Non-leptonic two-body B decays

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August 24, 2009

< Tevatron B physics workshop at KISTI >

Outline

Effective Hamiltonian

- The low-energy effects of the full theory can be described by the effective Hamiltonian approach in order to include QCD effects systematically.
- The low-energy effective Hamiltonian calculated within the framework of the operator product expansion (OPE) has a finite number of operators in a given order, which is dependent upon the structure of the model.
- Effective Hamiltonian for $\Delta B = 1$ and $\Delta S = 1$ transition with right-handed currents:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{\substack{i=1,2,11,12 \\ q=u,c}} \lambda_q^{LL} C_i^q O_i^q - \lambda_t^{LL} \left(\sum_{i=3}^{10} C_i O_i + C_7^\gamma O_7^\gamma + C_8^G O_8^G \right) \right] + (C_i O_i \to C_i' O_i')$$

Effective Hamiltonian

Operators

- Current-Current

$$O_1^u = (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}} (\bar{u}_\beta b_\alpha)_{\mathrm{V-A}}, \qquad O_2^u = (\bar{s}_\alpha u_\alpha)_{\mathrm{V-A}} (\bar{u}_\beta b_\beta)_{\mathrm{V-A}}$$
$$O_1^c = (\bar{s}_\alpha c_\beta)_{\mathrm{V-A}} (\bar{c}_\beta b_\alpha)_{\mathrm{V-A}}, \qquad O_2^c = (\bar{s}_\alpha c_\alpha)_{\mathrm{V-A}} (\bar{c}_\beta b_\beta)_{\mathrm{V-A}}$$

- QCD-Penguins

$$O_3 = (\bar{s}_\alpha b_\alpha)_{\mathrm{V-A}} \sum_q (\bar{q}_\beta q_\beta)_{\mathrm{V-A}}, \qquad O_4 = (\bar{s}_\alpha b_\beta)_{\mathrm{V-A}} \sum_q (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$
$$O_5 = (\bar{s}_\alpha b_\alpha)_{\mathrm{V-A}} \sum_q (\bar{q}_\beta q_\beta)_{\mathrm{V+A}}, \qquad O_6 = (\bar{s}_\alpha b_\beta)_{\mathrm{V-A}} \sum_q (\bar{q}_\beta q_\alpha)_{\mathrm{V+A}}$$

- Electroweak-Penguins

$$O_7 = \frac{3}{2} (\bar{s}_\alpha b_\alpha)_{\mathrm{V-A}} \sum_q e_q (\bar{q}_\beta q_\beta)_{\mathrm{V+A}}, \qquad O_8 = \frac{3}{2} (\bar{s}_\alpha b_\beta)_{\mathrm{V-A}} \sum_q e_q (\bar{q}_\beta q_\alpha)_{\mathrm{V+A}}$$
$$O_9 = \frac{3}{2} (\bar{s}_\alpha b_\alpha)_{\mathrm{V-A}} \sum_q e_q (\bar{q}_\beta q_\beta)_{\mathrm{V-A}}, \qquad O_{10} = \frac{3}{2} (\bar{s}_\alpha b_\beta)_{\mathrm{V-A}} \sum_q e_q (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$

- Magnetic-Penguins

$$O_7^\gamma = \frac{e}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1+\gamma_5) b_\alpha F_{\mu\nu}, \qquad O_8^G = \frac{g}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1+\gamma_5) T^a_{\alpha\beta} b_\beta G^a_{\mu\nu},$$

Effective Hamiltonian

Operators

- If we have additional $SU(2)_R$ group in the model, operator basis is doubled by O_i' which are the chiral conjugates of O_i.
- Also new operators $O_{11,12}$ and $O_{11,12}'$ arise with mixed chiral structure of $O_{1,2}$ and $O_{1,2}'$

$$O_{11}^u = (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}} (\bar{u}_\beta b_\alpha)_{\mathrm{V+A}} , \qquad O_{12}^u = (\bar{s}_\alpha u_\alpha)_{\mathrm{V-A}} (\bar{u}_\beta b_\beta)_{\mathrm{V+A}} ,$$
$$O_{11}^c = (\bar{s}_\alpha c_\beta)_{\mathrm{V-A}} (\bar{c}_\beta b_\alpha)_{\mathrm{V+A}} , \qquad O_{12}^c = (\bar{s}_\alpha c_\alpha)_{\mathrm{V-A}} (\bar{c}_\beta b_\beta)_{\mathrm{V+A}} ,$$

