Baryogenesis in a supersymmetric model

S.W. Ham

Center for High Energy Physics, Kyungpook National University

HPC Asia 2007, 2007. 9. 12.

Contents

1. Introduction

(a) Baryogenesis: A.D. Linde, Phase Transitions in Gauge Theories and Cosmology, Rep. Prog. Phys., 42, 389 (1979)

Matter-Antimatter Asymmetry of the Universe?.

Locally antimatter area $\rightarrow$ Photon Flux ($\times$)

$\rightarrow$ Baryon Asymmetry of the Universe?.

Sakharov: (1) The presence of baryon number violation

(2) The violation of Both C (Charge Conjugation) and CP

$\leftarrow$ (CTP Conservation); CP Vioaltion: CKM matrix

(3) A departure from thermal equilibrium

-A.D. Sakharov, JETP Lett. 5, 24 (1967)

(1) B-number violation process, axial anomaly: S.L. Adler, Phys. Rev. 177, 2426 (1969); J.S. Bell and R. Jackiw, Nuovo Cim. A 60, 47 (1969); G.'t Hooft, Phys. Lett. 37, 8 (1976); Phys. Rev. D14, 3432 (1976)

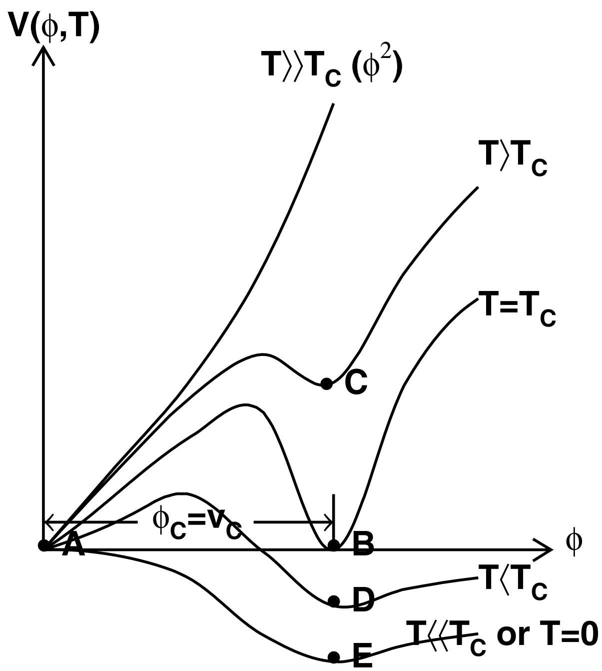
(2) Nature $\Longrightarrow$ C and P are both maximally violated !

$\Gamma(\pi^+ \rightarrow \mu^+\nu_L) \neq \Gamma(\pi^- \rightarrow \mu^-\bar{\nu}_L) = 0$: C-violation

$\Gamma(\pi^+ \rightarrow \mu^+\nu_L) \neq \Gamma(\pi^- \rightarrow \mu^-\nu_R) = 0$: P-violation

$\Gamma(\pi^+ \rightarrow \mu^+\nu_L) = \Gamma(\pi^- \rightarrow \mu^-\bar{\nu}_R) = 0$: CP-invariance

(3) The existence of thermal non-equilibrium during the evolution of Universe;
$\longrightarrow$ First order phase transition (Figure; 5 page)
$\star$ The nature of a symmetry-breaking phase transition (Generally)
{1} First order phase transition;
◯ There is a potential barrier (two degenerate vacua).
◯ The change in ϕ in going from one phase to the other must be discontinuous.
◯ The transition cannot take place classically, but must proceed either through quantum ($T = 0$) or thermal ($T \neq 0$) tunnelling.
{2} Second order phase transition;
◯ There is no potential barrier.
◯ The transition occurs smoothly.
◯ The transition may proceed classically.
$\star$ The Strength of the phase transition;
◯ Strongly first order phase transition: $\diamond$ Sphaleron Constraint; $\phi_C \geq T_C$
Sphaleron: Greek for ready to fall, E.W. Kolb and M.S. Tuner, The Early Universe, Addison-Wesley Publishing Company (1990)
◯ Weakly first order phase transition: $\phi_C \leq T_C$



Potential at Finite temperature ($T \neq 0$)
(Temperature: 1 GeV = 1.1605×10^{13} K)
$V = V(\phi, T) = V(\phi, 0) + V_1(\phi, 0) + V_1(\phi, T)$
Symmetry Restoration at high temperature
A: Symmetric phase state
B: Broken phase state
C: Local minimum ($T > T_C; \phi \neq 0$)
D,E: Global minimum ($T < T_C; \phi \neq 0$)

$V_1(\phi, 0)$: Coleman-Weinberg Effective Potential, S. Coleman and E. Weinberg, Radiative Corrections as the Origin of Spontaneous Symmetry Breaking, Phys. Rev. D **7**, 1888 (1973).
$V_1(\phi, T)$: Dolan-Jackiw Thermal Effective Potential, L. Dolan and R. Jackiw, Symmetry Behavior at Finite Temperature, Phys. Rev. D **9**, 3320 (1974).

(b) Supersymmetry (SUSY)
{1} Gauge Coupling Unification
{2} No-Go Theorem; Theory
Poincare group (space-time maximal symmetry)
$\rightarrow$ Super-Poincare group (Super Charge; Majorana Spinors)
{3} Gravity (Local SUSY)
{4} Superstring; SUSY was originally introduced into physics in the context of string theory, Ryder, Guantum Field Theory
{5} Particle $\longleftrightarrow$ Superparticle (spin 1/2)
{6} Two Higgs douplets
{7} SUSY breaking (soft terms); Phenomenology

$\star$ Minimal Supersymmetric Standard Model (MSSM)
Soft SUSY Breaking (by hand, 1 TeV-2 TeV) Models
Two Higgs doublets: Scalar Higgs bosons (h, H), Pseudoscalar Higgs boson (A), Charged Higgs bosons $(H^{\pm})$
SM particles + Superparticles (sfermions, charginos, neutralinos 4)

2. The explicit CP violation in the Higgs sector
Gauge theory of CP violation, S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976)
A supersymmetric $U(1)'$ model, the $U(1)$-extended supersymmetric standard model (USSM), a MSSM with an extra $U(1)'$
Explicit CP violation in a MSSM with an extra $U(1)'$, S.W. Ham, E.J. Yoo, and S.K. Oh, Phys. Rev. D76, 015004 (2007) (KISTI;"The Strategic Supercomputing Support Program").
Higgs doublet + Higgs singlet: Scalar Higgs bosons (S_1, S_2, S_3), Pseudoscalar Higgs boson (A), Charged Higgs bosons ($H^\pm$)
SM particles + Superparticles (sfermions, charginos, neutralinos)
Motivation; In the MSSM, μ-problem, J.E. Kim and H.P. Nilles, Phys. Lett. B138, 150 (1984)
Superstring-inspired E_6 Model, Higgs singlet 1 (effective rank 5 model) or Higgs singlet 2 (E_6 model)
μ-parameter; $\mu = \lambda s$, Vacuuum Expectation Value (VEV) of Higgs singlet
Explicit Breaking of the global U(1) Peccei-Quinn symmetry
The breaking terms of $U(1)'$ gauge symmetry in terms of the VEV of the Higgs singlet at a TeV scale, $m'_Z = 1$ TeV.

The E_6 gauge group may be decomposed into

$$E_6 \supset SU(10) \times U(1)_\psi \supset SU(5) \times U(1)_\chi \times U(1)_\psi \ ,$$

where $SU(5)$ is further broken down to the Standard Model (SM) gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$.
At the electroweak scale, the desired extra $U(1)'$ symmetry may be given as an orthogonal linear combination of $U(1)_\chi$ and $U(1)_\psi$ as

$$Q' = \cos\theta_E Q_\chi + \sin\theta_E Q_\psi \ ,$$

where Q', Q_χ, and Q_ψ are the $U(1)'$, $U(1)_\chi$, and $U(1)_\psi$ charges, respectively. Sometimes, for four different values of the angle θ_E, the four particular combinations of the $U(1)_\chi$ and $U(1)_\psi$ are called as follows:
the χ-model for $\theta_E = 0$, the ψ-model for $\theta_E = \pi/2$,
the η-model for $\theta_E = \tan^{-1}(-\sqrt{5/3})$, and the ν-model for $\theta_E = \tan^{-1}\sqrt{15}$.
The gauge group we consider is thus $G = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$.

The superpotential for the model we consider may be expressed as

$$W \approx \lambda S H_1 H_2 + h_t Q H_2 t_R^c + h_b Q H_1 b_R^c + k S D_L \bar{D}_R \ ,$$

where Q is the left-handed quark doublet, t_R^c and b_R^c are the charge conjugate of the right-handed top quark and bottom quark, and $H_1 H_2 = H_1^0 H_2^0 - H_1^- H_2^+$. The Higgs potential of this model at the tree level may expressed as a sum of the F-term, the D-term, and the soft breaking term, that is,

$$V_0 = V_F + V_D + V_{\rm S} \ ,$$

where

$$\begin{aligned}
V_F &= |\lambda|^2[(|H_1|^2 + |H_2|^2)|S|^2 + |H_1 H_2|^2] \ , \\
V_D &= \frac{g_2^2}{8}(H_1^\dagger \vec{\sigma} H_1 + H_2^\dagger \vec{\sigma} H_2)^2 + \frac{g_1^2}{8}(|H_1|^2 - |H_2|^2)^2 \\
&\quad + \frac{g_1^{'2}}{2}(\tilde{Q}_1 |H_1|^2 + \tilde{Q}_2 |H_2|^2 + \tilde{Q}_3 |S|^2)^2 \ , \\
V_{\rm S} &= m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 |S|^2 - [\lambda A_\lambda (H_1 H_2) S + {\rm H.c.}] \ ,
\end{aligned}$$

where $\tilde{Q}_1$, $\tilde{Q}_2$, and $\tilde{Q}_3$ are respectively the effective $U(1)'$ charges of H_1, H_2, and S, satisfying the condition of $\tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 = 0$ to obey the gauge invariance of the superpotential under $U(1)'$.

After the electroweak symmetry breakdown, these mass matrices yield the tree-level masses for the squarks as

$$
\begin{aligned}
m^2_{\tilde{t}_1,\ \tilde{t}_2} &= m^2_Q + m^2_t \mp m_t \sqrt{A^2_t + \lambda^2 s^2 \cot^2\beta - 2\lambda A_t s \cot\beta \cos\phi_t} \ , \\
m^2_{\tilde{b}_1,\ \tilde{b}_2} &= m^2_Q + m^2_b \mp m_b \sqrt{A^2_b + \lambda^2 s^2 \tan^2\beta - 2\lambda A_b s \tan\beta \cos\phi_b} \ , \\
m^2_{\tilde{k}_1,\ \tilde{k}_2} &= m^2_K + m^2_k \mp m_k \sqrt{A^2_k + \lambda^2 v^4 \sin^2 2\beta/(4s^2) - \lambda A_k v^2 \sin 2\beta \cos\phi_k/s} \ ,
\end{aligned}
$$

where $\tan\beta = v_2/v_1$ and $v^2 = v_1^2 + v_2^2$.

The full Higgs potential at the one-loop level may be written as

$$V = V_0 + V_1 \ ,$$

where V_1 is the radiative corrections due to the relevant particles and their superpartners. According to the effective potential method, V_1 is given as

$$V_1 = \sum_l \frac{n_l \mathcal{M}_l^4}{64\pi^2} \left[\log \frac{\mathcal{M}_l^2}{\Lambda^2} - \frac{3}{2} \right] \ ,$$

where Λ is the renormalization scale in the modified minimal subtraction scheme.

