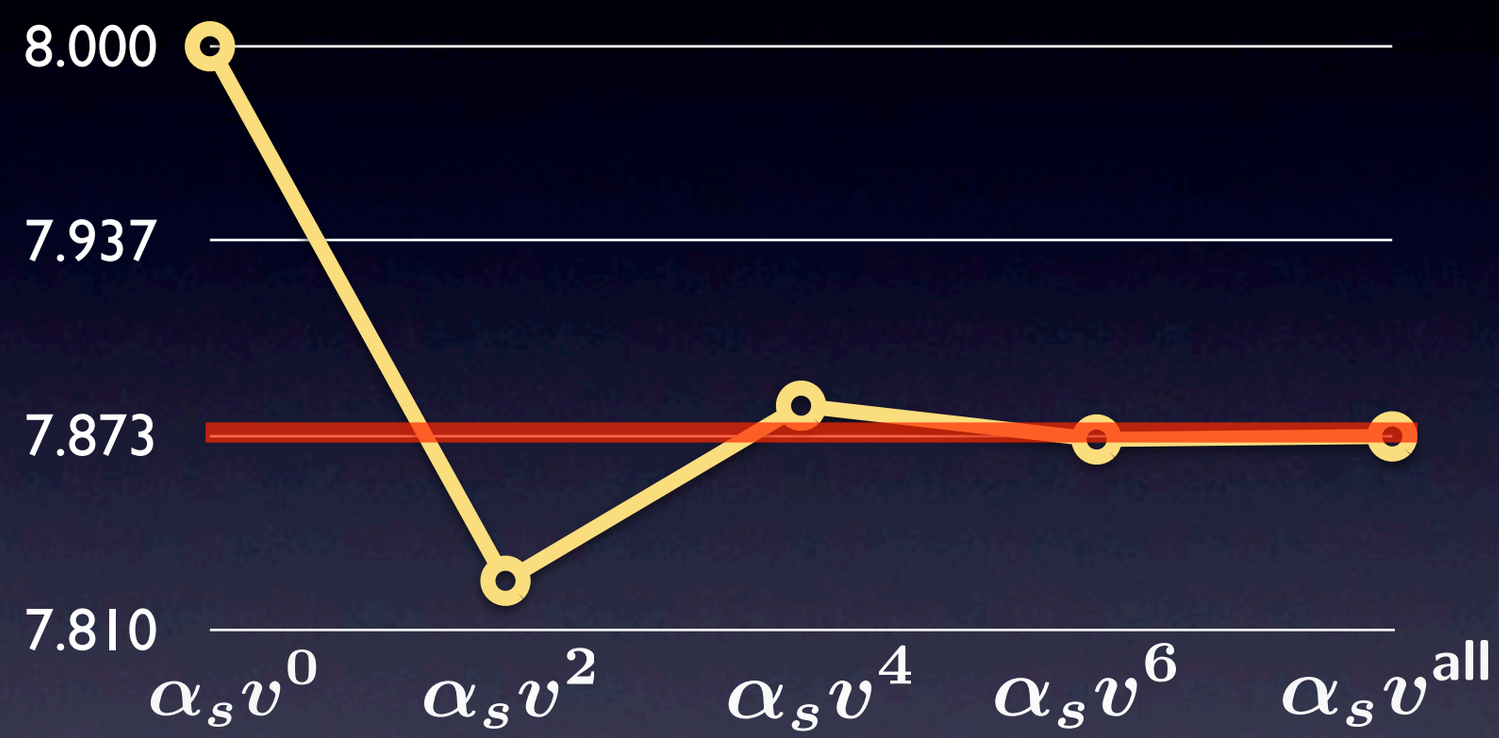
$$i\mathcal{A}^i_{H(^3S_1)} = \sqrt{2m_H}\; C(\delta)\; \langle 0|\chi^\dagger\sigma^i\psi|H\rangle$$

$$C(\delta) = \left[1 - \frac{\langle \boldsymbol{q}^2\rangle}{3E(E+m)}\right]\left(1+\Delta G^{(1)}_{\overline{\rm MS}}\right) - \frac{\langle \boldsymbol{q}^2\rangle}{3E}\Delta H^{(1)}$$

$$\delta^2 \Longrightarrow \frac{\langle \boldsymbol{q}^2\rangle}{m^2+\langle \boldsymbol{q}^2\rangle}$$

resummation

Order-α_s coefficient of EM current, resummed to v^{2n}



MEs are from Bodwin, Chung, Kang, Lee, and Yu (PRD77, 094017)

- From the closed-form results, we can show that there is a finite radius of convergence $v < 1$.