Effective Hamiltonian

- Wilson Coefficients ($\mu = 1.5$ GeV):

$$
\begin{aligned}
C_1^q &= -0.443, & C_1^{q\prime} &= C_1^q \zeta_g \lambda_q^{RR}/\lambda_q^{LL} \\
C_2^q &= 1.224, & C_2^{q\prime} &= C_2^q \zeta_g \lambda_q^{RR}/\lambda_q^{LL} \\
C_3 = 0.023, \quad C_4 &= -0.045, & C_5 &= 0.012, \quad C_6 = -0.064 \\
C_7 = 0.008\alpha, \quad C_8 &= 0.064\alpha, & C_9 &= -1.403\alpha, \quad C_{10} = 0.482\alpha \\
C_7^\gamma &= -0.385 - 17.07 A^{tb}, & C_7^{\gamma\prime} &= -17.07 A^{ts*} \\
C_8^G &= -0.175 - 7.506 A^{tb}, & C_8^{G\prime} &= -7.506 A^{ts*} \\
C_{11}^{u\prime} &= 0.623 A^{us*}, & C_{12}^{u\prime} &= 0.881 A^{us*}.
\end{aligned}
$$

where

$$
\lambda_q^{AB} \equiv V_{qs}^{A*} V_{qb}^B, \qquad A^{tD} = \xi_g \frac{V_{tD}^R}{V_{tD}^L} e^{i\alpha_\circ} \ (D = b, s)
$$

Effective Hamiltonian

- It is convenient to express the one-loop matrix elements of $\mathcal{H}_{eff}$ in terms of the tree-level matrix elements of the effective operators:

$$< sq\bar{q}|\mathcal{H}_{eff}|B >= -\frac{G_F}{\sqrt{2}}\lambda_t^{LL}\sum_{i=1}^{10} C_i^{eff} < sq\bar{q}|O_i|B >^{tree} +(C_iO_i \to C_i'O_i'),$$

with the effective WCs

$$c_1^{eff(\prime)} = c_1^{(\prime)}, \quad c_2^{eff(\prime)} = c_2^{(\prime)}, \quad c_3^{eff(\prime)} = c_3^{(\prime)} - \frac{1}{N_c}c_g^{(\prime)}, \quad c_4^{eff(\prime)} = c_4^{(\prime)} + c_g^{(\prime)}$$

$$c_5^{eff(\prime)} = c_3^{(\prime)} - \frac{1}{N_c}c_g^{(\prime)}, \quad c_6^{eff(\prime)} = c_4^{(\prime)} + c_g^{(\prime)}, \quad c_7^{eff(\prime)} = c_7^{(\prime)} + c_\gamma^{(\prime)}, \quad c_8^{eff(\prime)} = c_8^{(\prime)} + c_\gamma^{(\prime)}$$