The non-trivial tadpole minimum condition is given as

$$0 = A_\lambda \sin\phi_0 - \frac{3m_t^2 A_t \sin\phi_t}{16\pi^2 v^2 \sin^2\beta} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) - \frac{3m_b^2 A_b \sin\phi_b}{16\pi^2 v^2 \cos^2\beta} f(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2)$$
$$- \frac{3m_k^2 A_k \sin\phi_k}{16\pi^2 s^2} f(m_{\tilde{k}_1}^2,\ m_{\tilde{k}_2}^2)\ ,$$

where

$$f(m_x^2,\ m_y^2) = \frac{1}{(m_y^2 - m_x^2)}\left[m_x^2 \log\frac{m_x^2}{\Lambda^2} - m_y^2 \log\frac{m_y^2}{\Lambda^2}\right] + 1\ .$$

At the one-loop level, the squared mass matrix may be decomposed as

$$M = \bar{M}^0 + M^1$$

where

$$\bar{M}^0 = \begin{pmatrix} M_{11}^0 & M_{12}^0 & M_{13}^0 & 0 \\ M_{12}^0 & M_{22}^0 & M_{23}^0 & 0 \\ M_{13}^0 & M_{23}^0 & M_{33}^0 & 0 \\ 0 & 0 & 0 & \bar{m}_A^2 \end{pmatrix}\ ,$$

$$
\begin{aligned}
\bar{m}_A^2 \;=\; & \frac{2\lambda v}{\sin 2\alpha}\left[A_\lambda \cos\phi_0 - \frac{3m_t^2 A_t \cos\phi_t}{16\pi^2 v^2 \sin^2\beta} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \right. \\
& \left. -\frac{3m_b^2 A_b \cos\phi_b}{16\pi^2 v^2 \cos^2\beta} f(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2) - \frac{3m_k^2 A_k \cos\phi_k}{16\pi^2 s^2} f(m_{\tilde{k}_1}^2,\ m_{\tilde{k}_2}^2)\right] \ .
\end{aligned}
$$

One can decompose M^1 as

$$
M^1 = M^t + M^b + M^k \ .
$$

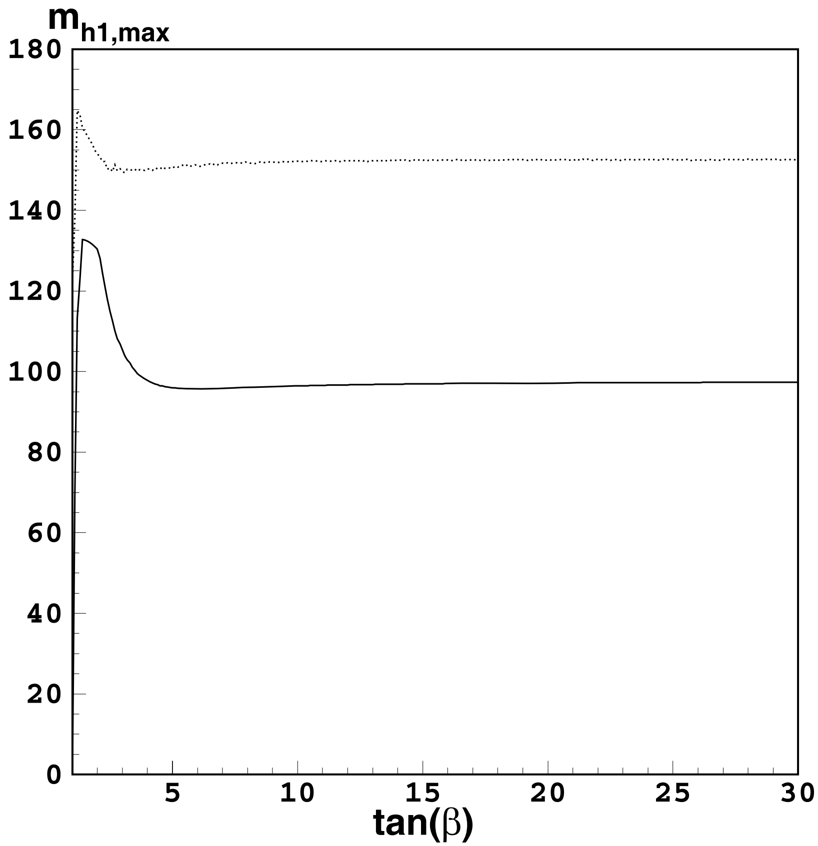
For numerical analysis, we take the ν-model, where $U(1)'$ is a mixture of $U(1)_\chi$ and $U(1)_\psi$ with $\theta_E = \tan^{-1}\sqrt{15}$.

The $U(1)'$ and $U(1)_Y$ charges of the Higgs fields in the present model are given in Table I of the following Refs: Theory and phenomenology of an exceptional supersymmetric standard model, S.F. King, S. Moretti, and R. Nevzorov, Phys. Rev. D73, 035009 (2006); Exceptional supersymmetric standard model, Phys. Lett. B634, 278 (2006).

Thus, the effective $U(1)'$ charges of H_1, H_2, and S are respectively given as $\tilde{Q}_1 \approx -0.4910123$, $\tilde{Q}_2 \approx -0.2995571$, and $\tilde{Q}_3 = -(\tilde{Q}_1 + \tilde{Q}_2)$.

The physical four neutral Higgs bosons are h_i (i= 1 to 4) and their masses are m_{h_i} ($i = 1$ to 4) with $m_{h_i} \leq m_{h_j}$ for $i < j$. The upper bound on $m^2_{h_1}$ at the one-loop level is given as; Supersymmetric Models with Extended Higgs Sector, M. Drees, Int. J. Mod. Phys. A4, 3635 (1989)

$$
\begin{aligned}
m^2_{h_1} \;\leq\; & \lambda^2 v^2 \sin^2 2\beta + m_Z^2 \cos^2 2\beta + 2g'^2 v^2 (\tilde{Q}_1 \cos^2\beta + \tilde{Q}_2 \sin^2\beta)^2 \\
& + \frac{3m_t^4}{8\pi^2 v^2} \frac{(\lambda s \cot\beta \Delta_{\tilde{t}_1} - A_t \Delta_{\tilde{t}_2})^2}{(m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1})^2} g(m^2_{\tilde{t}_1},\ m^2_{\tilde{t}_2}) \\
& + \frac{3m_t^4}{4\pi^2 v^2} \frac{(\lambda s \cot\beta \Delta_{\tilde{t}_1} - A_t \Delta_{\tilde{t}_2})}{(m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1})} \log\left(\frac{m^2_{\tilde{t}_2}}{m^2_{\tilde{t}_1}}\right) + \frac{3m_t^4}{8\pi^2 v^2} \log\left(\frac{m^2_{\tilde{t}_1} m^2_{\tilde{t}_2}}{m_t^4}\right) \\
& + \frac{3m_b^4}{8\pi^2 v^2} \frac{(\lambda s \tan\beta \Delta_{\tilde{b}_1} - A_b \Delta_{\tilde{b}_2})^2}{(m^2_{\tilde{b}_2} - m^2_{\tilde{b}_1})^2} g(m^2_{\tilde{b}_1},\ m^2_{\tilde{b}_2}) \\
& + \frac{3m_b^4}{4\pi^2 v^2} \frac{(\lambda s \tan\beta \Delta_{\tilde{b}_1} - A_t \Delta_{\tilde{b}_2})}{(m^2_{\tilde{b}_2} - m^2_{\tilde{b}_1})} \log\left(\frac{m^2_{\tilde{b}_2}}{m^2_{\tilde{b}_1}}\right) + \frac{3m_b^4}{8\pi^2 v^2} \log\left(\frac{m^2_{\tilde{b}_1} m^2_{\tilde{b}_2}}{m_b^4}\right) \\
& + \frac{3m_k^4 \lambda^2 v^2 \Delta^2_{\tilde{k}_1}}{8\pi^2 s^2} \frac{g(m^2_{\tilde{k}_1},\ m^2_{\tilde{k}_2})}{(m^2_{\tilde{k}_2} - m^2_{\tilde{k}_1})^2} - \frac{3m_k^2 \lambda^2 v^2 \sin^2 2\beta}{16\pi^2 s^2} f(m^2_{\tilde{k}_1},\ m^2_{\tilde{k}_2})\ .
\end{aligned}
$$



$A_t = A_b = A_k$
$m_{\rm SUSY} = m_Q = m_K$
The upper bound on the lightest neutral Higgs boson mass as a function of $\tan\beta$, where other relevant parameters are allowed to vary within their respective ranges:
$100 \le A_t,\ \bar{m}_A\ ({\rm GeV}) \le 2000$
$100 \le m_{\rm SUSY}\ ({\rm GeV}) \le 1000$,
$\mathbf{1500} \le s\ ({\rm GeV}) \le 2000$,
$0 \le \phi_t, \phi_b, \phi_k \le 2\pi$,
$1 < \tan\beta \le 30$, $0 < \lambda \le 0.85$
$m_t = 175$ GeV, $m_b = 4$ GeV,
$m_k = 700$ GeV, $\Lambda = 700$ GeV

The large s can satisfy the experimental constraint that the mixing angle between Z and Z' should be smaller than 2-3 $\times 10^{-3}$.

We introduce (i = 1,2,3)

$$\omega_i(\phi_k) = \frac{2|M_{i4}|}{|M_{ii}| + |M_{44}|} \ , \quad \text{for the CP mixing of } h_i A$$

which depend implicitly on other parameters as well. In order to express the relative contributions by ϕ_k, we introduce

$$\Omega_i(\phi_k) = \frac{\omega_i(\phi_k)}{\omega_i(\phi_k = 0)} \ .$$

In Figure, Ω_3 is calculated as a function of ϕ_k.