Summary

- Order-α_s corrections to the quarkonium EM current at all orders in v have been computed.
- Only $Q\bar{Q}$ contribution is considered. Neglect of gauge-field contribution results in errors of order v^4.
- A new method is introduced to compute NRQCD 1-loop corrections directly from QCD amplitude.
- This allows us to obtain very compact expressions for the $Q\bar{Q}$ contributions valid to all orders in v.
- We can use this method to resum a class of relativistic corrections.
- The method may be applied to compute short-distance coefficients for other effective theories.

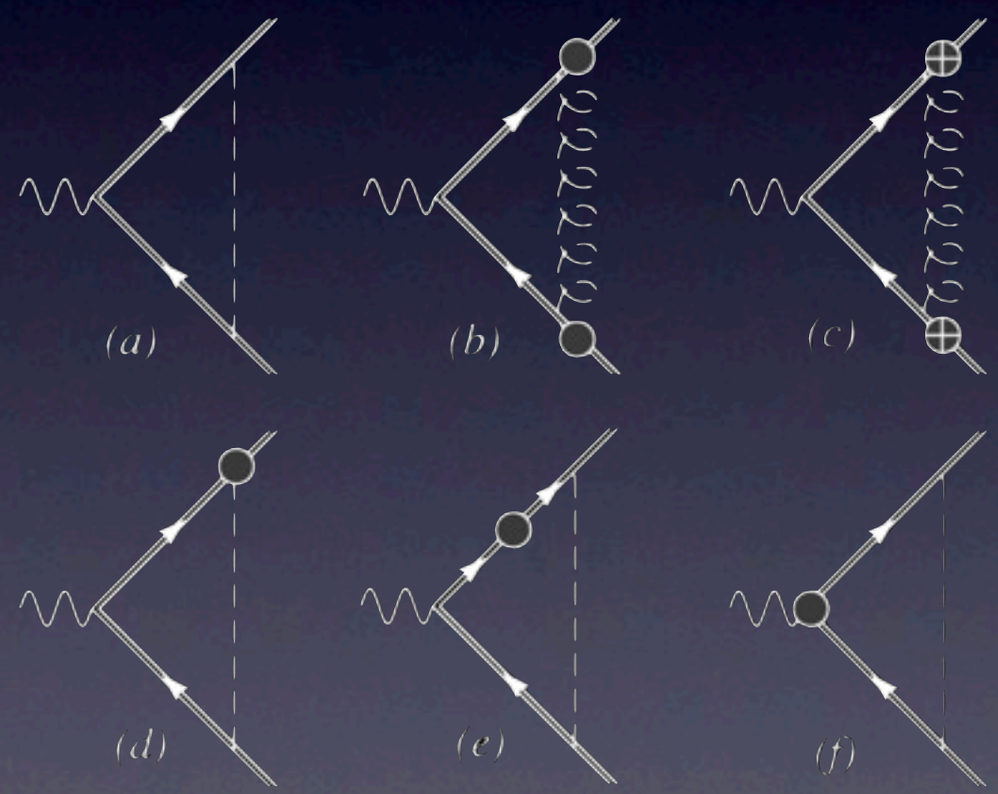
Supplementary

A glance at a traditional fixed order calculation...