where

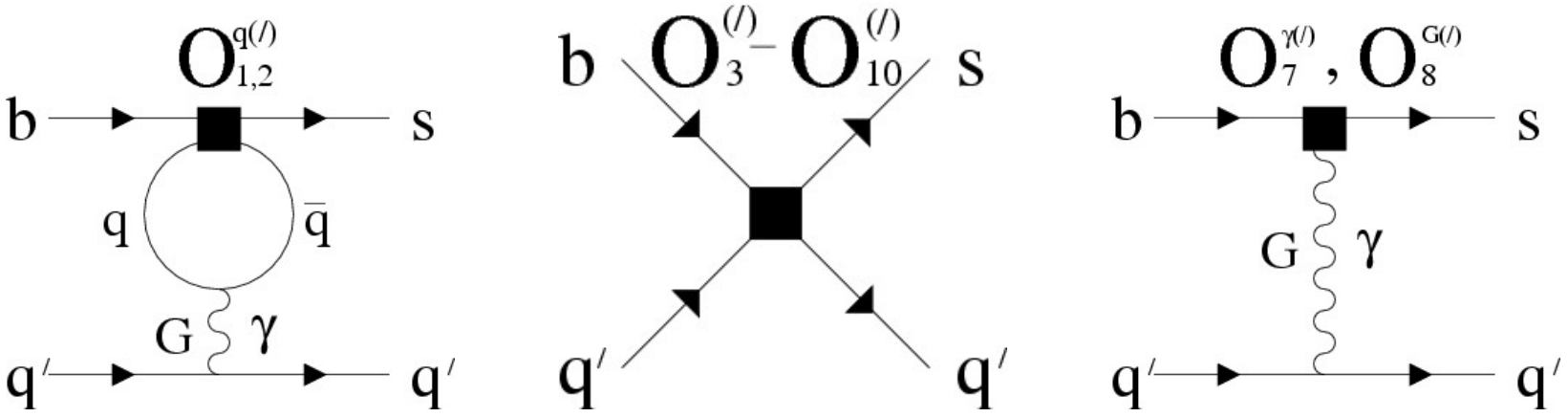
$$\begin{aligned} c_g^{(\prime)} &= -\frac{\alpha_s}{8\pi}\left[\frac{1}{\lambda_t^{LL}}\sum_{q=u,c}\lambda_q^{LL}c_2^{q(\prime)}\mathcal{I}(m_q,k,m_b) + 2c_8^{G(\prime)}\frac{m_b^2}{k^2}\right] \\ c_\gamma^{(\prime)} &= -\frac{\alpha_s}{3\pi}\left[\frac{1}{\lambda_t^{LL}}\sum_{q=u,c}\lambda_q^{LL}(c_1^{q(\prime)} + \frac{1}{N_c}c_2^{q(\prime)})\mathcal{I}(m_q,k,m_b) + c_7^{\gamma(\prime)}\frac{m_b^2}{k^2}\right] \end{aligned}$$

$$\underbrace{\mathcal{I}(m,k,\mu) = 4\int_0^1 dx x(1-x)\ln\left[\frac{m^2 - k^2x(1-x)}{\mu^2}\right]}$$

$\Longrightarrow$ Two different CP even phases arise!