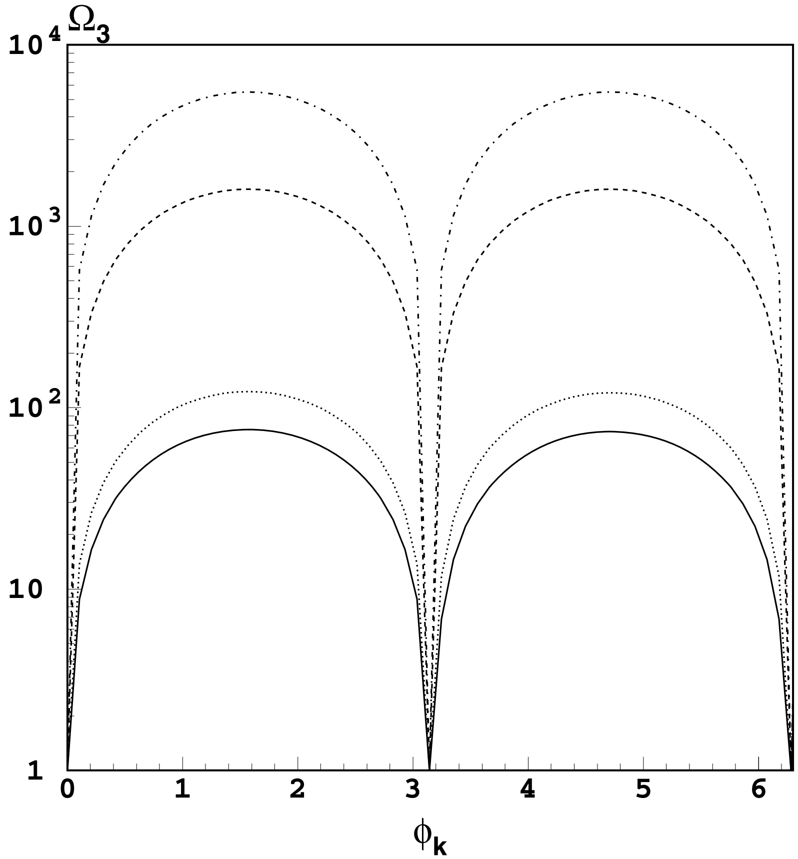
The absolute size of the CP violation can be evaluated by

$$\rho(\phi_t, \phi_b, \phi_k) = 4\sqrt{|O_{11}O_{12}O_{13}O_{14}|} \ .$$

In order to figure out the dependence of ρ on ϕ_k, we introduce

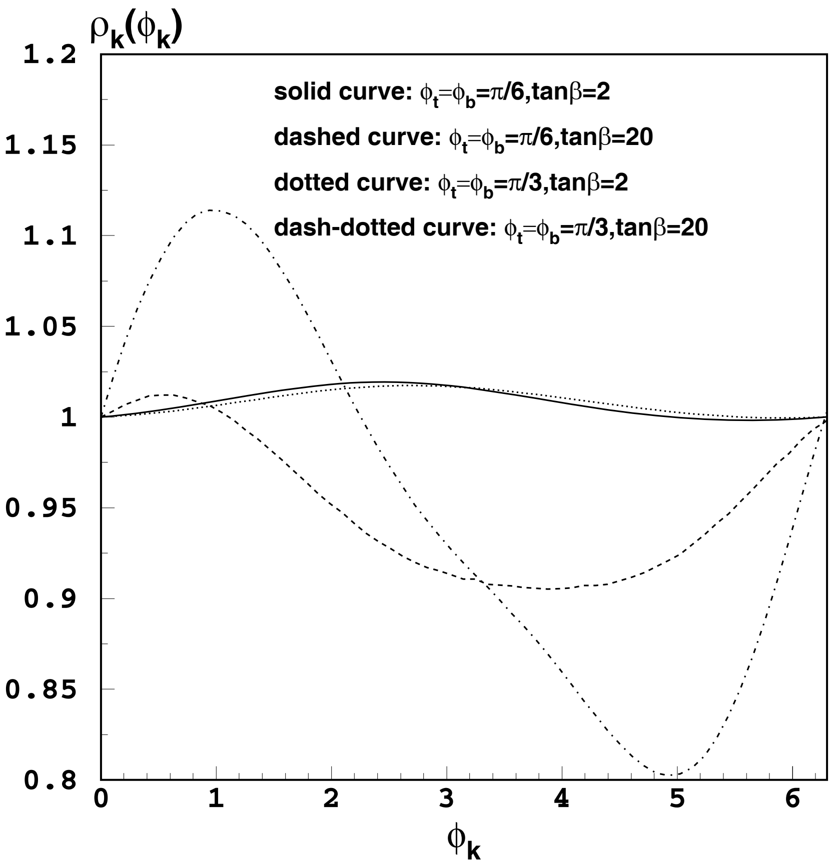
$$\rho_k(\phi_k) = \frac{\rho(\phi_t, \phi_b, \phi_k)}{\rho(\phi_t, \phi_b, \phi_k = 0)}$$

In figure, ρ_k is calculated as a function of ϕ_k, for the same parameter setting as previous Fig.



Plot of Ω_3 as a function of ϕ_k, for four different sets of $\phi_t = \phi_b$, $\tan\beta$:
$\phi_t = \pi/6$, $\tan\beta = 2$ (solid)
$\phi_t = \pi/6$, $\tan\beta = 20$ (dash)
$\phi_t = \pi/3$, $\tan\beta = 2$ (dot)
$\phi_t = \pi/3$, $\tan\beta = 20$(dash-dot)
The remaining parameters are set as $m_{\rm SUSY} = \bar{m}_A = A_t = s/2$ $= 1000$ GeV and $\lambda = 0.3$.

For the parameter values in Fig, $m_{Z'}$ is stable at approximately 1039 GeV. The mixing angle between Z and Z', denoted as $\alpha_{ZZ'}$, depends on both $\tan\beta$ and s. We find that $\alpha_{ZZ'} = 1.39 \times 10^{-3}$ for $\tan\beta = 2$, and and slightly increases for larger values of $\tan\beta$ as 2.93×10^{-3} for $\tan\beta = 20$.



Plot of ρ_k as a function of ϕ_k for the same parameter setting as previous Fig.

Fig. shows that the fluctuation of ρ_k is larger for $\tan\beta = 20$ than for $\tan\beta = 2$. Thus, ϕ_k play an important role in ρ_k for large $\tan\beta$. The complex phase of the exotic quark sector may change up to 20 % of the CP violation.

-Spontaneous CP violation

At the one-loop level, the minimum condition for the vacuum with respect to ϕ_s is obtained by

$$
\begin{aligned}
A_\lambda \;=\; & \frac{3m_t^2 A_t}{16\pi^2 v^2 \sin^2\beta} f(m_{\tilde{t}_1}^2,\; m_{\tilde{t}_2}^2) + \frac{3m_b^2 A_b}{16\pi^2 v^2 \cos^2\beta} f(m_{\tilde{b}_1}^2,\; m_{\tilde{b}_2}^2) \\
& + \frac{3m_k^2 A_k}{16\pi^2 s^2} f(m_{\tilde{k}_1}^2,\; m_{\tilde{k}_2}^2)
\end{aligned}
$$

for $\phi_s \neq 0, \pi$. The axion mass at the one-loop level is obtained by

$$
\begin{aligned}
m_a^2 \;=\; & \frac{3m_t^4 \lambda^2 A_t^2 s^2 \sin^2\phi_t}{8\pi^2 v^2 \sin^4\beta \cos^2\alpha} \frac{g(m_{\tilde{t}_1}^2,\; m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2} + \frac{3m_b^4 \lambda^2 A_b^2 s^2 \sin^2\phi_b}{8\pi^2 v^2 \cos^4\beta \cos^2\alpha} \frac{g(m_{\tilde{b}_1}^2,\; m_{\tilde{b}_2}^2)}{(m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2)^2} \\
& + \frac{3m_k^4 \lambda^2 A_k^2 v^2 \sin^2\phi_k}{8\pi^2 s^2 \cos^2\alpha} \frac{g(m_{\tilde{k}_1}^2,\; m_{\tilde{k}_2}^2)}{(m_{\tilde{k}_2}^2 - m_{\tilde{k}_1}^2)^2} \,,
\end{aligned}
$$

where

$$
g(m_x^2, m_y^2) = \frac{m_y^2 + m_x^2}{m_x^2 - m_y^2} \log\frac{m_y^2}{m_x^2} + 2 \,.
$$

3. ElectroWeak Phase Transition (EWPT) in a supersymmetric model

Electroweak phase Transition in the Standard Model (SM)

$$V(\phi,T) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + V_1(\phi,0) + V_1(\phi,T) \ .$$

$$V_1(\phi,0) = \sum_i \frac{n_i}{64\pi^2}\left[m_i^4(\phi)\log\left(\frac{m_i^2(\phi)}{m_i^2(v)}\right) - \frac{3}{2}m_i^4(\phi) + 2m_i^2(v)m_i^2(\phi)\right] \ ,$$

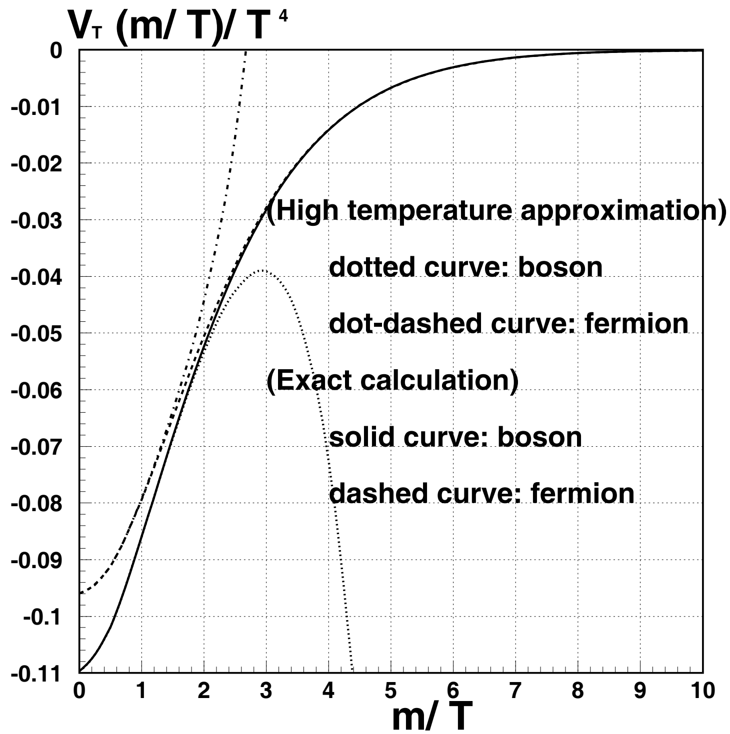
$$V_1(\phi,T) = \sum_i \frac{n_i T^4}{2\pi^2}\int_0^\infty dx\ x^2\ \log\left[1 \pm \exp\left(-\sqrt{x^2 + m_i^2(\phi)/T^2}\right)\right] \ ,$$

where $i = W, Z, t, \phi, G$; Boson($-$) and Fermion ($+$).

High Temperature Approximation on $V_1(\phi,T)$;

$$\begin{aligned} V_1^{(\text{high }T)}(\phi,T) \ =\ & -n_t\left[\frac{T^2 m_t^2(\phi)}{48} + \frac{m_t^4(\phi)}{64\pi^2}\log\left(\frac{m_t^2(\phi)}{c_f T^2}\right)\right] \\ & + \sum_i n_i\left[\frac{T^2 m_i^2(\phi)}{24} - \frac{T m_i^3(\phi)}{12\pi} - \frac{m_i^4(\phi)}{64\pi^2}\log\left(\frac{m_i^2(\phi)}{c_b T^2}\right)\right] \ , \end{aligned}$$

where $\log c_f = 2.64$ and $\log c_b = 5.41$.



In the SM, the high temperature approximation is consistent with the exact calculation of the integrals within 5 % for $m_f/T < 1.6$ in the fermion case and for $m_b/T < 2.2$ in the boson case.

-The electroweak phase transition and baryogenesis, G.W. Anderson and L.J. Hall, Phys. Rev. D45, 2685 (1992).