Luke and Savage (PRD 1998), order $\alpha_s v^2$

$\alpha_s v^0$

$\alpha_s v^2$



Corrections to $\langle 0|\chi^\dagger \boldsymbol{\sigma}^i \psi|Q\bar{Q}\rangle$

(a) A_0 temporal

(b) $\boldsymbol{\nabla}\cdot\boldsymbol{A}$ spatial

(c) $\boldsymbol{\sigma}\cdot\boldsymbol{B}$ spatial

(d) $\boldsymbol{D}\cdot\boldsymbol{E}$ temporal

(e) $\boldsymbol{D}^4$ quark propagator correction

Corrections to $\langle 0|\chi^\dagger \left(\overrightarrow{\boldsymbol{\nabla}}\cdot\boldsymbol{\sigma}\overrightarrow{\boldsymbol{\nabla}}^i + \overleftarrow{\boldsymbol{\nabla}}\cdot\boldsymbol{\sigma}\overleftarrow{\boldsymbol{\nabla}}^i\right)\psi|Q\bar{Q}\rangle$

(f) A_0 temporal

- It is non-trivial to extend to all orders in v

A glance at a traditional fixed order calculation...

Luke and Savage (PRD 1998), order $\alpha_s v^2$

$$(a)=c_1\frac{g^2}{12\pi^2}u_h^\dagger\boldsymbol{\sigma}^j v_h\left\{\frac{m}{|\mathbf{p}|}\left[\pi^2+i\pi\left(\gamma_E+\frac{2}{d-4}+\ln\frac{\mathbf{p}^2}{\pi\mu^2}\right)\right]-\frac{i\pi}{4}\frac{|\mathbf{p}|}{m}\right\}. \tag{4.22}$$

This reproduces the $O(1/v)$ term in the full amplitude.

As discussed above, there are no graphs at $O(g^2v^{2n})$ in NRQCD from radiation gluon loops. At $O(g^2v)$ there are contributions from the leading relativistic corrections to Coulomb scattering. In addition, since Coulomb exchange scales as v^{-1}, the dressing of $\mathbf{O}_2$ with a single A_P^0 exchange also contributes at $O(g^2v)$. A_P^i exchange contributes both via the $\mathbf{p}\cdot\mathbf{A}$ coupling [Fig. 5(b)]

$$(b)=c_1\frac{g^2}{12\pi^2}u_h^\dagger\boldsymbol{\sigma}^j v_h\frac{|\mathbf{p}|}{m}\left[\pi^2+i\pi\left(\gamma_E-1+\frac{2}{d-4}+\ln\frac{\mathbf{p}^2}{\pi\mu^2}\right)\right] \tag{4.23}$$

and the Fermi coupling

$$(c)=c_1\frac{g^2}{12\pi^2}\left(u_h^\dagger\boldsymbol{\sigma}^j v_h\frac{|\mathbf{p}|}{m}+\frac{m}{|\mathbf{p}|}\frac{u_h^\dagger\mathbf{p}\cdot\boldsymbol{\sigma}\mathbf{p}^i v_h}{m^2}\right)\left(-\frac{i\pi}{2}\right). \tag{4.24}$$

Coulomb exchange is corrected by the Darwin vertex,

$$(d)=c_1\frac{g^2}{12\pi^2}u_h^\dagger\boldsymbol{\sigma}^j v_h\frac{|\mathbf{p}|}{m}(-i\pi), \tag{4.25}$$

while the spin-orbit coupling does not contribute. The relativistic corrections to the quark and antiquark propagators give

$$(e)=c_1\frac{g^2}{12\pi^2}u_h^\dagger\boldsymbol{\sigma}^j v_h\frac{|\mathbf{p}|}{2m}\left[\pi^2+i\pi\left(\gamma_E+\frac{1}{2}+\frac{2}{d-4}+\ln\frac{\mathbf{p}^2}{\pi\mu^2}\right)\right], \tag{4.26}$$

and finally, the one-loop correction to $\mathbf{O}_2$ in Fig. 5(f) gives

$$(f)=-c_2\frac{g^2}{12\pi^2}\frac{u_h^\dagger\mathbf{p}\cdot\boldsymbol{\sigma}\mathbf{p}^i v_h}{m^2}\frac{m}{2|\mathbf{p}|}\left[\pi^2+i\pi\left(\gamma_E-3+\frac{2}{d-4}+\ln\frac{\mathbf{p}^2}{\pi\mu^2}\right)\right]-c_2\frac{g^2}{12\pi^2}u_h^\dagger\boldsymbol{\sigma}^j v_h\frac{|\mathbf{p}|}{m}\left(\frac{i\pi}{2}\right). \tag{4.27}$$

Our results agree with Luke & Savage at order $\alpha_s v^2$