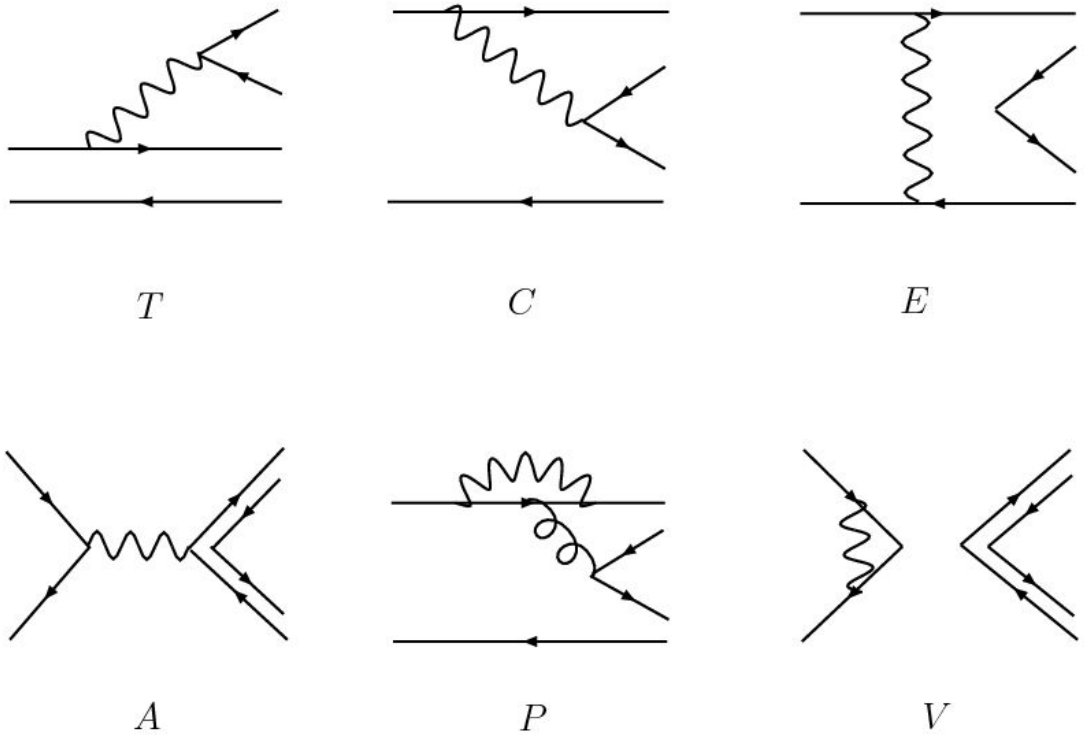
Effective Hamiltonian

- Diagrams for penguin-induced $b \to s\bar{s}s$ decays



Two body hadronic B decays

- Various topological diagrams* for $B \to M1M2$ decays



T: the color allowed external W emission tree diagram;
C: the color-suppressed internal W emission diagram;
E: the W-exchange diagram; A: the W-annihilation diagram;
P: the penguin diagram; V: the vertical W-loop diagram

*bollowed from arXiv:0901.4396

Two body hadronic B decays

Factorization approximation

- Factorization approximation for the matrix elements of the operators using the vacuum insertion method (Gaillard and Lee, Phys. Rev. D **10** 897 (1974)) :

$$
\begin{aligned}
< M_1 M_2 | \mathcal{H}_{eff} | B > \quad = \quad & Z_1 < M_1 | J_1^\mu | 0 >< M_2 | J'_{1\mu} | B > \\
& + Z_2 < M_2 | J_2^\mu | 0 >< M_1 | J'_{2\mu} | B > \\
& + Z_3 < M_1 M_2 | J_3^\mu | 0 >< 0 | J'_{3\mu} | B >
\end{aligned}
$$

- From Lorentz invariance one can obtain the decomposition of the hadronic matrix element in terms of decay constants and formfactors (Bauer and Wirbel, Z. Phys. C. **42** 671 (1989)) $\mapsto$ The relevant decay constants and form factors can be obtained from various leptonic and semi-leptonic decay experiments, respectively.

Two body hadronic B decays

Factorization approximation

- Consider the matrix element of the operator O_6 for the process $B^- \to \phi K^{*-}$:

$$
\begin{aligned}
< \phi K^{*-} | O_6 | B^- > &= \frac{1}{N_c} < \phi | \bar{s}\gamma^\mu s | 0 >< K^{*-} | \bar{s}\gamma_\mu (1-\gamma_5) b | B^- > \\
&+ \underbrace{2 < \phi K^{*-} | \bar{s}(1+\gamma_5) u | 0 >< 0 | \bar{u}\gamma_5 b | B^- >}_{\text{annihilation contribution, usually neglected in FA}}
\end{aligned}
$$

- "Annihilation contribution" to decay rates may be small (or may not, depending on specific decay modes), but could be important in *CP* asymmetry because it contains *strong* phases! $\Rightarrow$ We need to reduce "hadronic uncertainty" before considering any "new physics".

- *CP* violating asymmetry originates from the superposition of *CP*-odd(violating) phases in CKM matrix and *CP*-even(conserving) phases.