EWPT in the SM without $\phi + G(H)$ contributions,

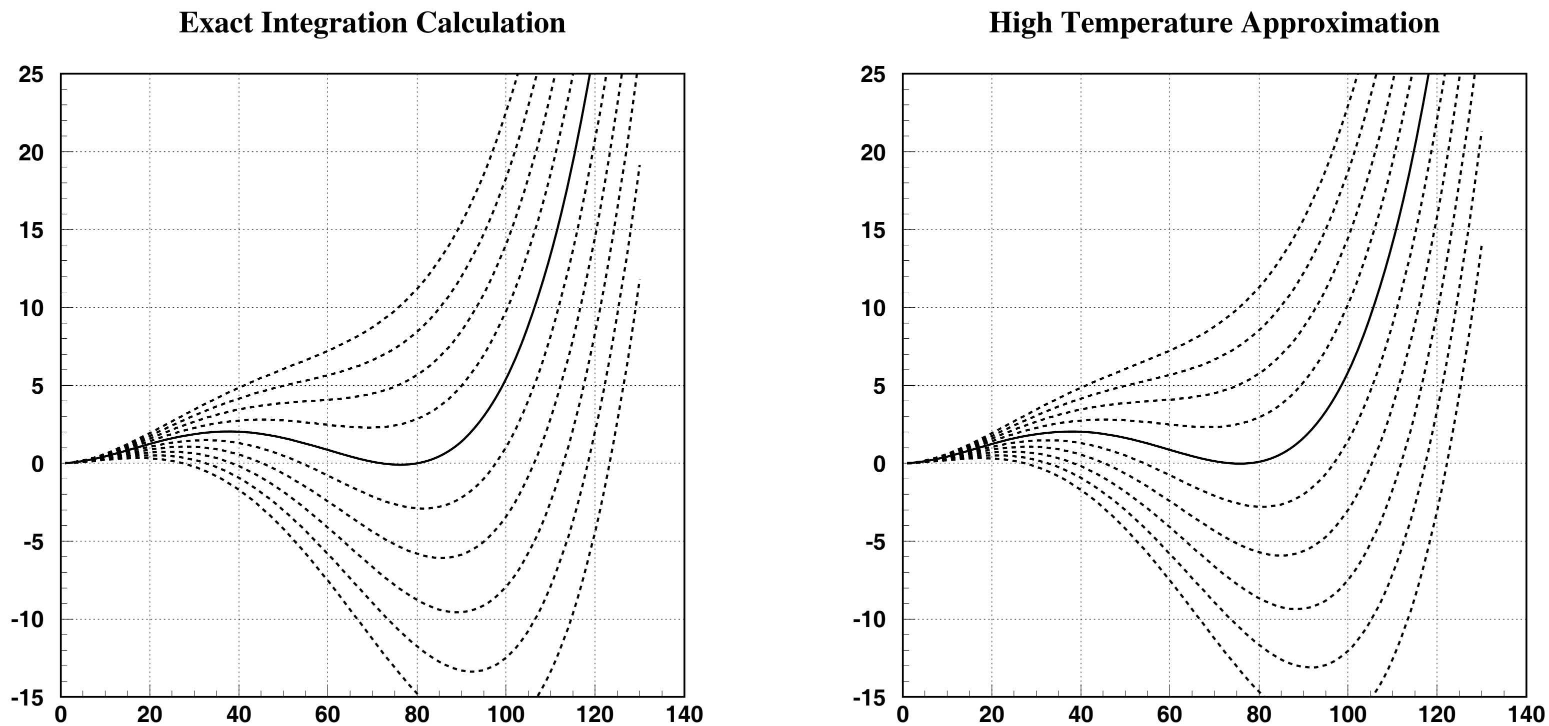
$$
\begin{aligned}
V_{\rm eff}(\phi,T) &\simeq V_0(\phi,0)+V_1(\phi,0)+V_1^{({\rm high}T)}(\phi,T) \\
& (DT^2-B)\phi^2 - ET\phi^3 + \frac{\lambda_T}{4}\phi^4 \ ,
\end{aligned}
$$

$$
\begin{aligned}
D &= \frac{6m_W^2+3m_Z^2+6m_t^2}{24v^2} \ , \quad B = \frac{m_H^2}{4} - \frac{6m_W^4+3m_Z^4-12m_t^4}{32\pi^2v^2} \ , \\
E &= \frac{6m_W^3+3m_Z^3}{12\pi v^3} \ , \\
\lambda_T &= \frac{m_H^2}{2v^2}\{1-\frac{1}{8\pi^2v^2m_H^2}\{6m_W^4\log\left(\frac{m_W^2}{a_bT^2}\right)+3m_Z^4\log\left(\frac{m_Z^2}{a_bT^2}\right) \\
& -12m_t^4\log\left(\frac{m_t^2}{a_fT^2}\right)\}\} \ ,
\end{aligned}
$$

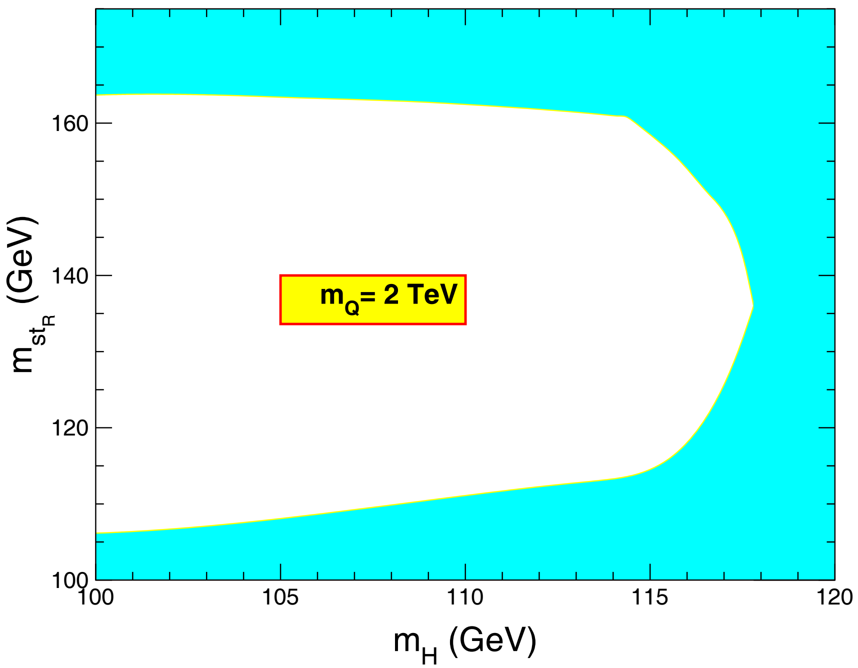
where $\log a_f = 1.14$; $\log a_b = 3.91$.

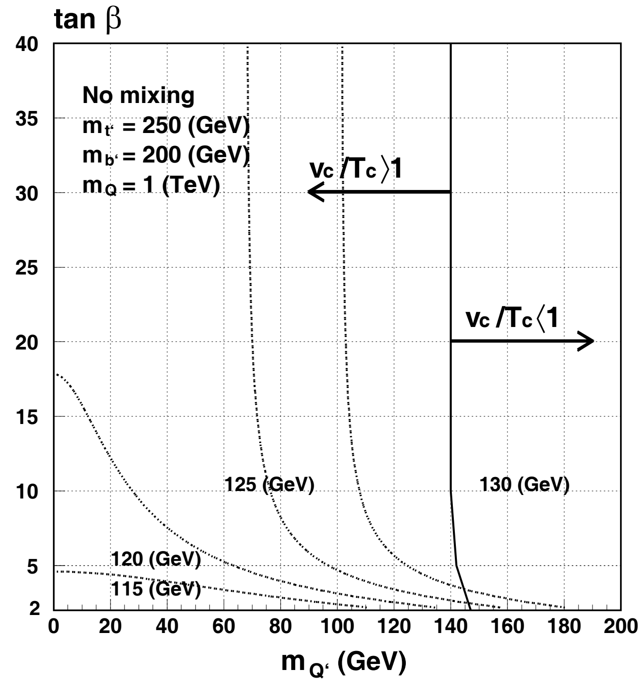
On the SM in the context of the EWPT;

$$
\begin{aligned}
&\text{If } m_h < 35 \text{ GeV, Then } \phi_C/T_C \geq 1 \\
&\text{If } m_h \geq 35 \text{ GeV, Then } \phi_C/T_C < 1
\end{aligned}
$$



The Thermal Effective Potential in the Standard Model
$V(\phi, T)$ as a function of ϕ





EWPT in the MSSM

Left: Electroweak baryogenesis and the Higgs and stop masses, M. Quiros, Nucl. Phys. Proc. Suppl. 101, 401 (2001)

Right: Electroweak phase transition in the MSSM with four generations, S.W. Ham, S.K. Oh, and D. Son, Phys. Rev. D71, 015001 (2005)

EWPT in the NMSSM, nMSSM
The mass of stop quark need not be smaller than the top quark mass to ensure the first-order EWPT be strong.
NMSSM: trilinear terms $(\lambda A_\lambda H_1 H_2 N + k A_k N^3/3 + \text{H.c.})$
CP-conserving case: The Electroweak Phase Transition in a Nonminimal Supersymmetric Model, M. Pietroni, Nucl. Phys. B402, 27 (1993)
Does LEP prefer the NMSSM?, M. Bastero-Gil, C. Hugonie, S.F. King, D.P. Roy, S. Vempati, Phys. Lett. B489, 359 (2000)
CP-violating case: Phase transitions in the NMSSM, K. Funakubo, S. Tao, F. Toyoda, Prog. Theor. Phys. 114, 369 (2005)
{In fact, we have not encountered an extremely deep minimum or high barriar between the minima of the effective potential in the numerical studies}
nMSSM: trilinear terms $(\lambda A_\lambda H_1 H_2 N + \text{H.c.})$
CP-conserving case: Electroweak phase transition in a nonminimal supersymmetric model, S.W. Ham, S.K. OH, C.M. Kim, E.J. Yoo, D. Son, Phys. Rev. D70, 075001 (2004).
CP-violating case: Electroweak phase transition in the MNMSSM with explicit CP violation, S.W. Ham, J.O. Im, and S.K. Oh, arXiv:hep-ph/0707.4543

Electroweak phase transitions in the MSSM with an extra $U(1)'$, S.W. Ham, E.J. Yoo, S.K. OH, arXiv:hep-ph/0704.0328 (PRD, in press), (KISTI;"The Strategic Supercomputing Support Program")

Approximation A: In terms of these temperature-dependent VEVs, the vacuum at finite temperature is defined as the minimum of $V(T)$ as

$$\langle V(v_1, v_2, s, T)\rangle = \langle V_0\rangle + \langle V_1\rangle + \langle V_T\rangle \ .$$

The T^2 terms in the high T approximation of V_T can be expressed as

$$\begin{aligned}
\langle V_0\rangle &= m_1^2 v_1^2 + m_2^2 v_2^2 + m_3^2 s^2 + \frac{g_1^2+g_2^2}{8}(v_1^2 - v_2^2)^2 + \lambda^2(v_1^2 v_2^2 + v_1^2 s^2 + v_2^2 s^2) \\
&\quad - 2\lambda A_\lambda v_1 v_2 s + \frac{g_1^{'2}}{2}(\tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_3 s^2)^2 \ , \\
\langle V_1\rangle &= f_b(m_t^2) + f_b(m_b^2) + f_b(m_k^2) \ , \textbf{degenerate squarks} \\
\langle V_T\rangle &= \frac{T^2}{24}\Big[4m_1^2 + 4m_2^2 + 2m_3^2 + (2g_1^2 + 6g_2^2 + 6\lambda^2)(v_1^2 + v_2^2) + 12\lambda^2 s^2 \\
&\quad + 12 g_1^{'2}(\tilde{Q}_1^2 v_1^2 + \tilde{Q}_2^2 v_2^2 + \tilde{Q}_3^2 s^2) + 2 g_1^{'2}\tilde{Q}_1\tilde{Q}_2(v_1^2 + v_2^2) \\
&\quad + 2 g_1^{'2}\tilde{Q}_2\tilde{Q}_3(v_2^2 + s^2) + 2 g_1^{'2}\tilde{Q}_1\tilde{Q}_3(v_1^2 + s^2) \\
&\quad + 8 g_1^{'2}(\tilde{Q}_1 + \tilde{Q}_2)(\tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_3 s^2) + 6(h_t^2 v_2^2 + h_b^2 v_1^2 + k^2 s^2)\Big] \ ,
\end{aligned}$$