Two body hadronic B decays

Form Factors

- Decay Constants
  - Pseudoscalar: $< P(p)|\bar{q}\gamma^\mu\gamma_5 q'|0 >= if_P p^\mu$
  - Vector: $< V|\bar{q}\gamma_\mu q|0 >= f_V m_V \epsilon^*_\mu$
- Form Factors
  - $P \to P$:

$$\langle P_1(p_1)|\bar{q}\gamma_\mu b|B(p_B)\rangle = \left[(p_B+p_1)_\mu - \frac{m_B^2-m_1^2}{q^2}q_\mu\right]F_1(q^2) + \frac{m_B^2-m_1^2}{q^2}q_\mu F_0(q^2), \qquad (F_1(0)=F_0(0))$$

  - $P \to V$:

$$\langle V(p_V)|V_\mu - A_\mu|\bar{B}^0(p_B)\rangle = -\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu *}p_B^\alpha p_V^\beta \frac{2V(q^2)}{(m_B+m_V)} - i\left(\epsilon^*_\mu - \frac{\epsilon^*\cdot q}{q^2}q_\mu\right)(m_B+m_V)A_1(q^2) +$$

$$i\left((p_B+p_V)_\mu - \frac{(m_B^2-m_V^2)}{q^2}q_\mu\right)(\epsilon^*\cdot q)\frac{A_2(q^2)}{m_B+m_V} - i\frac{2m_V(\epsilon^*\cdot q)}{q^2}q_\mu A_0(q^2),$$

$$(2m_V A_0(0) = (m_B+m_V)A_1(0) - (m_B-m_V)A_2(0))$$

  - $V \to V$:

$$P_B^\mu < V_1 V_2|\bar{q}\gamma_\mu(1\pm\gamma_5)q'|0 >= (\epsilon_1^*\cdot\epsilon_2^*)P_B^2 V_1(P_B^2) - (\epsilon_2^*\cdot p_1)(\epsilon_1^*\cdot p_2)V_2(P_B^2) \pm i\epsilon^{\mu\nu\alpha\beta}\epsilon^*_{1\mu}\epsilon^*_{1\nu}p_{1\alpha}p_{2\beta}A(P_B^2)$$

Two body hadronic B decays

Form Factors

- There have been numerous efforts to obtain the decay constants and the form factors (or hadronic matrix elements) theoretically in various ways.
- Decay Constants: Lattice QCD (quenched and/or unquenched QCD), QCD Sum Rules, Light-front QCD (LQCD), Chiral Perturbation Theory (χ PT), etc...
- Form Factors: Monopole(dipole) approximation, QCD-improved Factorization (QCDF), Perturbative QCD (pQCD, aka k_T factorization), Soft-Collinear Effective Theory (SCET), Light-Cone Sum Rules (LCSR, QCD Sum Rules), etc...

Two body hadronic B decays

Matrix Elements

- Example of $B \to PP$ transitions

$$
\begin{aligned}
\mathcal{A}(B^- \to \pi^0 K^-) &= \frac{G_F}{2}\Bigg\{ \left[\lambda_u^{LL}(a_1 + \rho_u^\pi a_{11}) + \frac{3}{2}\lambda_t^{LL}(a_7 - a_9)\right] X^{(BK,\pi)} \\
&+ \left[\lambda_u^{LL}(a_2 + a_{12}) - \lambda_t^{LL}\Big(a_4 + a_{10} + 2\rho_s^K(a_6 + a_8)\Big)\right] X^{(B\pi,K)} \\
&+ \left[\lambda_u^{LL}(a_2 - a_{12}) - \lambda_t^{LL}\Big(a_4 + a_{10} + 2\rho_s^B(a_6 + a_8)\Big)\right] X^{(B,\pi K)} \Bigg\} \\
&+ (a_i \to -a_i'), \\
\mathcal{A}(B^- \to \pi^- \bar{K}^0) &= \frac{G_F}{\sqrt{2}}\Bigg\{ -\lambda_t^{LL}\left[a_4 - \frac{1}{2}a_{10} + 2\rho_s^K\left(a_6 - \frac{1}{2}a_8\right)\right] X^{(B\pi,K)} \\
&+ \left[\lambda_u^{LL}(a_2 - a_{12}) - \lambda_t^{LL}\Big(a_4 + a_{10} + 2\rho_s^B(a_6 + a_8)\Big)\right] X^{(B,\pi K)} \Bigg\} \\
&+ (a_i \to -a_i'),
\end{aligned}
$$

$$
\begin{aligned}
X^{(BK,\pi)} &= -\sqrt{2} < \pi^0|\bar{u}\gamma^\mu\gamma_5 u|0 >< \bar{K}^0|\bar{s}\gamma_\mu b|\bar{B}^0 > = if_\pi F_0^{B\to K}(m_\pi^2)(m_B^2 - m_K^2), \\
X^{(B\pi,K)} &= +\sqrt{2} < \bar{K}^0|\bar{s}\gamma^\mu\gamma_5 d|0 >< \pi^0|\bar{d}\gamma_\mu b|\bar{B}^0 > = if_K F_0^{B\to\pi}(m_K^2)(m_B^2 - m_\pi^2), \\
X^{(B,\pi K)} &= +\sqrt{2} < \pi^0\bar{K}^0|\bar{s}\gamma^\mu d|0 >< 0|\bar{d}\gamma_\mu\gamma_5 b|\bar{B}^0 > = if_B F_0^{\pi K}(m_B^2)(m_K^2 - m_\pi^2), \\
\rho_q^H &\equiv \frac{m_H^2}{m_b m_q}\ (H = \pi, K, B,\ q = u, s)
\end{aligned}
$$

Two body hadronic B decays

Matrix Elements

- Example of $B \to PV$ transitions

$$
\begin{aligned}
\mathcal{A}(B^- \to \pi^0 K^{*-}) &= \frac{G_F}{2}\Bigg\{ \left[\lambda_u^{LL}(a_1 + \rho_u^\pi a_{11}) + \frac{3}{2}\lambda_t^{LL}(a_7 - a_9)\right] X_-^{(BK^*,\pi)} \\
&+ \left[\lambda_u^{LL}(a_2 + a_{12}) - \lambda_t^{LL}(a_4 + a_{10})\right] X_-^{(B\pi,K^*)} \\
&+ \left[\lambda_u^{LL}(a_2 - a_{12}) - \lambda_t^{LL}(a_4 + a_{10})\right] X_-^{(B,\pi K^*)} \\
&+ 2\lambda_t^{LL}\rho_s^B(a_6 + a_8) X_+^{(B,\pi K^*)} \Bigg\} + (a_i \to a_i',\ X_\mp \to X_\pm), \\
\mathcal{A}(B^- \to \pi^- \bar{K}^{*0}) &= \frac{G_F}{\sqrt{2}}\Bigg\{ -\lambda_t^{LL}\left(a_4 - \frac{1}{2}a_{10}\right) X_-^{(B\pi,K^*)} \\
&+ \left[\lambda_u^{LL}(a_2 - a_{12}) - \lambda_t^{LL}(a_4 + a_{10})\right] X_-^{(B,\pi K^*)} + 2\lambda_t^{LL}\rho_s^B(a_6 + a_8) X_+^{(B,\pi K^*)} \Bigg\} \\
&+ (a_i \to a_i',\ X_\mp \to X_\pm),
\end{aligned}
$$

$$
\begin{aligned}
X_\pm^{(BK^*,\pi)} &= \pm\sqrt{2} < \pi^0|\bar{u}\gamma^\mu\gamma_5 u|0 >< \bar{K}^{*0}|\bar{s}\gamma_\mu(1 \pm \gamma_5)b|\bar{B}^0 > = 2f_\pi m_{K^*}(\epsilon \cdot p_\pi) A_0^{B\to K^*}(m_\pi^2), \\
X_\pm^{(B\pi,K^*)} &= -\sqrt{2} < \bar{K}^{*0}|\bar{s}\gamma^\mu d|0 >< \pi^0|\bar{d}\gamma_\mu b|\bar{B}^0 > = 2f_{K^*} m_{K^*}(\epsilon \cdot p_\pi) F_1^{B\to\pi}(m_{K^*}^2), \\
X_\pm^{(B,\pi K^*)} &= \mp\sqrt{2} < \pi^0\bar{K}^{*0}|\bar{s}\gamma^\mu(1 \pm \gamma_5)d|0 >< 0|\bar{d}\gamma_\mu\gamma_5 b|\bar{B}^0 > = 2f_B m_{K^*}(\epsilon \cdot p_\pi) A_0^{\pi K^*}(m_B^2)
\end{aligned}
$$