where

$$f_b(m_q^2) = \frac{3m_q^4}{16\pi^2}\left[\frac{3}{2} + \log\left(\frac{\tilde{m}^2 + m_q^2}{m_q^2}\right)\right] \ ,$$

$$\begin{aligned}
m_1^2 \ = & \ -\frac{m_Z^2}{2}\cos 2\beta - \lambda^2(s(0)^2 + v(0)^2\sin^2\beta) + \lambda A_\lambda s(0)\tan\beta \\
& - g_1'^2\tilde{Q}_1(\tilde{Q}_1 v(0)^2\cos^2\beta + \tilde{Q}_2 v(0)^2\sin^2\beta + \tilde{Q}_3 s(0)^2) - f_c(m_b^2(0)) \\
m_2^2 \ = & \ \frac{m_Z^2}{2}\cos 2\beta - \lambda^2(s(0)^2 + v(0)^2\cos^2\beta) + \lambda A_\lambda s(0)\cot\beta \\
& - g_1'^2\tilde{Q}_2(\tilde{Q}_1 v(0)^2\cos^2\beta + \tilde{Q}_2 v(0)^2\sin^2\beta + \tilde{Q}_3 s(0)^2) - f_c(m_t^2(0)) \\
m_3^2 \ = & \ -\lambda^2 v(0)^2 + \frac{\lambda}{2s(0)}v(0)^2 A_\lambda \sin 2\beta \\
& - g_1'^2\tilde{Q}_3(\tilde{Q}_1 v(0)^2\cos^2\beta + \tilde{Q}_2 v(0)^2\sin^2\beta + \tilde{Q}_3 s(0)^2) - f_c(m_k^2(0)) \ ,
\end{aligned}$$

$$f_c(m_q^2) = \frac{3h_q^2 m_q^2}{16\pi^2}\left[2 + 2\log\left(\frac{\tilde{m}^2 + m_q^2}{m_q^2}\right) + \frac{m_q^2}{\tilde{m}^2 + m_q^2}\right] \ .$$

Now, $\langle V(v_1, v_2, s, T)\rangle \longrightarrow \langle V(v_1, v_2, T)\rangle$ via

$$\uparrow$$

$$\begin{aligned}
0 &= 2m_3^2 s - 2\lambda A_\lambda v_1 v_2 + 2\lambda^2(v_1^2 + v_2^2)s \\
&\quad + 2g_1^{'2}\tilde{Q}_3 s(\tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_3 s^2) + 2h_k^2 m_k f_c(m_k^2) \\
&\quad + \frac{T^2}{24}s[24\lambda^2 + 24g_1^{'2}\tilde{Q}_3^2 + 20g_1^{'2}\tilde{Q}_3(\tilde{Q}_1 + \tilde{Q}_2) + 12k^2] \ ,
\end{aligned}$$

which is obtained by calculating the first derivative of the full effective potential at the finite temperature with respect to s.
The solution of the above Nonlinear Equation: we calculate s using bisection method.

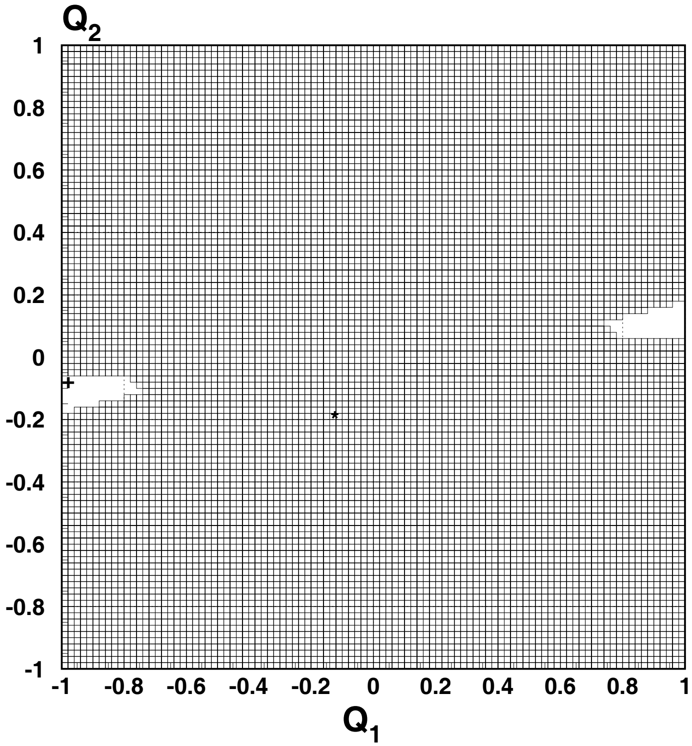
Approximation B: The exact integral expression for $\langle V_T\rangle$

$$\begin{aligned}
\langle V_T\rangle &= -\sum_{l=t,b,k}\frac{6T^4}{\pi^2}\int_0^\infty dx\ x^2\ \log\left[1+\exp\left(-\sqrt{x^2+\frac{m_l^2(v_1,v_2,s)}{T^2}}\right)\right] \\
&\quad + \sum_{l=\tilde{t},\tilde{b},\tilde{k}}\frac{6T^4}{\pi^2}\int_0^\infty dx\ x^2\ \log\left[1-\exp\left(-\sqrt{x^2+\frac{\tilde{m}^2+m_l^2(v_1,v_2,s)}{T^2}}\right)\right] \ .
\end{aligned}$$

Now, $\langle V(v_1, v_2, s, T)\rangle \longrightarrow \langle V(v_1, v_2, T)\rangle$ via

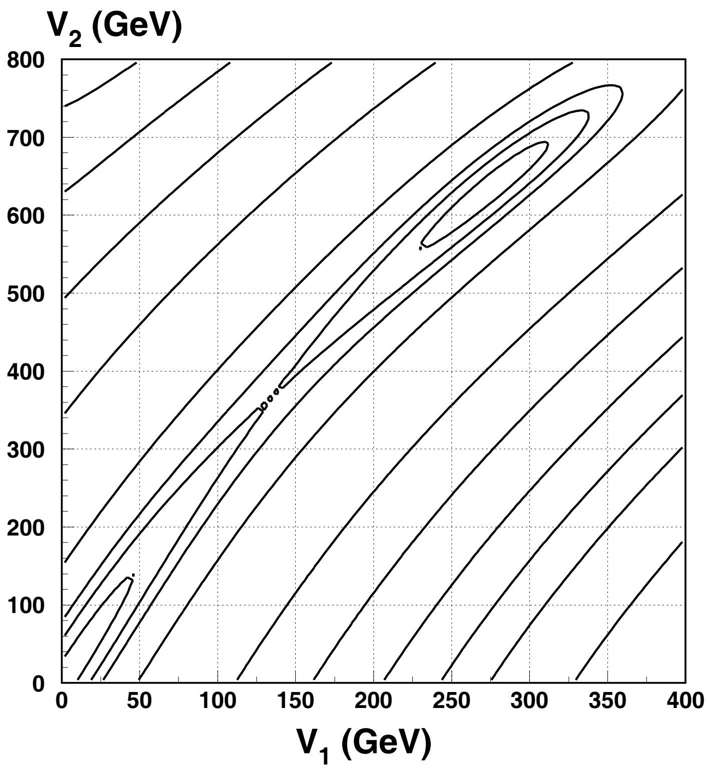
$$\uparrow$$

$$\begin{aligned}
0 \;=\; & 2m_3^2 s - 2\lambda A_\lambda v_1 v_2 + 2\lambda^2(v_1^2+v_2^2)s + 2{g_1'}^2 Q_3 s(\tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_3 s^2) \\
& + 2h_k^2 m_k f_c(m_k^2) \\
& + \frac{3T^2}{\pi^2}\int_0^\infty dx\; x^2\, \frac{2h_k^2 s \exp(-\sqrt{x^2+m_k^2/T^2})}{\sqrt{x^2+m_k^2/T^2}\left[1+\exp(-\sqrt{x^2+m_k^2/T^2})\right]} \\
& - \frac{3T^2}{\pi^2}\int_0^\infty dx\; x^2 \\
& \times \frac{2h_k^2 s \exp(-\sqrt{x^2+(\tilde{m}^2+m_k^2)/T^2})}{\sqrt{x^2+(\tilde{m}^2+m_k^2)/T^2}\left[1-\exp(-\sqrt{x^2+(\tilde{m}^2+m_k^2)/T^2})\right]}\;.
\end{aligned}$$



$|\alpha_{ZZ'}| < 2 \times 10^{-3}$
$m_{Z'} > 600$ GeV
$Q_i = g'_1 \tilde{Q}_i$

The allowed area in the (Q_1, Q_2)-plane for $\tan\beta = 3$ and $s(T = 0) = 500$ GeV. The values of Q_1 and Q_2 at the star-marked point correspond to the ν-model of E_6 gauge group realizations. The values of Q_1 and Q_2 at the cross-marked point are $(Q_1, Q_2) = (-1, -0.1)$, and hence Q_3 =1.1.

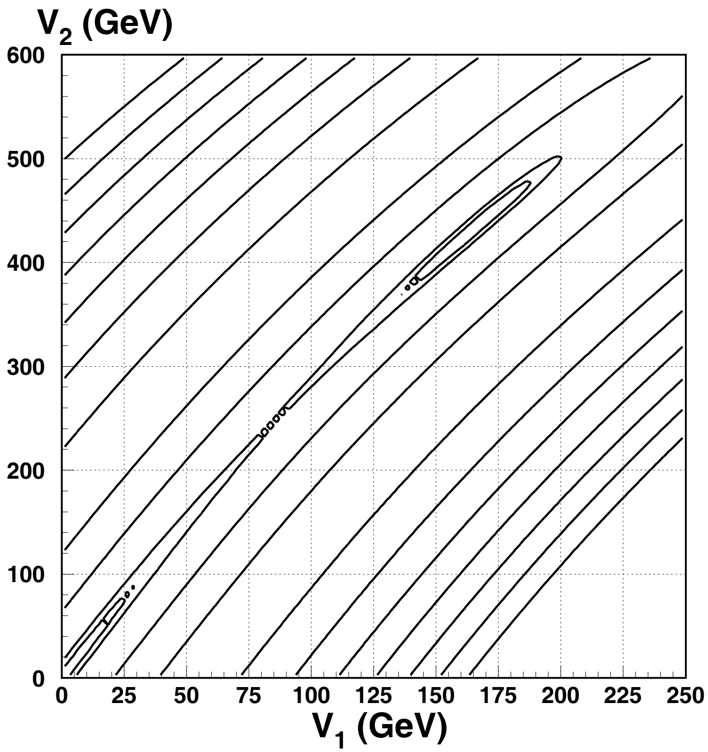


$\tan\beta = 3$, $\lambda = 0.8$, $s(0) = 500$ GeV
$m_A = 1830$ GeV, $T_c = 100$ GeV
$m_{S_i} = 56, 807, 1827$ GeV
$v_c = 696$, $v_c/T_c > 1$

Notice two distinct minima of $\langle V(v_1, v_2, T)\rangle$ on the (v_1, v_2)-plane: $(0, 0)$ where the phase of the state is symmetric, and $(275, 640)$ GeV, where the phase of the state is broken. The electroweak phase transition may take place from $(0, 0)$ to $(275, 640)$ GeV on the (v_1, v_2)-plane, which is evidently discontinuous and therefore it is first order.

λ	m_A (GeV)	(v_1, v_2) (GeV)	m_{S_1}, m_{S_2}, m_{S_3} (GeV)	v_c/T_c
0.1	478	(1750, 1650)	120, 524, 792	26
0.2	675	(1400, 1500)	118, 674, 796	23
0.3	900	(1200, 1400)	112, 786, 908	18
0.4	1109	(870, 1200)	104, 792, 1112	15
0.5	1306	(600, 1000)	93, 796, 1307	12
0.6	1486	(430, 850)	82, 800, 1485	8
0.7	1660	(340, 700)	70, 803, 1658	7
0.8	1830	(275, 640)	56, 807, 1827	6.9

The values of other parameters are fixed as $\tan\beta = 3$, $s(0) = 500$ GeV, $\tilde{m} = 1000$ GeV, and $T_c = 100$ GeV. The coordinates of its symmetric-phase minimum is $(0, 0)$ for all sets.



$\tan\beta = 3$, $\lambda = 0.8$, $s(0) = 500$ GeV
$m_A = 1780$ GeV, $T_c = 100$ GeV
$m_{S_i} = 82, 804, 1777$ GeV
$v_c = 470$, $v_c/T_c > 1$

Notice two distinct minima of $\langle V(v_1, v_2, T)\rangle$ on the (v_1, v_2)-plane: $(0, 0)$ where the phase of the state is symmetric, and $(165, 440)$ GeV, where the phase of the state is broken. The electroweak phase transition may take place from $(0, 0)$ to $(165, 440)$ GeV on the (v_1, v_2)-plane, which is evidently discontinuous and therefore it is first order.

λ	m_A GeV	(v_1, v_2) GeV	m_{S_i} GeV	v_c/T_c
0.1	462	(1600, 1600)	121, 468, 791	22
0.2	663	(1400, 1400)	118, 662, 795	19
0.3	885	(1100, 1100)	113, 785, 894	15
0.4	1095	(800, 1200)	106, 792, 1098	14
0.5	1287	(680, 990)	97, 796, 1288	12
0.6	1457	(400, 750)	91, 799, 1456	8
0.7	1620	(300, 600)	86, 801, 1618	6
0.8	1780	(165, 440)	82, 804, 1777	4.7

Some sets of λ and m_A that allow strongly first-order electroweak phase transitions in the MSSM with an extra $U(1)'$, obtained by Approximation B. Other descriptions are the same as previous Table.

4. The effect of CP phase on the strength of EWPT in a supersymmetric $U(1)'$ model (USSM)
Electroweak phase transition in MSSM with $U(1)'$ in explicit CP violation scenario, S.W. Ham and S.K. OH arXiv:hep-ph/0708.1785; (KISTI;"The Strategic Supercomputing Support Program")

Non-degenerate Stop Quark, one CP phase (ϕ_t)

In case of explicit CP violation, at the one-loop level, the non-trivial tadpole minimum condition with respect to the pseudoscalar component of the Higgs field is given as

$$0 = A_\lambda \sin\phi_0 - \frac{3m_t^2 A_t \sin\phi_t}{16\pi^2 v^2 \sin^2\beta} f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \ ,$$

$$m_{\tilde{t}_1,\tilde{t}_2}^2 = m_T^2 + m_t^2 \mp \sqrt{h_t^2 A_t^2 v_2^2 + h_t^2 \lambda^2 v_1^2 s^2 - 2h_t^2 \lambda A_t v_1 v_2 s \cos\phi_t} \ ,$$

$$\begin{aligned}\langle V_1(T)\rangle \ = \ & -\frac{6T^4}{\pi^2}\int_0^\infty dx\ x^2\ \log\left[1+\exp\left(-\sqrt{x^2+m_t^2(v_2)/T^2}\right)\right] \\ & +\sum_{i=1}^{2}\frac{3T^4}{\pi^2}\int_0^\infty dx\ x^2\ \log\left[1-\exp\left(-\sqrt{x^2+m_{\tilde{t}_i}^2(v_1,v_2,s)/T^2}\right)\right] \ .\end{aligned}$$

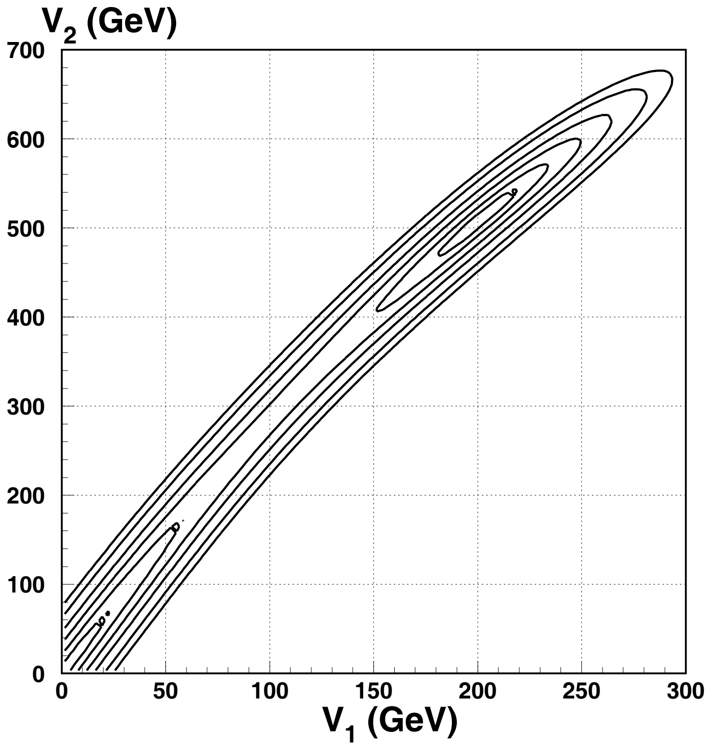
Now, $\langle V(v_1, v_2, s, T)\rangle \longrightarrow \langle V(v_1, v_2, T)\rangle$ via
$$\uparrow$$

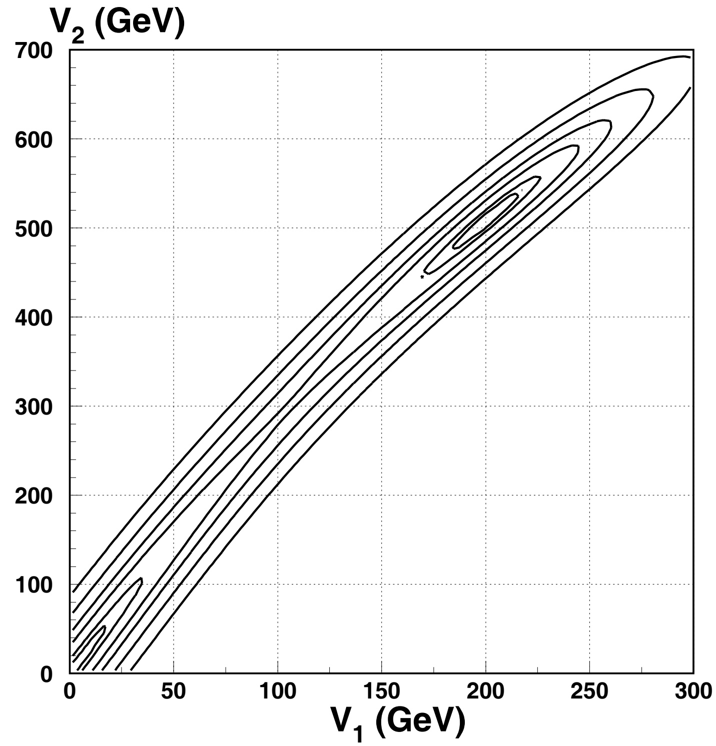
$$
\begin{aligned}
0 \; = \; & 2m_3^2 s - 2\lambda A_\lambda v_1 v_2 \cos\phi_0 + 2\lambda^2(v_1^2+v_2^2)s + 2g_1^{\prime 2}\tilde{Q}_3 s(\tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_3 s^2) \\
& - \frac{3h_t^2\lambda v_1}{8\pi^2}(\lambda v_1 s - A_t v_2\cos\phi_t) f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \\
& - \frac{3T^2}{2\pi^2}\frac{2h_t^2\lambda v_1}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)}(\lambda s v_1 - A_t v_2\cos\phi_t) \\
& \times \int_0^\infty dx\; x^2\, \frac{\exp(-\sqrt{x^2+m_{\tilde{t}_1}^2/T^2})}{\sqrt{x^2+m_{\tilde{t}_1}^2/T^2}\{1-\exp(-\sqrt{x^2+m_{\tilde{t}_1}^2/T^2})\}} \\
& + \frac{3T^2}{2\pi^2}\frac{2h_t^2\lambda v_1}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)}(\lambda s v_1 - A_t v_2\cos\phi_t) \\
& \times \int_0^\infty dx\; x^2 \frac{\exp(-\sqrt{x^2+m_{\tilde{t}_2}^2/T^2})}{\sqrt{x^2+m_{\tilde{t}_2}^2/T^2}\{1-\exp(-\sqrt{x^2+m_{\tilde{t}_2}^2/T^2})\}} \; .
\end{aligned}
$$

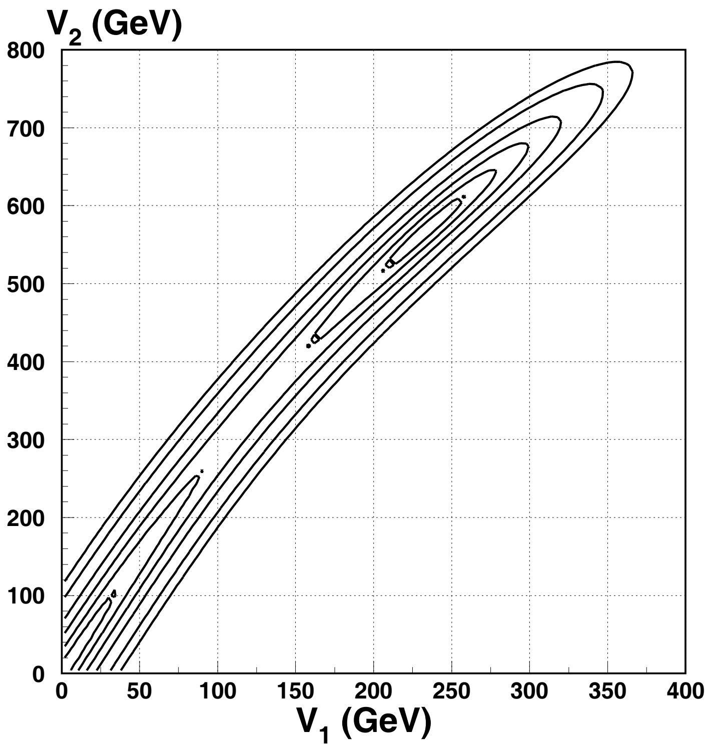
figures	Fig. 1	Fig. 2	Fig. 3	Fig. 4
ϕ_t	$\pi/1000$	$\pi/2$	$\pi/1000$	$\pi/2$
λ	0.8	$\leftarrow$	0.7	$\leftarrow$
A_λ (GeV)	2271	$\leftarrow$	2115	$\leftarrow$
T_c (GeV)	100	147	100	143
m_{h_1} (GeV)	73	75	79	80
m_{h_2} (GeV)	792	792	789	789
m_{h_3} (GeV)	1748	1747	1579	1578
m_{h_4} (GeV)	1750	1749	1580	1579
(v_{1A}, v_{2A}, s_A) (GeV)	(1,3,483)	$\leftarrow$	(2,4,484)	$\leftarrow$
(v_{1B}, v_{2B}, s_B) (GeV)	(199,507,617)	$\leftarrow$	(230,564,639)	$\leftarrow$
v_c/T_c	5.57	3.79	6.24	4.36