Two body hadronic B decays

Matrix Elements

- Example of $B \to PV$ transitions

$$
\begin{aligned}
\mathcal{A}(B^- \to \rho K^-) &= \frac{G_F}{2}\Bigg\{ \left[\lambda_u^{LL} a_1 - \frac{3}{2}\lambda_t^{LL}(a_7 + a_9)\right] X_-^{(BK,\rho)} \\
&+ \left[\lambda_u^{LL}(a_2 - a_{12}) - \lambda_t^{LL}\left(a_4 + a_{10}\right)\right]\left[X_-^{(B\rho,K)} + X_-^{(B,\rho K)}\right] \\
&+ 2\lambda_t^{LL}\left(a_6 + a_8\right)\left[\rho_s^K X_+^{(B\rho,K)} + \rho_s^B X_+^{(B,\rho K)}\right]\Bigg\} \\
&+ (a_i \to a_i',\ X_\mp \to X_\pm), \qquad (1) \\
\mathcal{A}(B^- \to \rho^- \bar{K}^0) &= \frac{G_F}{\sqrt{2}}\Bigg\{ -\lambda_t^{LL}\left(a_4 - \frac{1}{2}a_{10}\right) X_-^{(B\rho,K)} + \left[\lambda_u^{LL}(a_2 - a_{12}) - \lambda_t^{LL}(a_4 + a_{10})\right] X_-^{(B,\rho K)} \\
&+ 2\lambda_t^{LL}\left[\rho_s^K\left(a_6 - \frac{1}{2}a_8\right) X_+^{(B\rho,K)} + \rho_s^B\left(a_6 + a_8\right) X_+^{(B,\rho K)}\right]\Bigg\} \\
&+ (a_i \to a_i',\ X_\mp \to X_\pm),
\end{aligned}
$$

$$
\begin{aligned}
X_\pm^{(BK,\rho)} &= \sqrt{2} < \rho|\bar{u}\gamma^\mu u|0 >< \bar{K}^0|\bar{s}\gamma_\mu b|\bar{B}^0 > = 2 f_\rho m_\rho(\epsilon \cdot p_K) F_1^{B\to K}(m_\rho^2), \\
X_\pm^{(B\rho,K)} &= \mp\sqrt{2} < \bar{K}^0|\bar{s}\gamma^\mu\gamma_5 d|0 >< \rho|\bar{d}\gamma_\mu(1 \pm \gamma_5) b|\bar{B}^0 > = 2 f_K m_\rho(\epsilon \cdot p_K) A_0^{B\to\rho}(m_K^2), \\
X_\pm^{(B,\rho K)} &= \mp\sqrt{2} < \rho\bar{K}^0|\bar{s}\gamma^\mu(1 \pm \gamma_5) d|0 >< 0|\bar{d}\gamma_\mu\gamma_5 b|\bar{B}^0 > = 2 f_B m_\rho(\epsilon \cdot p_K) A_0^{\rho K}(m_B^2)
\end{aligned}
$$

Two body hadronic B decays

Matrix Elements

- The decay $B \to V_1 V_2$ is described by the amplitude

$$\mathcal{A}(B(p) \to V_1(p_1, \varepsilon_1) V_2(p_2, \varepsilon_2)) = \mathcal{A}_0\, \varepsilon_1^* \cdot \varepsilon_2^* + \mathcal{A}_1\, (\varepsilon_1^* \cdot p_2)(\varepsilon_2^* \cdot p_1) + i\mathcal{A}_2\, \epsilon^{\alpha\beta\gamma\delta} \varepsilon_{1\alpha}^* \varepsilon_{2\beta}^* p_{1\gamma} p_{2\delta}$$

- The three helicity amplitudes can be rewritten in the transversity basis as:

$$\mathcal{A}_L = -x\mathcal{A}_0 - m_1 m_2 (x^2 - 1)\mathcal{A}_1, \qquad \mathcal{A}_\parallel = -\sqrt{2}\mathcal{A}_0$$
$$\mathcal{A}_\perp = -\sqrt{2} m_1 m_2 \sqrt{x^2 - 1}\mathcal{A}_2, \qquad x \equiv \frac{p_1 \cdot p_2}{m_1 m_2}$$