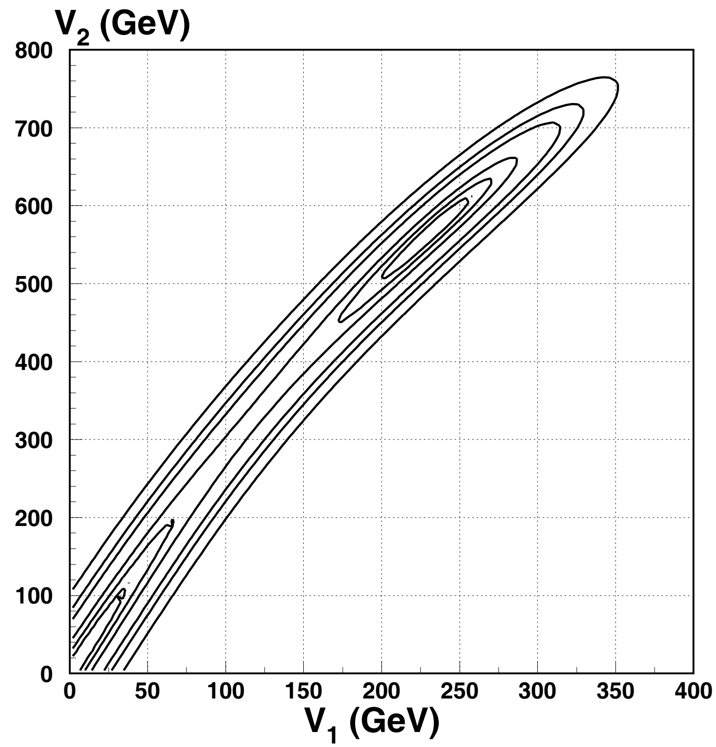
$$v_c = \sqrt{(v_{1B} - v_{1A})^2 + (v_{2B} - v_{2A})^2 + (s_B - s_A)^2} \ ,$$

$\Lambda = 300$ GeV, $Q_1 = -1$, $Q_2 = -0.1$, $Q_3 = 1.1$. $\tan\beta = 3$, $s(0) = m_T = 500$ GeV, $A_t = 100$ GeV. $\tan\beta = 3$, $s(0) = m_T = 500$ GeV, $A_t = 100$ GeV


V2 (GeV)
700
600
500
400
300
200
100
0
0
50
100
150
200
250
300
V1 (GeV)


V2 (GeV)
700
600
500
400
300
200
100
0
0
50
100
150
200
250
300
V1 (GeV)


V2 (GeV)
800
700
600
500
400
300
200
100
0
0
50
100
150
200
250
300
350
400
V1 (GeV)


V2 (GeV)
800
700
600
500
400
300
200
100
0
0
50
100
150
200
250
300
350
400
V1 (GeV)

5. Summary and Conclusions

For Electroweak Baryogenesis,

(1) B number violation (2) C and CP violation (3) Thermal non-equilibrium

(1) B number violation; It is axiomatic, Axial Anomaly

In the SM,

{2} the size of the CP violation in CKM is very small. Magnitude of the cosmological baryon asymmetry, S. Barr, G. Segre, and H. A. Weldon, Phys. Rev. D20, 2494 (1979); in the CKM $n_B/n_\gamma \approx \mathrm{O}(10^{-20}) \ll \mathrm{O}(10^{-10})$.

{3} Weakly first order EWPT

In the MSSM,

{2} Explicit CP violation, CP phase arising from soft terms.

Effects of large CP violating soft phases on supersymmetric electroweak baryogenesis, M. Brhlik, G.J. Good, and G.L. Kane, Phys. Rev. D63, 035002 (2000)

{3} A light stop scenario for a strongly first order EWPT ($m_{\tilde{t}_1} < m_t$).

In the NMSSM and nMSSM

{2} Explicit CP violation, CP phase arising from soft terms.

{3} A strongly first order EWPT for $m_{\tilde{t}_1} > m_t$ because of the trilinear terms (A_λ, A_k).

In a supersymmetric $U(1)'$ model,
- Explicit CP violation is possible for the radiatively corrected Higgs potential
Radiative corrections due to Exotic-quark system
- Spontaneous CP violation is impossible
- Strongly first-order EWPT is possible for a wide area of parameter space
Increasing Phase Transition with increasing the lightest Higgs boson mass.
Very strong first-order EWPT

IBM

```
ssh user-id@nobela.ksc.re.kr
ftp nobela.ksc.re.kr
```

xlf -c pythia.f → pythia.o
xlf -o extraw0 extraw0.f pythia.o
./extraw0 → extraw0.dat

```
llsubmit extraw0.sh
llq
llq -s nobel2.ksc.re.kr.157217.0
llq -x nobel2.ksc.re.kr.157381.0
llcancel nobel.ksc.re.kr.157217.0
```

extraw0.sh

```
#!/bin/ksh
#@ job_type = serial
#@ job_name = extraw0
#@ class = normal
#@ resources = ConsumableCpus(1) ConsumableMemory(1gb)
#@ wall_clock_limit = 10:00:00
#@ output = $(job_name).out
#@ error = $(job_name).err
#@ queue
cp extraw0 /ytmp/a116hsw
cd /ytmp/a116hsw
./extraw0
cp extraw0.dat /edun/a116hsw/pythia/extra
rm extraw0
rm extraw0.dat
```

LINUX
ssh user-id@hamel.ksc.re.kr
g77 -c file.f $\rightarrow$ file.c
g77 -o file.o file.f $\rightarrow$ file.o
./file.o $\rightarrow$ file.dat

qsub hmass0.qsub
qstat -a

sched:

Job ID	Username	Queue	Jobname	SessID	NDS	TSK	Req'd Memory	Eeq'd Time	S	Elap Time
107349.sched	x217hsw	N1	hmass0	9615	1	2	–	300:0	R	70:41
107350.sched	x217hsw	N1	hmass0	23172	1	2	–	300:0	R	19:16

qdel 107350
$PP \rightarrow Z \rightarrow Zh$ for Higgs-strahlung process; PDF: CTEQ6M, LO
Neutralino contribution on the Higgs production of the MSSM at the LHC

hmass0.qsub

```
#!/bin/bash
#PBS -l nodes=1:cpp=2
#PBS -l walltime=300:00:00
#PBS -N hmass0
#PBS -r n
### queue list: N1 (for serial job), N16, N32, N128, N256
#PBS -q N1
#PBS -j oe

cd mssm/lhc/v2hv

/opt/mpich/1.2.5..10/gm-2.0.12-2.4.20-28.7smp-i686/smp/intel32/ssh/bin
/mpirun.ch_gm–gm-kill 5 –gm-no-shmem -np 32 -machinefile $PBS_NODEFILE
PBS_JOBID=$PBS_JOBID hmass0.exe
```

Study in the future.

2008 The Strategic Supercomputing Support Program
- Explicit CP violation in a supersymmetric E_6 model
- Electroweak Phase Transition in a supersymmetric E_6 model
- Higgs production of a supersymmetric E_6 model (CP violation) at the ILC
- Higgs production of a supersymmetric E_6 model (CP violation) at the LHC
- Higgs Decay of a supersymmetric E_6 model (CP violation)
- Title: Baryogenesis in a supersymmetric model
- IBM p690 (1000 CPU time)
- Tera Cluster (9000 CPU time)

Acknowledgments
This research is supported by KOSEF through CHEP. The authors would like to acknowledge the support from KISTI (Korea Institute of Science and Technology Information) under "The Strategic Supercomputing Support Program" with Dr. Kihyeon Cho as the technical supporter. The use of the computing system of the Supercomputing Center is also greatly appreciated.

Backups

The elements of the tree level mass matrix for the neutral Higgs bosons are

$$\begin{aligned}
M^0_{11} &= m_Z^2 \cos^2\beta + 2g_1'^2 \tilde{Q}_1^2 v^2 \cos^2\beta + m_A^2 \sin^2\beta \cos^2\alpha \ , \\
M^0_{22} &= m_Z^2 \sin^2\beta + 2g_1'^2 \tilde{Q}_2^2 v^2 \sin^2\beta + m_A^2 \cos^2\beta \cos^2\alpha \ , \\
M^0_{33} &= 2g_1'^2 \tilde{Q}_3^2 s^2 + A^2 \sin^2\alpha \ , \\
M^0_{12} &= g_1'^2 \tilde{Q}_1 \tilde{Q}_2 v^2 \sin 2\beta + (\lambda^2 v^2 - m_Z^2/2) \sin 2\beta - m_A^2 \cos\beta \sin\beta \cos^2\alpha \ , \\
M^0_{13} &= 2g_1'^2 \tilde{Q}_1 \tilde{Q}_3 vs \cos\beta + 2\lambda^2 vs \cos\beta - m_A^2 \sin\beta \cos\alpha \sin\alpha \ , \\
M^0_{23} &= 2g_1'^2 \tilde{Q}_2 \tilde{Q}_3 vs \sin\beta + 2\lambda^2 vs \sin\beta - m_A^2 \cos\beta \cos\alpha \sin\alpha \ . \qquad (1)
\end{aligned}$$

At the one-lop level, the elements of the mass matrix for the neutral Higgs boson due to the contribution of the top and stop quarks are

$$\begin{aligned}
M^t_{11} &= \frac{3m_t^4 \lambda^2 s^2 \Delta_{\tilde{t}_1}^2}{8\pi^2 v^2 \sin^2\beta} \frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2} \ , \\
M^t_{22} &= \frac{3m_t^4 A_t^2 \Delta_{\tilde{t}_2}^2}{8\pi^2 v^2 \sin^2\beta} \frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2} + \frac{3m_t^4 A_t \Delta_{\tilde{t}_2}}{4\pi^2 v^2 \sin^2\beta} \frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)}
\end{aligned}$$

$$
+\frac{3m_t^4}{8\pi^2 v^2 \sin^2\beta}\log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right) ,
$$

$$
M_{33}^t = \frac{3m_t^4\lambda^2\Delta_{\tilde{t}_1}^2}{8\pi^2\tan^2\beta}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2} ,
$$

$$
M_{44}^t = \frac{3m_t^4\lambda^2 A_t^2 s^2\sin^2\phi_t}{8\pi^2 v^2\sin^4\beta\cos^2\alpha}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2} ,
$$

$$
M_{12}^t = -\frac{3m_t^4\lambda A_t s\Delta_{\tilde{t}_1}\Delta_{\tilde{t}_2}}{8\pi^2 v^2\sin^2\beta}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}-\frac{3m_t^4\lambda s\Delta_{\tilde{t}_1}}{8\pi^2 v^2\sin^2\beta}\frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)}
$$

$$
M_{13}^t = \frac{3m_t^4\lambda^2 s\Delta_{\tilde{t}_1}^2}{8\pi^2 v\sin\beta\tan\beta}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}-\frac{3m_t^2\lambda^2 s\cos\beta}{8\pi^2 v\sin^2\beta}f(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2),
$$

$$
M_{14}^t = -\frac{3m_t^4\lambda^2 A_t s^2\Delta_{\tilde{t}_1}\sin\phi_t}{8\pi^2 v^2\sin^3\beta\cos\alpha}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2} ,
$$

$$
M_{23}^t = -\frac{3m_t^4\lambda A_t\Delta_{\tilde{t}_1}\Delta_{\tilde{t}_2}}{8\pi^2 v\sin\beta\tan\beta}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}-\frac{3m_t^4\lambda\cos\beta\Delta_{\tilde{t}_1}}{8\pi^2 v\sin^2\beta}\frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)},
$$

$$
\begin{aligned}
M_{24}^t &= \frac{3m_t^4\lambda A_t^2 s\Delta_{\tilde{t}_2}\sin\phi_t}{8\pi^2 v^2\sin^3\beta\cos\alpha}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}+\frac{3m_t^4\lambda A_t s\sin\phi_t}{8\pi^2 v^2\sin^3\beta\cos\alpha}\frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)}\ , \\
M_{34}^t &= -\frac{3m_t^4\lambda^2 A_t s\Delta_{\tilde{t}_1}\sin\phi_t}{8\pi^2 v\sin^2\beta\tan\beta\cos\alpha}\frac{g(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2-m_{\tilde{t}_1}^2)^2}\ , \qquad (2)
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta_{\tilde{t}_1} &= A_t\cos\phi_t-\lambda s\cot\beta\ , \\
\Delta_{\tilde{t}_2} &= A_t-\lambda s\cot\beta\cos\phi_t\ . \qquad (3)
\end{aligned}
$$