- In the LRM ,

$$\mathcal{A}(B \to V_1 V_2) = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \lambda_q^{LL} \left[C_\pm^I x_\pm^{(BV_1, V_2)} + C_\pm^A x_\pm^{(B, V_1 V_2)} \right] \Rightarrow |\mathcal{A}(B \to V_1 V_2)|^2 = |\mathcal{A}_L|^2 + |\mathcal{A}_\perp|^2 + |\mathcal{A}_\parallel|^2$$

- In the helicity basis,

$$\begin{aligned}
\mathcal{A}_0 &= \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \lambda_q^{LL} \left[f_2 m_2 (m_B + m_1) \left(C_-^I - C_+^I \right) A_1(m_2^2) - f_B m_B^2 \left(C_-^A + C_+^A \right) V_1(m_B^2) \right] \\
\mathcal{A}_1 &= \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \lambda_q^{LL} \left[-\frac{2 f_2 m_2}{m_B + m_1} \left(C_-^I - C_+^I \right) A_2(m_2^2) + f_B \left(C_-^A + C_+^A \right) V_2(m_B^2) \right] \\
\mathcal{A}_2 &= \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \lambda_q^{LL} \left[-\frac{2 f_2 m_2}{m_B + m_1} \left(C_-^I + C_+^I \right) V(m_2^2) + f_B \left(C_-^A - C_+^A \right) A(m_B^2) \right]
\end{aligned}$$

$\Rightarrow$ Right-handed contribution can enhance $\mathcal{A}_\perp$ and $\mathcal{A}_\parallel$.

Two body hadronic B decays

Matrix Elements

- Example of $B \to VV$ transitions

$$
\begin{aligned}
\mathcal{A}(B^- \to \rho K^{*-})_H &= \frac{G_F}{2} m_B^2 \Big\{ f_\rho \left[\lambda_u^{LL} a_1^H - \frac{3}{2} \lambda_t^{LL} (a_7^H + a_9^H) \right] F_H^{B\to K^*} \\
&\quad + f_{K^*} \left[\lambda_u^{LL} (a_2^H - a_{12}^H) - \lambda_t^{LL} \left(a_4^H + a_{10}^H \right) \right] F_H^{B\to\rho} \\
&\quad + f_B \left[\lambda_u^{LL} (a_2^H - a_{12}^H) - \lambda_t^{LL} \left(a_4^H + a_{10}^H - 2 p_H \rho_s^B \left(a_6^H + a_8^H \right) \right) \right] F_H^{\rho K^*} \Big\}, \\
\mathcal{A}(B^- \to \rho^- \bar{K}^{*0})_H &= \frac{G_F}{\sqrt{2}} m_B^2 \Big\{ - f_{K^*} \lambda_t^{LL} \left(a_4^H - \frac{1}{2} a_{10}^H \right) F_H^{B\to\rho} \\
&\quad + f_B \left[\lambda_u^{LL} (a_2^H - a_{12}^H) - \lambda_t^{LL} \left(a_4^H + a_{10}^H - 2 p_H \rho_s^B \left(a_6^H + a_8^H \right) \right) \right] F_H^{\rho K^*} \Big\},
\end{aligned}
$$

where

$$
a_i^H = a_i + p_H a_i', \quad p_L = p_{||} = -1 \text{ and } p_\perp = 1
$$

Summary

- For a given effective weak Hamiltonian, there are two important issues in the study of the hadronic matrix elements for nonleptonic decays of heavy mesons: One is the renormalization scale and scheme dependence of the matrix element, and the other is the nonfactorizable effect.
- Factorization approach is well poised to becoming a useful theoretical tool in studying nonleptonic B decays in many(not all) decays.
- Predictive power of the factorization approaches comes from universality of nonperturbative inputs, such as meson wave functions.
- One should reduce "hadronic uncertainty" before considering any "new physics".