At the one-lop level, the elements of the mass matrix for the neutral Higgs boson due to the contribution of the bottom and sbottom quarks are

$$
\begin{aligned}
M_{11}^b &= \frac{3m_b^4 A_b^2\Delta_{\tilde{b}_1}^2}{8\pi^2 v^2\cos^2\beta}\frac{g(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)^2}+\frac{3m_b^4 A_b\Delta_{\tilde{b}_1}}{4\pi^2 v^2\cos^2\beta}\frac{\log(m_{\tilde{b}_2}^2/m_{\tilde{b}_1}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)} \\
&\quad+\frac{3m_b^4}{8\pi^2 v^2\cos^2\beta}\log\left(\frac{m_{\tilde{b}_1}^2 m_{\tilde{b}_2}^2}{m_b^4}\right)\ ,
\end{aligned}
$$

$$
\begin{aligned}
M_{22}^b &= \frac{3m_b^4\lambda^2 s^2\Delta_{\tilde{b}_2}^2}{8\pi^2 v^2\cos^2\beta}\frac{g(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)^2}\ , \\
M_{33}^b &= \frac{3m_b^4\lambda^2\Delta_{\tilde{b}_2}^2}{8\pi^2\cot^2\beta}\frac{g(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)^2}\ , \\
M_{44}^b &= \frac{3m_b^4\lambda^2 A_b^2 s^2\sin^2\phi_b}{8\pi^2 v^2\cos^4\beta\cos^2\alpha}\frac{g(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)^2}\ , \\
M_{12}^b &= -\frac{3m_b^4\lambda A_b s\Delta_{\tilde{b}_1}\Delta_{\tilde{b}_2}}{8\pi^2 v^2\cos^2\beta}\frac{g(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)^2}-\frac{3m_b^4\lambda s\Delta_{\tilde{b}_2}}{8\pi^2 v^2\cos^2\beta}\frac{\log(m_{\tilde{b}_2}^2/m_{\tilde{b}_1}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)} \\
M_{13}^b &= -\frac{3m_b^4\lambda A_b\Delta_{\tilde{b}_1}\Delta_{\tilde{b}_2}}{8\pi^2 v\cos\beta\cot\beta}\frac{g(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)^2}-\frac{3m_b^4\lambda\sin\beta\Delta_{\tilde{b}_2}}{8\pi^2 v\cos^2\beta}\frac{\log(m_{\tilde{b}_2}^2/m_{\tilde{b}_1}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)}, \\
M_{14}^b &= \frac{3m_b^4\lambda A_b^2 s\Delta_{\tilde{b}_1}\sin\phi_b}{8\pi^2 v^2\cos^3\beta\cos\alpha}\frac{g(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)^2}+\frac{3m_b^4\lambda A_b s\sin\phi_b}{8\pi^2 v^2\cos^3\beta\cos\alpha}\frac{\log(m_{\tilde{b}_2}^2/m_{\tilde{b}_1}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)}\ , \\
M_{23}^b &= \frac{3m_b^4\lambda^2 s\Delta_{\tilde{b}_2}^2}{8\pi^2 v\cos\beta\cot\beta}\frac{g(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2)}{(m_{\tilde{b}_2}^2-m_{\tilde{b}_1}^2)^2}-\frac{3m_b^2\lambda^2 s\tan\beta}{8\pi^2 v\cos\beta}f(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2),
\end{aligned}
$$

$$
\begin{aligned}
M^b_{24} &= -\frac{3m_b^4\lambda^2 A_b s^2 \Delta_{\tilde{b}_2}\sin\phi_b}{8\pi^2 v^2\cos^3\beta\cos\alpha}\frac{g(m^2_{\tilde{b}_1},\ m^2_{\tilde{b}_2})}{(m^2_{\tilde{b}_2}-m^2_{\tilde{b}_1})^2}\ , \\
M^b_{34} &= -\frac{3m_b^4\lambda^2 A_b s \Delta_{\tilde{b}_2}\sin\phi_b}{8\pi^2 v\cos^2\beta\cot\beta\cos\alpha}\frac{g(m^2_{\tilde{b}_1},\ m^2_{\tilde{b}_2})}{(m^2_{\tilde{b}_2}-m^2_{\tilde{b}_1})^2}\ ,
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
\Delta_{\tilde{b}_1} &= A_b - \lambda s\tan\beta\cos\phi_b\ , \\
\Delta_{\tilde{b}_2} &= A_b\cos\phi_b - \lambda s\tan\beta\ .
\end{aligned}
\tag{5}
$$

At the one-lop level, the elements of the mass matrix for the neutral Higgs boson due to the contribution of the exotic quarks and squarks are

$$
\begin{aligned}
M^k_{11} &= \frac{3m_k^4\lambda^2 v^2\sin^2\beta\Delta^2_{\tilde{k}_1}}{8\pi^2 s^2}\frac{g(m^2_{\tilde{k}_1},\ m^2_{\tilde{k}_2})}{(m^2_{\tilde{k}_2}-m^2_{\tilde{k}_1})^2}\ , \\
M^k_{22} &= \frac{3m_k^4\lambda^2 v^2\cos^2\beta\Delta^2_{\tilde{k}_1}}{8\pi^2 s^2}\frac{g(m^2_{\tilde{k}_1},\ m^2_{\tilde{k}_2})}{(m^2_{\tilde{k}_2}-m^2_{\tilde{k}_1})^2}\ ,
\end{aligned}
$$

$$M_{33}^k = \frac{3m_k^4 A_k^2 \Delta_{\tilde{k}_2}^2}{8\pi^2 s^2} \frac{g(m_{\tilde{k}_1}^2,\ m_{\tilde{k}_2}^2)}{(m_{\tilde{k}_2}^2 - m_{\tilde{k}_1}^2)^2} + \frac{3m_k^4 A_k \Delta_{\tilde{k}_2}}{4\pi^2 s^2} \frac{\log(m_{\tilde{k}_2}^2/m_{\tilde{k}_1}^2)}{(m_{\tilde{k}_2}^2 - m_{\tilde{k}_1}^2)}$$

$$+ \frac{3m_k^4}{8\pi^2 s^2} \log\left(\frac{m_{\tilde{k}_1}^2 m_{\tilde{k}_2}^2}{m_k^4}\right) ,$$

$$M_{44}^k = \frac{3m_k^4 \lambda^2 A_k^2 v^2 \sin^2\phi_k}{8\pi^2 s^2 \cos^2\alpha} \frac{g(m_{\tilde{k}_1}^2,\ m_{\tilde{k}_2}^2)}{(m_{\tilde{k}_2}^2 - m_{\tilde{k}_1}^2)^2} ,$$

$$M_{12}^k = \frac{3m_k^4 \lambda^2 v^2 \sin 2\beta \Delta_{\tilde{k}_1}^2}{16\pi^2 s^2} \frac{g(m_{\tilde{k}_1}^2,\ m_{\tilde{k}_2}^2)}{(m_{\tilde{k}_2}^2 - m_{\tilde{k}_1}^2)^2} - \frac{3m_k^2 \lambda^2 v^2 \sin 2\beta}{16\pi^2 s^2} f(m_{\tilde{k}_1}^2,\ m_{\tilde{k}_2}^2),$$

$$M_{13}^k = -\frac{3m_k^4 \lambda A_k v \sin\beta \Delta_{\tilde{k}_1} \Delta_{\tilde{k}_2}}{8\pi^2 s^2} \frac{g(m_{\tilde{k}_1}^2,\ m_{\tilde{k}_2}^2)}{(m_{\tilde{k}_2}^2 - m_{\tilde{k}_1}^2)^2}$$

$$- \frac{3m_k^4 \lambda v \sin\beta \Delta_{\tilde{k}_1}}{8\pi^2 s^2} \frac{\log(m_{\tilde{k}_2}^2/m_{\tilde{k}_1}^2)}{(m_{\tilde{k}_2}^2 - m_{\tilde{k}_1}^2)}$$

$$M_{14}^k = -\frac{3m_k^4 \lambda^2 A_k v^2 \sin\beta \Delta_{\tilde{k}_1} \sin\phi_k}{8\pi^2 s^2 \cos\alpha} \frac{g(m_{\tilde{k}_1}^2,\ m_{\tilde{k}_2}^2)}{(m_{\tilde{k}_2}^2 - m_{\tilde{k}_1}^2)^2} ,$$

$$
\begin{aligned}
M^k_{23} &= -\frac{3m_k^4\lambda A_k v\cos\beta\Delta_{\tilde{k}_1}\Delta_{\tilde{k}_2}}{8\pi^2 s^2}\frac{g(m^2_{\tilde{k}_1},\ m^2_{\tilde{k}_2})}{(m^2_{\tilde{k}_2}-m^2_{\tilde{k}_1})^2} \\
&\quad -\frac{3m_k^4\lambda v\cos\beta\Delta_{\tilde{k}_1}}{8\pi^2 s^2}\frac{\log(m^2_{\tilde{k}_2}/m^2_{\tilde{k}_1})}{(m^2_{\tilde{k}_2}-m^2_{\tilde{k}_1})}, \\
M^k_{24} &= -\frac{3m_k^4\lambda^2 A_k v^2\cos\beta\Delta_{\tilde{k}_1}\sin\phi_k}{8\pi^2 s^2\cos\alpha}\frac{g(m^2_{\tilde{k}_1},\ m^2_{\tilde{k}_2})}{(m^2_{\tilde{k}_2}-m^2_{\tilde{k}_1})^2}\ , \\
M^k_{34} &= \frac{3m_k^4\lambda A_k^2 v\Delta_{\tilde{k}_2}\sin\phi_k}{8\pi^2 s^2\cos\alpha}\frac{g(m^2_{\tilde{k}_1},\ m^2_{\tilde{k}_2})}{(m^2_{\tilde{k}_2}-m^2_{\tilde{k}_1})^2}+\frac{3m_k^4\lambda A_k v\sin\phi_k}{8\pi^2 s^2\cos\alpha}\frac{\log(m^2_{\tilde{k}_2}/m^2_{\tilde{k}_1})}{(m^2_{\tilde{k}_2}-m^2_{\tilde{k}_1})}, \qquad (6)
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta_{\tilde{k}_1} &= A_k\cos\phi_k-\lambda v\tan\alpha\ , \\
\Delta_{\tilde{k}_2} &= A_k-\lambda v\tan\alpha\cos\phi_k\ . \qquad (7)
\end{aligned}
$$